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VENABLE'S ARITHMETICS—NEW TWO-BOOK SERIES.

PRACTICAL ARITHMETIC

BY

CHARLES S. VENABLE, LL.D.

PROFESSOR OF MATHEMATICS IN THE UNIVERSITY OF VIRGINIA

NEW YORK AND NEW ORLEANS:

UNIVERSITY PUBLISHING COMPANY.

1894.

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Press of J. J. Little & Co.
Astor Place, New York.

PREFACE.

VENABLE'S PRACTICAL ARITHMETIC is the more advanced of a two-book series of arithmetics.

The constant aim of the author has been not only to provide the pupil with a work thoroughly graded, so that each step shall be the sufficient training for the step that follows; but, also, to present the work in such variety of form, as at once to maintain his interest and to render his judgment independent. The theoretical and the technical have been subordinated to the practical.

The series is not surpassed in the number of its concrete problems. The oral problems in each subject precede the written, and include work in the applications of percentage and interest. Miscellaneous problems are distributed throughout the book, at the end of the different subjects, and involve all the preceding subjects.

The concrete problems are supplemented by groups of abstract examples under suggestive headings, the numbers in these problems to be replaced by those in the groups of abstract examples, each concrete problem serving as a model for others thus suggested to the teacher. In the multiplicity of problems given and suggested, the author believes that good service has been rendered the teacher, in time saved in the class-*room.*

Methods that are new and progressive have been adopted in the Elementary and in the Practical, so far as they have been generally accepted by teachers. In the more advanced part of the Practical Arithmetic several ways of treating a subject are sometimes presented, so that teachers may compare different methods as a useful exercise in training pupils to think for themselves.

The author has endeavored to associate, in the mind of the pupil, the rules and forms in the more advanced subjects with those in the less advanced, reducing the reasoning in each subject, as nearly as possible, to first principles. Percentage, Interest, and their applications are thus treated under four general problems which have been developed inductively in the study of Common and Decimal Fractions.

The two books of the series, for unity of method, have the definitions and drill exercises of the one corresponding with those of the other. Each, however, is provided with business forms, etc., and is complete in itself.

In presenting this new revised series of arithmetics to the attention of the public, the author begs leave to acknowledge his great obligations to his accomplished collaborator, Mr. S. T. Stewart of Brooklyn, New York, for valuable services rendered in the work of its preparation, and also to other skilled and able teachers for helpful suggestions.

C. S. V.

UNIVERSITY OF VIRGINIA,
August, 1888.

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PRACTICAL ARITHMETIC.

1. A **unit** is a single thing, or one.

One man, *one* dollar, *one* regiment, *one*, are units.

2. A **number** is a unit, or a collection of units.

Three men, *five* dollars, *nine* regiments, are numbers.

3. An **abstract number** is a number the name of whose unit is not given.

Seven, fifteen, forty-five, one hundred six, are abstract numbers.

4. A **concrete number** is a number the name of whose unit is given.

Six *horses*, nine *pounds*, ten *dollars*, are concrete numbers.

5. A **simple** number is a number which expresses one or more units of the same *name* or *denomination ;* as, 5 yards ; 3 dollars ; 7.

6. A **compound** (*or denominate*) **number** is a number containing units of two or more different names or denominations, but the same in kind ; as, nine pounds twelve ounces ; and five bushels three pecks two quarts.

7. Arithmetic teaches the use of numbers.

NOTATION AND NUMERATION.

8. Notation is the *writing* of numbers with figures, or with letters.

9. Numeration is the *reading* of numbers expressed by figures, or by letters.

10. Figures are not numbers, but characters or marks that *represent numbers.*

ARABIC NOTATION.

11. Arabic Notation represents numbers by ten figures:

1	2	3	4	5	6	7	8	9	0
one	two	three	four	five	six	seven	eight	nine	naught

12. The first nine figures are called *digits,* or *significant figures.* They represent, in their order, the first nine numbers.

13. The last figure is called *naught, zero,* or *cipher*. It is used with the significant figures to give them their proper place and value.

14. A figure standing alone is said to occupy the *first* place, and it then represents the corresponding number of *ones,* or units of the *first* order.

15. A figure standing with another figure at its right is said to occupy the *second* place, and it then represents the corresponding number of *tens,* or units of the *second* order.

16. A figure standing with two figures at its right is said to occupy the *third* place, and it then represents the corresponding number of *hundreds,* or units of the *third* order.

17. Numbers containing units of the first, second, or third place or order are read as follows:

1st. 2d. 3d. Orders.

Units.	Tens.	Hundreds.
1st. Order.	2d. 1st. Order.	3d. 2d. 1st. Order.
1, *one* unit or *one.*	1 0, *one* ten or *ten.*	1 0 0, *one* hundred.
2, *two* units " *two.*	2 0, *two* tens " ***twenty.***	2 0 0, *two* "
3, *three* " " *three.*	3 0, *three* " " ***thirty.***	3 0 0, *three* "
4, *four* " " *four.*	4 0, *four* " " ***forty.***	4 0 0, *four* "
5, *five* " " *five.*	5 0, *five* " " ***fifty.***	5 0 0, *five* "
6, *six* " " *six.*	6 0, *six* " " ***sixty.***	6 0 0, *six* "
7, *seven* " " *seven.*	7 0, *seven* " " ***seventy.***	7 0 0, *seven* "
eight " " *eight.*	8 0, *eight* " " ***eighty.***	8 0 0, *eight* "
" " *nine.*	9 0, *nine* " " ***ninety.***	9 0 0, *nine* "

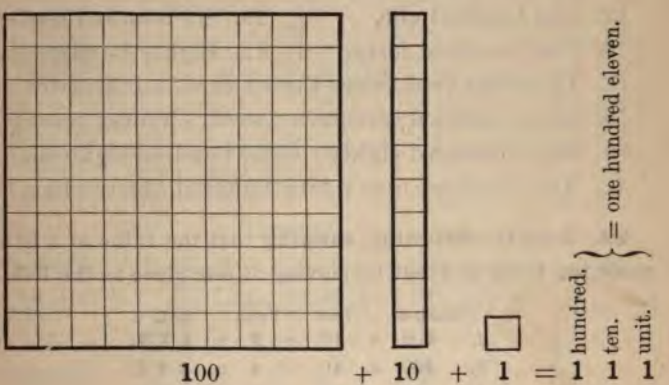
18. A number may contain units of two or more orders.

Two Orders.

Ten to Twenty.		Twenty to One Hundred.	
2d. 1st. Orders.	2d. 1st. Orders.	2d. 1st. Orders.	2d. 1st. Orders.
1 1, *eleven.*	1 6, ***six**teen.*	2 1, *twenty-one.*	7 6, *seventy-six.*
1 2, ***twelve.***	1 7, ***seven**teen.*	2 2, *twenty-two.*	9 7, *ninety-seven.*
1 3, ***thir**teen.*	1 8, ***eight**teen.*	2 3, *twenty-three.*	6 8, *sixty-eight.*
1 4, ***four**teen.*	1 9, ***nine**teen.*	3 4, *thirty-four.*	5 9, *fifty-nine.*
1 5, *fifteen.*	2 0, ***twenty.***	4 5, *forty-five.*	8 4, *eighty-four.*

Three Orders.

104, *one hundred four;* **560,** *five hundred sixty;* **789,** *seven hundred eighty-nine.*



Three Places.

19. 1. In 111 what does the 1 in the first or right-hand place stand for? The 1 in the second place? The 1 in the third or left-hand place?

2. What does 63 stand for? 91? 125? 152? 523? 600? 900?

3. In 218 how many hundreds are there? How many tens? How many units?

4. How many hundreds, tens, and units are there in 289? In *476? In 418? In* 801? In 760? In 927?

20. Read and explain the meaning of the following:

1. 11; 14; 13; 12.	*4.* 107; 207; 680.	*7.* 808; 497; 305.
2. 17; 19; 26; 38.	*5.* 362; 359; 475.	*8.* 642; 85; 900.
3. 51; 68; 94; 99.	*6.* 520; 811; 120.	*9.* 729; 902; 999.

21. Write with figures the following numbers:

1. Fourteen.
2. Forty.
3. Forty-four.
4. Seventeen.
5. Seventy.
6. Seventy-seven.
7. Ninety.
8. Nineteen.
9. Fifty.
10. One hundred eleven.
11. One hundred ten.
12. One hundred one.
13. Four hundred forty.
14. Five hundred fifty.
15. Five hundred five.
16. Six hundred sixteen.
17. Eighty hundred two.
18. Three hundred thirty-three; three hundred three.
19. Seven hundred seventeen; seven hundred seventy.
20. Eight hundred eighty; eight hundred eighteen.
21. Nine hundred nine; nine hundred ninety-nine.

22. Read the following, showing that the value of a figure is made ten times as great by moving it one place to the left.

	Hundreds.		Tens.		Units.		h. t. u.
1.	**1**00	+	**1**0	+	**1**	=	**1 1 1.**
2.	**4**00	+	**4**0	+	**4**	=	**4 4 4.**
3.	**7**00	+	**7**0	+	**7**	=	**7 7 7.**

Thus: 1 is 1 unit; 10 is 1 ten or 10 units; 100 is 1 hundred, 10 tens, or 100 units; 111 is 1 hundred, 1 ten, and 1 unit, or one hundred eleven.

23. The three places, hundreds, tens, and units, constitute a **period.**

24. The largest number expressed by one figure is 9, by two figures, 99; and, by three figures, or the *first* period, 999.

25. To write ten hundred, write **10** with two ciphers (**1**e ciphers) at the right, thus, **1000**; it is read ten *or* **one thousand.**

26. The fourth place expresses *thousands;* it is the place of the lowest, or units' order, in the *second* period, or period of **thousands.**

27. 1. Beginning at the right, the first three places, units, tens, and hundreds, form the **first period** or the *period of units.*

2. The three places of the second group, towards the *left,* are also named units, tens, and hundreds, and they form the **second period** or the *period of thousands.*

3. The three places of the third group are also named units, tens, and hundreds, and they form the **third period** or the *period of millions.*

4. Each group of three places, units, tens, and hundreds, towards the left, is a separate period, the names of other periods, in their order being : billions, trillions, quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions, etc.; as in the following :

Table.

PERIODS.	BILLIONS.				MILLIONS.				THOUSANDS.				UNITS.		
ORDERS.	12th.	11th.	10th.		9th.	8th.	7th.		6th.	5th.	4th.		3d.	2d.	1st.
PLACES.	Hundreds.	Tens.	Units.		Hundreds.	Tens.	Units.		Hundreds.	Tens.	Units.		Hundreds.	Tens.	Units.
NUMBERS TO BE READ.	4	0	5	,	6	2	1	,	5	7	0	,	6	3	2.
	2	7	3	,	4	4	5	,	6	1	3	,	7	2	0.
	1	1	1	,	2	5	3	,	4	2	9	,	8	0	0.
	7	2	8	,	0	0	0	,	6	0	5	,	0	5	8.
					9	8	7	,	5	6	8	,	0	4	1.
	3	9	7	,	6	4	0	,	2	3	6	,	4	2	9.

The first of the above numbers is read : four hundred five billion, six hundred twenty-one million, five hundred seventy thousand, six *hundred thirty-two.*

28. Rule for Numeration: 1. Beginning at the right, separate the figures into periods of three places each.

2. Beginning at the left, read each period, as if it formed a number by itself, giving the name of the period.

3. Periods and orders represented by ciphers only, are not read; and the name of units' period is not read.

4. The final *s* is omitted in reading the names of periods, and the word *and* should not be used in reading any whole number.

29. Read, and show the meaning of the following numbers:

	Millions.			Thousands.			Units.		
	h.	t.	u.	h.	t.	u.	h.	t.	u.
1.	1	0	5,	1	0	5,	1	0	5.
2.	2	3	0,	2	3	0,	2	3	0.
3.	4	7	5,	4	7	5,	4	7	5.
4.	7	6	8,	7	6	8,	7	6	8.

	Millions.			Thousands.			Units.		
	h.	t.	u.	h.	t.	u.	h.	t.	u.
5.	7	1	2,	3	0	6,	4	2	7.
6.	4	9	6,	0	9	0,	0	5	0.
7.	9	0	0,	6	5	7,	4	1	3.
8.	8	0	4,	5	0	9,	7	6	1.

30. Rule for Notation: 1. Beginning at the left, write each period, as if it formed a number by itself.

2. Periods and orders not represented by significant figures are indicated by ciphers.

A *comma* may separate successive periods; and a *period* may be placed at the end of each number.

31. Write the following numbers:

1. Five hundred fifty thousand, five hundred fifty.
2. Six hundred six thousand, six hundred six.
3. One hundred ten million, one hundred ten.
4. Four hundred billion, seven hundred million, sixty-five.
5. One hundred twenty-seven million, one hundred twenty-seven thousand.
6. Seven million, seventy thousand, seven hundred ...en.

... *million*, seven thousand, seventy-seven.

8. Sixteen billion, one hundred six thousand.
9. Eighty million, eight hundred thousand, eight.
10. Ninety-seven billion, ninety-seven thousand, ninety.
11. Thirty trillion, three hundred million, thirty-three.

32. Write from dictation the numbers in Arts. 27 and 29.

33. In expressing **United States Money** the dollar mark, **$**, is written before the number, and the dollar point, or period, after the number of dollars. Two places at the right of the period or dollar point, are reserved for cents or for dimes and cents, and a third place for mills.

Ciphers are used in the place of cents, or of tens of cents (dimes), when there are no significant figures of these orders; thus,

Fifty dollars and fifty cents,	$50.50
Fifty dollars and five cents,	$50.05
Fifty cents,	$.50
Five cents,	$.05
Five mills,	$.005

34. Read, and show the value of the different places in the following expressions of United States money:

$500.00	$.50	$.009	$9.00
$50.00	$.05	$.09	$90.00
$5.00	$.005	$.90	$900.00

Ten mills make one cent; ten cents make one dime; ten dimes make one dollar. The word *and* may be read for the dollar point, or period, between dollars and cents. Cents alone may be indicated by ¢, ct., or cts. written after the number; thus, 5 ¢; 1 ct.; 33 cts.

35. Read the following sums of money:

1. $1,000.10.	5. $476.25.	9. $608.80.	13. $10.105.	
2. $875.50.	6. $327.08.	10. $740.40.	14. $100.000.	
3. $909.09.	7. $500.05.	11. $933.33.	15. [illegible]	
4. $700.75.	8. $999.00.	12. $801.01.	16.	

36. Write the following sums of money:

1. One hundred dollars.
2. Ten dollars *and* ten cents.
3. One dollar *and* ten mills.
4. Forty dollars a
5. Four dollars a
6. Forty cents a
7. One thousand five hundred dollars *and* fift
8. Sixteen hundred sixty dollars *and* six cent
9. Two thousand fifty dollars *and* five cents.
10. One hundred sixty dollars and sixteen cent

ROMAN NOTATION.

37. Roman Notation represents numbers by s
viz.:

Letters:	I	V	X	L	C	
Values:	1	5	10	50	100	5

38. The letters, except V, L, and D, are repe
peat their values. I or X may be used three time
and C or M four times: thus, II, 2; III, 3;
XXX, 30; CCCC, 400; MMMM, 4,000.

39. Letters of less value written *after* those
value indicate the *sum* of the values; thus, VII,
13; LXV, 65; CLI, 151.

40. Letters of less value written *before* those o
value indicate the *difference* between the two valu
IV, 4; IX, 9; XL, 40; LIX, 59; CXLI, 141.

41. A dash written over any number makes the
thousand times as great; thus, $\overline{\text{V}}$, 5,000; $\overline{\text{L}}$, 50
500,000.

the following numbers:

4. XXVIII. 7. LXXXI. 10. CIX. 13. I
XXXIX. 8. LXXIX. 11. CXLI. 14. I
II. 9. XCIX. 12. CCII. 15. M

13, 14, 15, 16, 17, and 18.

g columns:

1	5	8	5
9	1	6	7
4	7	5	6
1	8	3	4
	2	9	2
	6	4	3
	9	7	8
	4	2	4

ive sums only, treating
gures as single figures;

l or exceed 10.

53; 648,578; and,

—1. Write the num-
t figures representing
e same order stand in
olumn.
1, 21 units, or 2 tens and
Write the 1 under the
d add the 2 with the tens.
10, 15, 23 tens; or, 2 hun-
3 tens. Write the 3
ns and add the 2 with

imilar manner, the
der that column,
d with the units

any
nsw

te the results,

51. SIGHT EXERCISES. **Read sums at sight:**

2	2	9	1	8	3	7	4	6	5
3	9	9	9	9	9	9	9	9	9
9	2	9	1	8	3	7	4	6	5
3	7	7	7	7	7	7	7	7	7
1	2	9	1	8	3	7	4	6	5
3	4	4	4	4	4	4	4	4	4
8	2	9	1	8	3	7	4	6	5
3	6	6	6	6	6	6	6	6	6
3	2	9	1	8	3	7	4	6	5
3	8	8	8	8	8	8	8	8	8
4	2	9	1	8	3	7	4	6	5
3	5	5	5	5	5	5	5	5	5
6	2	9	7	8	3	7	4	6	5
3	2	2	3	2	2	2	2	2	2
5	2	9	1	8	3	7	4	6	5
3	1	1	1	1	1	1	1	1	1

These exercises comprise all the possible additions of two numbers, each *expressed* by one digit. By daily practice learn to tell the sums *rapidly, reading* them in as many different directions as possible.

52. Illustrate the PRINCIPLES OF ADDITION by using the three following forms:

$6	**6**	**4**
$4	**4**	**6**
$10	**10**	**10**

I. *Only like numbers and units of the same order can be added.*

II. *The sum is the same in kind or denomination as the numbers to be added.*

III. *The order of the numbers to be added does not affect the sum.*

53. Oral Problems. 1. John walked 6 miles the first hour, 4 miles the second, and 9 miles the third hour. How far did he walk in the three hours?

2. Ellen paid 11 cents for lace, 9 cents for ribbon, and 15 cents for muslin. How much did she pay for all?

3. If Mrs. James spends $5 for an umbrella, $17 for a piece of silk, and $8 for a bonnet, how much altogether does she spend?

4. James had 7 marbles; Henry had 14 more than James; and Edward had as many as both together. How many marbles did Edward have?

5. One stable contains 15 horses; a second stable, 9 horses more than the first; and a third stable, 11 horses more than the other two together. How many horses are there in the third stable?

6. A farmer gathered 9 quarts of berries on Monday; and on Tuesday 9 quarts more than he gathered on Monday; and so on, each day 9 quarts more than on the day before. How many quarts did he gather on Friday?

7. Frank paid $12 for an overcoat, and $18 for a suit of clothes. *How much did* both cost him?

54. To find the sum, when each column is less than 10.

1. Find the sum of \$203.21; \$5,460.50; \$105.05, and \$210.13.

Explanation.—1. The numbers are written so that figures representing units of the same order stand in the same column.

2. Each column is then added by itself, and its sum is placed underneath the column; thus, 8, **9**; 6, **8**; 5, **8**; **7**; 3, 7, **9**; **5**.

3. Write the dollar sign before the first number and before the answer. Answer, \$5,978.89.

\$203.21
5,460.50
105.05
210.13
\$5,978.89

2.	3.	4.	5.
\$20,416.13	\$10,010.10	\$55,005.25	\$40,400.40
5,021.41	9,507.08	30,850.70	7,070.07
13,200.04	20,160.30	12,132.03	31,310.31
351.20	40,212.50	1,001.01	1,208.10

55. Read sums at sight:

9	19	29	39	49	59	69	79	89
2	2	2	2	2	2	2	2	2
8	18	28	38	48	58	68	78	88
3	3	3	3	3	3	3	3	3
7	17	27	37	47	57	67	77	87
4	4	4	4	4	4	4	4	4
6	16	26	66	46	56	66	76	86
5	5	5	5	5	5	5	5	5

56. Construct, and read for 12, 13, 14, 15, 16, 17, and 18, tables similar to those in Art. 55.

57. Find the sums of the following columns:

6	1	1	8	0	1	5	8	5
5	2	4	3	5	9	1	6	7
5	3	5	7	4	4	7	5	6
7	4	3	6	6	3	8	3	4
3	2	2	5	2	7	2	9	2
0	4	6	4	3	5	6	4	3
8	3	4	2	8	4	9	7	8
2	2	5	4	2	3	4	2	4

Think and read in each column the successive sums only, treating familiar combinations of two or three figures as single figures; thus, in the first column: 10, 20, 36.

58. To find the sum, when columns equal or exceed 10.

1. What is the sum of 307,987; 24,653; 648,578; and, 32,813?

307,987
24,653
648,578
32,813
1,014,031

Explanation.—1. Write the numbers so that figures representing units of the same order stand in the same column.

2. Units: 11, 21 units, or 2 tens and 1 unit. Write the 1 under the units and add the 2 with the tens.

3. Tens: 10, 15, 23 tens; or, 2 hundreds and 3 tens. Write the 3 under the tens and add the 2 with the hundreds.

4. The other columns in succession are added in similar manner, the units in the sum of any column being written under that column, and the tens in the sum of any column being added with the units of the next higher order. Answer, 1,014,031.

NOTE.—In practice, merely think the additions, and write the results, *thus, 11, 21—1; 10, 15, 23—3;* 10, 15, 21, 30—0, etc.

59. Rule for Addition. 1. Write the numbers so that figures representing units of the same order stand in the same column.

2. Beginning at the right, add the columns, separately.

3. If the sum of any column is less than ten, write under that column the figure representing the sum.

4. If the sum of any column is equal to or greater than ten, write the figure representing the units in the sum under that column, and add the tens with the units in the column of the next higher order.

5. Write the entire sum of the last or left-hand column.

Proof.—Add the columns down instead of up, and find the same result.

60. Find the sums:

1.	2.	3.	4.
$88,640.25	$15,540.32	$50,500.50	$10,100.01
9,075.80	700,657.14	879,879.79	412,214.14
44,068.25	32,789.48	42,086.43	59,059.59
13,463.47	563,320.96	754,702.91	651,301.42
6,687.63	86,475.32	89,611.40	800,891.90

5.	6.	7.	8.
478,209,312.	10,700,641.	505,330,003.	440,875,620.
100,010,860.	504,540,504.	27,027,270.	505,407,032.
59,764,327.	80,710,642.	816,978,423.	36,728,487.
241,605,083.	232,000,232.	583,064,390.	600,060,600.
27,432,247.	45,000,450.	77,805,428.	97,758,043.

61. Oral Problems. 1. Matthew had 9 tops and Edgar 7 tops. How many tops did both have?

Fred had 8 marbles and Thomas gave him 6. How then have?

6	16
5	5

nt $.20 one day and $.25 the next day. How she spend in both days?

4. Mr. Jameson earned $14, $16, and $15 in three weeks. How much money altogether did he earn ?

5. A farmer has 50 sheep in one flock, 25 in a second flock, and 9 in a third. How many sheep in all has he ?

6. Mr. Johns has three farms containing respectively 30, 40, and 25 acres. How many acres altogether has he ?

7. Frank spent $.25 for a reader, $.15 for a blank book, and $.05 for a pencil. How much money did he spend ?

8. Mary bought a pound of coffee, $.32 ; a pound of butter, $.33 ; and two bags of salt, $.10. How much money did she spend ?

9. Mrs. Brown paid $36 for a cloak and $14 for a dress. How much did both cost her ?

10. A grocer paid $100 for a horse, $35 for a wagon, and $15 for harness. How much did he pay for all ?

11. A park contains 40 elm trees, 30 maple trees, 10 oak trees, and 5 pine trees. How many trees are there ?

12. Mrs. Thomas is 47 years old. How old will she be in 23 years ?

13. Mr. Myers travelled 35 miles by carriage and 115 miles by railroad. How far did he travel altogether ?

14. A man bought a horse for $75, and a wagon for $25. He paid $10 for repairing the wagon and then sold horse and wagon at a profit of $15. How much did he receive ?

62. Written Problems. 1. How many square miles are there in Great Britain ; England and Wales containing 55,100 square miles, and Scotland, 29,600 ?

2. The first of three numbers is 5,564 ; the second, 9,765 ; the third, as much as the first and second together. What is the sum of the three numbers ?

3. Mrs. Mabie spent $18.64 at one store; $20.33 at a second store ; $19.20 at a third store ; and $17.19 at a fourth store. *How much did* she spend at the four stores together

4. On a ranch there were 1,589 cows; 2,407 sheep; 150 horses. How many in all were there?

5. Mr. King bought a house and lot for $3,750; it increased in value $825 the first year; $730 the second year; and $1,129 the third year. How much was the house worth at the end of the third year?

6. A commission merchant sold in one year 15,809 barrels of apples; 23,640 barrels of potatoes; 14,096 barrels of flour; and 17,007 barrels of sugar. How many in all did he sell?

7. Mrs. Jenkins spent in shopping: $16.30; $25.14; $19.28; $9.08; and $37.06. How much did she spend?

8. A landlord's rents for a year from tenants in four houses are as follows: $970.50; $1,685.25; $27,860.33; and $45,500. How much in all does he receive?

9. A grocer's purchases in a week amounted to $192.26; $230.05; $100; $346.35; $525.50; and $375.75. How much was the amount?

10. A grocer's sales for one week were as follows: $250.60; $179.83; $428.06; $350.75; $270.68; and $500.50. What was the amount of his sales for the week?

11. A broker sold wheat to four men as follows: 25,079; 17,463; 35,608; and 29,009 bushels. How many bushels in all did he sell?

12. An estate was divided as follows: $35,750 to the wife; $27,675 to the eldest son; $20,000 to each of 4 other sons; and $25,000 to the daughter. What was the estate worth?

13. A builder built 5 houses; the first for $5,125; the second for $125 more than the first; the third for $125 more than the second, and so on. What was the cost of the five houses together?

14. Mr. Baker began business with a capital of $2,500. He increased his capital $250.25 each year, for four years. What was his capital at the end of the fourth year?

SUBTRACTION.

63. 1. James had $.45 and spent $.07. How many cents had he left?

2. Mr. Marsh bought a horse for $75, and sold it at a loss of $8. What was his selling price?

3. If a merchant buys cloth for $.82 a yard, and in selling it loses $.08 a yard, what is the selling price?

64. To **subtract** is to take one number from another, as a part from the whole.

65. The **remainder** is that which is left after taking away a part of a number.

The *difference* between two numbers is the number which added to the less equals the greater. It is the same as the *remainder* found by taking the one number, as a part, from the other.

66. Subtraction is the process of finding a remainder, or of finding the difference between two numbers.

67. The **minuend** is the number *to be diminished*, or the number from which the other is to be taken.

68. The **subtrahend** is the number to be taken away, as a part.

69. The **sign of subtraction** is −; called *minus*, meaning *less;* thus 40 − 30 is read 40 *minus* 30, or 40 *less* 30.

From	7845	**Minuend.**
Take	5324	**Subtrahend.**
	2521	**Remainder.**

70. A **parenthesis** is made thus ( ); it signifies that the numbers within it are to be treated as a single quantity. Thus, 54 − (40 + 12), is read 54 less the parenthesis 40 + 12, and it means that the sum of 40 and 12, or, 52, having been found, it is to be subtracted from 54.

A brace { }, a bracket [ ], or a vinculum —— is used with the same effect as a parenthesis.

71. SLATE AND SIGHT EXERCISES. **Find the remainders:**

11	21	31	41	51	61	71	81	91
2	2	2	2	2	2	2	2	2
11	21	31	41	51	61	71	81	91
3	3	3	3	3	3	3	3	3
11	21	31	41	51	61	71	81	91
4	4	4	4	4	4	4	4	4
11	21	31	41	51	61	71	81	91
5	5	5	5	5	5	5	5	5
11	21	31	41	51	61	71	81	91
6	6	6	6	6	6	6	6	6
11	21	31	41	51	61	71	81	91
7	7	7	7	7	7	7	7	7
11	21	31	41	51	61	71	81	91
8	8	8	8	8	8	8	8	8
11	21	31	41	51	61	71	81	91
9	9	9	9	9	9	9	9	9
11	21	31	41	51	61	71	81	91
10	10	10	10	10	10	10	10	10

ay be used as a sight exercise from the book or from the

72. Construct for 12, 13, 14, 15, 16, 17, and 18, tables similar to the one in Art. 71.

73. DRILL EXERCISES. **Count:**

1. By 1's from 100 to 0.
2. By 2's from 100 to 0; from 99 to 1.
3. By 3's from 99 to 0; from 98, 97 to 2, 1.
4. By 4's from 100 to 0; from 99, 98, 97 to 3, 2, 1.
5. By 5's from 100 to 0; from 99, 98, 97, 96 to 4, 3, 2, 1.

74. Illustrate the PRINCIPLES OF SUBTRACTION by using the two following forms:

\$6	**6**		**16**
4	**4**	**Prin. V.; thus,**	**14**
\$2	**2**		**2**

I. *The minuend, subtrahend, and remainder are like numbers.*

II. *Increasing or decreasing the minuend produces a like effect on the remainder.*

III. *Increasing or decreasing the subtrahend produces an opposite effect on the remainder.*

IV. *Increasing and decreasing either minuend or subtrahend by the same number produces no effect on the remainder.*

V. *Increasing or decreasing both minuend and subtrahend by the same number produces no effect on the remainder.*

75. Oral Problems. 1. At 8 P.M., how many hours are there before midnight?

2. A man owed \$30. How much did he still owe after paying \$14?

3. Mr. Brown had \$50. How much money did he have left after spending \$14 for a pair of boots, \$8 for a hat, and \$5 for fur gloves?

4. The minuend is 60, and the subtrahend 35. What is the remainder?

76. To find the difference between any two numbers.

1. What is the difference between 9,257 and 5,379 ?

Explanation.—1. Write the less number under the greater, units under units, etc.

2. Units : As 9 units cannot be taken from 7 units, take 1 ten from the 5 tens ; adding the 1 ten, or 10 units, to the 7 units gives 17 units : 9 units from 17 units leaves 8 units. Write 8 under the units.

3. Tens : Since 1 ten was taken from the 5 tens, 4 tens are left. As 7 tens cannot be taken from 4 tens, take 1 hundred from the 2 hundreds; adding the 1 hundred, or 10 tens, to the 4 tens give 14 tens; 7 tens from 14 tens leaves 7 tens. Write 7 under the tens.

4. In similar manner, 3 hundreds from 11 hundreds leaves 8 hundreds; and 5 thousands from 8 thousands leaves 3 thousands.

8 11 14 17
9,257
5,379
3,878

NOTE.—In practice, merely think the subtractions, and write the results ; thus, 9 from 17, **8**; 7 from 14, **7**; 3 from 11, **8**; 5 from 8, **3**. Answer, 3,878.

Or, when any figure in the minuend is less than the figure of the subtrahend, 1 unit of the next higher order may be added to that figure in the minuend and to the figure of the next higher order in the subtrahend: thus, 9 from 17—**8**; 8 from 15—**7**; 4 from 12—**8**; and 6 from 9—**3**. Answer, 3,878. (Prin. V.)

1 1 1
9,257
1 1 1
5,379
3,878

2. What is the difference between 7,000 and 539 ?

Explanation.—1. Write the less number, etc.

2. Take 1 from the 7 thousands, leaving 6 thousands: change the 1 thousand to hundreds, making 10 hundreds. Take 1 from the 10 hundreds, leaving 9 hundreds: change the 1 hundred to tens, making 10 tens. Taking 1 from the 10 tens, etc., making the minuend 6 thousands, [illegible]hundreds, 9 tens, and 10 units.

6 9 9 10
7,000
539
6,461

9 from 10—**1**; 3 from 9—**6**; 5 from 9—**4**; 0 from 6—**6**. 6,461.

77. Rule for Subtraction.—1. Write the less number under the greater, units under units, tens under tens, etc.

2. Beginning at the right, subtract the number of units in the successive orders of the subtrahend from the number of units in the corresponding orders of the minuend, and underneath write the results.

3. If the number of units of any order in the minuend is less than the number of units of the same order in the subtrahend, add 10 to the number of units of that order in the minuend, and subtract 1 from the number of units in the next higher order of the minuend; then proceed as before.

***Proof.**—The remainder added to the subtrahend equals the minuend.*

78. Find the remainders:

1.	2.	3.	4.	5.
$3,085	$4,704	$5,057	$6,600	$7,070
2,466	3,998	989	5,049	2,509
6.	**7.**	**8.**	**9.**	**10.**
6,803	2,000	6,090	7,000	4,805
3,048	1,909	1,508	5,051	2,079
11.	**12.**	**13.**	**14.**	**15.**
27,091	13,506	20,806	17,700	30,010
15,412	9,087	14,059	12,899	19,806
16.	**17.**	**18.**	**19.**	**20.**
$125.48	$200.50	$350.09	$700.75	$909.09
68.17	189.09	259.60	548.86	89.90

79. Oral Problems. 1. Edward bought a knife for $.75 and sold it at a loss of $.26. What was the selling price?

2. James earned $.60 a day, which was $.25 more than *Edward earned.* How much did Edward earn?

3. Mr. Rogers earned $43 a week, and his expenses were $29 a week. How much could he save in one week?

4. Jane had $.48; her father gave her $.12, and she spent $.30. How much did she have left?

5. Mr. Arnold bought a horse for $175. He paid $25 for transporting the horse, and then sold him for $150. What was his loss?

6. A merchant having two pieces of cloth containing 25 and 35 yards, sold 39 yards. How many yards did he have left?

7. In a Second Reader there are 160 pages, and in a Third Reader, 220 pages. The Third Reader has how many more pages than the Second Reader?

8. A city in Delaware has 20,000 inhabitants, and a city in Maryland 12,000. Which city has the greater population, and how much?

9. A ship sailed 45 miles. How much further did it have to sail to complete a distance of 144 miles?

80. Written Problems. 1. How many years is it since the United States Government was organized in 1789?

2. In 1884, Dakota produced 13,950,000 bushels of corn and 22,330,000 bushels of wheat. How many more bushels of wheat than of corn did it produce?

3. What number must be added to 2,269 to make 5,000?

4. The minuend is 7,000; the remainder, 3,009; what is the subtrahend?

5. A merchant started business with a capital of $1,750; in two years his capital amounted to $2,349. How much had he gained?

6. What number added to 3,089 will make 4,906?

7. A $1,000 bill was given in payment of a debt of $799. What sum should be returned to the debtor?

8. *A farm bought* for $20,100 was sold for $19,899. What

9. The population of New York City in 1880 was 1,206,299, and of Brooklyn 556,663. How much the greater was the population of New York City?

10. A contractor made a contract to build a mansion for $200,000. In the construction, he spent $169,850.65. What were his profits?

11. The difference between the money owned by two brothers is $189.06. The one with the more money has $1,011.75. How much money has the other?

81. Copy, and complete the following bills:*

NEW ORLEANS, *Oct.* 12, 1888.

MR. JAMES THOMPSON,

Bought of MASON, BROWN & CO.

25	lb.	Sugar, ...	@ .06	$1	50
10	gal.	Molasses, ...	@ .18		
2	bl.	Flour, ...	@ 4.50		
				$	

Received payment,
MASON, BROWN & CO.

ST. LOUIS, MO., *Dec.*, 15, 1888.

MESSRS. JOHNSON & CO.,

Bought of FREDERICK TYLER.

250	bu.	Wheat, ...	@ $1.17	$	
125	"	Potatoes, ...	@ .78		
80	"	Corn, ...	@ .45		
				$	

Received Payment,
FREDERICK TYLER,
per C. W.

* These forms are to be taught at this time, only to pupils who are *familiar with multiplication.*

82. Make a receipted bill for each of the following forms:

Groceries.		Groceries.	
14 lb. Sugar, . .	@ 9 ¢.	4 qt. Strawberries, .	@ 8 ¢.
10 " Coffee, .	@ 32 ¢.	3 " Vinegar, . .	@ 5 ¢.
8 " Tea, . .	@ 70 ¢.	2 gal. Oil, . . .	@ 15 ¢.
50 " Butter, .	@ 25 ¢.	4 pk. Potatoes, . .	@ 25 ¢.

Dry Goods.		Notions.	
15 yd. Calico, .	@ 6 ¢.	10 yd. Ribbon, . .	@ 12 ¢.
20 " Gingham,	@ 12 ¢.	4 doz. Buttons, . .	@ 28 ¢.
35 " Muslin, .	@ 11 ¢.	2 " Spool Cotton,	@ 60 ¢.
12 " Cashmere,	@ 75 ¢.	5 yd. Lace, . . .	@ 37 ¢.

MISCELLANEOUS PROBLEMS.

83. Oral Problems. 1. A pleasure boat contains 40 ladies, 50 men, and 49 children. How many people does the pleasure boat contain ?

2. A merchant made $18 by selling an overcoat for $60. How much did the coat cost him ?

3. There were 35 girls and 36 boys in a certain grade at school. How many boys and girls in all were there in the grade ?

4. Mr. Howe had $100 and spent $40 for travelling and $35 for hotel expenses. How much money did he then have ?

5. A barrel of flour was bought for $4.25 and sold at a gain of $.75. What was the selling price ?

6. From a piece of cloth containing 75 yards, 51 yards were cut. How many yards were left in the piece ?

7. A farmer had 80 bushels of wheat, 40 bushels of oats, and 25 bushels of barley. How many bushels in all did he ?

contained 1,000 yards of thread. How many
t on the spool after 550 yards had been cut off ?

9. Mr. Jacobs bought one book for $.48, and another book for $1.00. How much did both books cost him ?

10. A horse costs $100, and a wagon $40. They were sold together at a loss of $80. What was the selling price ?

11. If one book cost $.48 and another $.99, how much will both books cost ?

12. Sarah had 40 grapes ; she gave to three children, 10, 5, and 15 grapes. How many did she have left ?

13. Edgar spent $.48 for a bat and $.49 for a cap. How much did he pay for both ?

14. The minuend is 45, and the remainder 29. What is the subtrahend ?

15. John bought a slate for $.12, a reader for $.38, and a drawing book for $.35. He gave in payment $1.00. How much change is due him ?

16. Mrs. Morrow had two $10 bills and a $5 bill. She spent $15 for a bonnet, and $8 for a parasol. How many dollars did she then have ?

17. Hannah bought ribbon for $.18, lace for $.17, and buttons for $.10. How much change is due her on paying with $.50 ?

18. Frank gave $.60 for a book worth $.45 and a slate. How much did the slate cost him ?

84. Written Problems. 1. Mrs. Edwards paid $45.35 for a cloak, $28.75 for one dress, $19.82 for another dress, and $26.95 for lace. How much did she pay for all ?

2. A man bought a horse and wagon for $325. He sold the horse for $275 and thereby gained $15. What did the wagon cost him ?

3. What is the entire surface of the great lakes, with respectively the following number of square miles : Superior, 31,500 ; Michigan, 23,150 ; Huron, 23,100 ; Erie, 7,900 ; *Ontario, 6,900 ?*

4. The population of five cities is as follows: 21,480, 39,604, 270,299, 175,618; and 199,089. What is their total population?

5. A house and two lots were bought for $10,000. One of the lots was sold for $1,500 at a loss of $500. What did the other lot with the house cost?

6. A farmer sold four loads of hay weighing respectively, 1,905, 1,826, 2,309, and 1,998 pounds. What was their entire weight?

7. Mr. Francis bought two horses and a carriage for $650. One of the horses was sold for $300 at a gain of $75; the other for $200 at a loss of $50. What did the carriage cost Mr. Francis?

8. Mr. Thomas started business with $5,000 capital. His profits the first year were $1,906 and the third year $929. His losses the second and fourth years were respectively $1,581 and $2,390. What was his capital at the end of the fourth year?

9. Mrs. Brown had $1,000; she paid $250 for a horse, $175 for a city lot, and $500 for furniture. How much money did she then have?

10. A man bought a house for $5,500, and expended $4,700 in repairs and alterations, and $147 in taxes. He sold the house for $7,000. Did he gain or lose, and how much?

11. Mr. Boyd started on a journey, having $1,500. His travelling expenses were $478; hotel bills, $708; and his purchases, $279. How many dollars did he have left?

12. A farm of 500 acres valued at $37 an acre was sold with live stock at $1,250 for $35,000. What was the profit?

13. If a farm, costing $15,700, a mill, $3,895, a house *nd lot*, $23,750, and some live stock together cost one hun- *usand dollars*, what is the value of the live stock?

MULTIPLICATION.

85. 1. Frank is 9 years old. His father is 7 times as old as Frank. How old is the father?

2. Mr. Thomas bought 8 tons of coal at $5 a ton. What was the entire cost of the coal?

3. Mr. Samuels worked 12 hours a day for 6 days in the week. How many hours in all did he work?

4. If 7 men can do a piece of work in 5 days, in what time can one man do the work?

86. To **multiply** one number by another is to take the one number as many times as there are units in the other.

87. Multiplication is a short method of finding the sum of several equal numbers.

Thus: John earned $25 a week. How much money did he earn in 5 weeks?

	Multiplication.
$25	$25
25	5
25	
25	
25	
$125	$125

88. The **multiplicand** is the number *to be repeated*, or *multiplied*.

89. The **multiplier** is the number by which to multiply. It shows how many times the other number is to be taken, or repeated. In practice the smaller number, abstract or concrete, is frequently used as the multiplier.

90. The **product** is the result obtained by multiplying.

91. Numbers which, multiplied together, give the product, are called its **factors** or *makers;* thus, 6 and 5 are *factors* of 30.

$25 **Multiplicand.**
5 **Multiplier.**
$125 **Product.**

92. Factors that cannot be separated into smaller factors are called **prime factors**, as 2, 3, 5, 7, etc.

93. The **sign of multiplication** is $\times$; it is read *times*, or *multiplied by;* thus, 9×7 is read 9 *times* 7, or it may be read 9 *multiplied by* 7.

94. Copy, and commit to memory:

2	2	2	2	2	2	2	2	2	2
1	2	3	4	5	6	7	8	9	10
2)2	2)4	2)6	2)8	2)10	2)12	2)14	2)16	2)18	2)20
1	2	3	4	5	6	7	8	9	10

Read thus: 7 times 2 are 14; $\frac{1}{2}$ of 14 is 7, or 2 in 14, 7: etc.

95. Construct a table similar to the preceding, for each of the other numbers to and including 12.

96. Copy, and commit to memory the following table:

×	1	2	3	4	5	6	7	8	9	10	11	12
1	**1**	2	3	4	5	6	7	8	9	10	11	12
2	2	**4**	6	8	10	12	14	16	18	20	22	24
3	3	6	**9**	12	15	18	21	24	27	30	33	36
4	4	8	12	**16**	20	24	28	32	36	40	44	48
5	5	10	15	20	**25**	30	35	40	45	50	55	60
6	6	12	18	24	30	**36**	42	48	54	60	66	72
7	7	14	21	28	35	42	**49**	56	63	70	77	84
8	8	16	24	32	40	48	56	**64**	72	80	88	96
9	9	18	27	36	45	54	63	72	**81**	90	99	108
10	10	20	30	40	50	60	70	80	90	**100**	110	120
11	11	22	33	44	55	66	77	88	99	110	**121**	132
12	12	24	36	48	60	72	84	96	108	120	132	**144**

In each square is the product of the number at the top multiplied by the number at the left in the first column.

97. Find the cost of the following:

	lb.		¢.		T.		$		yd.		¢.
1.	8	@	8.	*5.*	9	@	5.	*9.*	9	@	6.
2.	7	@	9.	*6.*	4	@	8.	*10.*	5	@	8.
3.	8	@	6.	*7.*	7	@	6.	*11.*	7	@	7.
4.	5	@	9.	*8.*	8	@	9.	*12.*	9	@	9.

98 oblems for the numbers in Art. 97.

ow much change should Frank receive, if he gives 3 *lollar in* payment for 8 pounds of sugar at 8 cents?

99. SLATE AND SIGHT EXERCISES. **Find products:**

0	1	2	3	4	5	6	7	8	9	10
2	2	2	2	2	2	2	2	2	2	2

0	1	2	3	4	5	6	7	8	9	10
3	3	3	3	3	3	3	3	3	3	3

0	1	2	3	4	5	6	7	8	9	10
4	4	4	4	4	4	4	4	4	4	4

0	1	2	3	4	5	6	7	8	9	10
5	5	5	5	5	5	5	5	5	5	5

0	1	2	3	4	5	6	7	8	9	10
6	6	6	6	6	6	6	6	6	6	6

0	1	2	3	4	5	6	7	8	9	10
7	7	7	7	7	7	7	7	7	7	7

0	1	2	3	4	5	6	7	8	9	10
8	8	8	8	8	8	8	8	8	8	8

0	1	2	3	4	5	6	7	8	9	10
9	9	9	9	9	9	9	9	9	9	9

0	1	2	3	4	5	6	7	8	9	10
10	10	10	10	10	10	10	10	10	10	10

100. ***Extend the*** **preceding tables to 12 times 12.**

101. Illustrate the PRINCIPLES OF MULTIPLICATION by using the three following forms:

$6	6	4
4	4	6
$24	24	24

I. *The multiplicand is an abstract or a concrete number.*

II. *The multiplier is an abstract number.*

III. *The product is the same in kind or denomination as the multiplicand.*

IV. *The product is the same, in whatever order the numbers are multiplied.*

V. *Multiplying either multiplicand or multiplier by a number, multiplies the product by the same number.*

VI. *Dividing either multiplicand or multiplier by a number, divides the product by the same number.*

102. Oral Problems. 1. What will 12 pounds of sugar cost at $.08 a pound?

2. How much must Mr. Smith pay for 11 yards of cloth at $.50 a yard?

3. If a train travels 60 miles an hour, how far will it travel in 7 hours?

4. What is the product of (15 − 9) times (8 − 4)?

5. If Grace commits to memory 50 lines of Paradise Lost each day, how many lines will she learn in a week of 6 days?

6. If oranges cost 3 cents each, what cost 10 dozen oranges?

7. In a bushel there are 32 quarts. How many quarts are there in 3 bushels?

8. If a pint of milk is worth 4 cents, what are 10 gallons of milk worth?

9. Matthew gave 10 dimes in payment for 8 pounds of sugar at 9 cents a pound, and 5 pounds of corn meal at 4 cents a pound. How much change is due him?

yard, how much will 30 yards of lace cost?

11. If Frank works 8 hours a day at $1.00 an hour, how much will he earn in 12 days?

12. At 4 cents each, how much will 500 lemons cost?

13. If a $5 bill is given in payment for a bushel of berries at 10 cents a quart, how much change shall be returned?

32 quarts make a bushel.

14. How much change shall be returned if a $10 bill is given in payment for 3 bushels of berries at 10 cents a quart?

15. What will 5 pairs of shoes cost, if each pair costs $5?

16. If a man earns $2 in a day, how much will he earn in 15 days? In 40 days? In 100 days?

17. How much will 7 thousand oiled bricks cost, if each thousand cost $20?

18. If each broom cost $.30, how much will 12 brooms cost? How much will 11 brooms cost?

19. What will a dozen pails cost, if each pail costs $.40?

20. How much does a man earn in 8 hours, if he earns $.50 each hour?

21. What is the cost of 12 dozen oranges, if each orange costs 5 cents?

22. If Mr. Brown pays $2 a day for board, how much will three weeks' board cost?

23. If it costs 3 cents to launder each cuff, how much will it cost to launder 6 pairs of cuffs?

24. What is the entire cost of 7 sets of encyclopædias at $80 a set?

25. A merchant's expenses were $90 per month. How much were they per year?

26. Mr. James bought a dozen lawn-tennis sets at $30 a set. What was the entire cost?

27. If each banjo is worth $22.20, what are 3 banjos worth?

28. What is the product of $(75 + 5)$ multiplied by $(12 - 9)$? $(900 - 81) \times (68 - 59) = ?$

103. To find the product when the multiplier is expressed by one figure.

1. Find the product of $24.35 multiplied by 9.

$$\begin{array}{r} \$24.35 \\ 9 \\ \hline \$219.15 \end{array}$$

Explanation.—1. Write the multiplier under the first figure at the right of the multiplicand.

2. 9 times 5 cents are 45 cents, or 4 dimes and 5 cents; write 5 under the cents; and reserve the 4 dimes to be added to the product of the dimes.

3. 9 times 3 dimes are 27 dimes, which with the 4 dimes make 31 dimes, or 3 dollars and 1 dime; write 1 under the dimes, and reserve the 3 dollars to be added to the product of the dollars.

4. 9 times 4 dollars are 36 dollars, which with the 3 dollars, make 39 dollars, or 3 tens of dollars, and 9; write the 9 under the dollars, and reserve the 3 tens of dollars to be added to the product of the tens of dollars.

5. 9 times 2 tens of dollars are 18 tens of dollars, which with the 3 tens of dollars make 21 tens of dollars, or 2 hundreds, and 1 ten of dollars; write the 1 under the tens of dollars, and the 2 in the place for hundreds of dollars. Answer, $219.15.

NOTE.—In practice, merely think the products, and write their unit figures, adding the other figures to the next product; thus: 45, **5**; 27, 31, **1**; 36, 39, **9**; 18, **21**.

2.	**3.**	**4.**	**5.**	**6.**
$34.27	$52.09	$60.75	$87.23	$52.36
6	7	4	5	3

7.	**8.**	**9.**	**10.**	**11.**
$46.28	$76.53	$42.08	$35.27	$69.85
9	8	6	9	7

12.	**13.**	**14.**	**15.**	**16.**
	18	$80.89	$77.65	$89.09
	9	8	9	7

104. Written Problems. 1. How many days are there in 3 years of 365 days each ?

2. If a steamship burns 325 tons of coal a day, how much coal will it burn in 7 days ?

3. Mr. James bought 9 city lots at $1,875. What was the entire cost ?

4. A merchant sold 175 barrels of flour at $5 a barrel. How much should he receive in payment ?

5. What is the cost of a bag of flour containing 75 pounds at 4 cents a pound ?

6. A city contained 15,063 inhabitants. In 10 years its population was multiplied by 7. What was the number of its inhabitants then ?

7. A gentleman left his estate to be divided equally among his 8 children. Each child received $25,000. What was the entire value of the estate ?

8. In each of 6 coal pockets there were 13,051 tons of coal. How much coal altogether was there ?

9. A merchant made $9,078 on each of 7 consignments of goods. How much in all did he make ?

105. Read products, taking the numbers in any order:

2 ×	5	12	3	8	11	10	2	7
3 ×	8	5	9	2	10	12	6	3
4 ×	12	10	5	9	2	7	3	4
5 ×	2	4	12	3	9	5	11	6
6 ×	7	11	4	12	8	3	5	10
7 ×	6	2	8	7	3	4	9	5
8 ×	3	6	10	11	7	2	8	9
9 ×	9	7	11	6	4	8	12	8
10 ×	11	3	7	4	6	9	10	12
11 ×	4	8	6	10	5	11	7	2
12 ×	*10*	*9*	2	5	12	6	4	1

3

106. To find the product when the multiplier is expressed by two figures.

1. Find the product of $2.03 multiplied by 38.

Explanation.—1. Write the multiplier under the multiplicand, placing units under cents, and tens under dimes, etc.

2. Multiply by the 8 in units' place. This gives the first partial product, 1,624 cents. Write the 4 in cents' place.

3. Multiply by the 3 in tens' place. This gives the second partial product, 609 dimes. Write the 9 in dimes' place.

4. Add the partial products. Answer, $77.14.

$2.03
38
1624
609
$77.14

2.	3.	4.	5.	6.	7.
$20.68	$35.09	$26.83	$40.06	$39.08	$56.09
32	45	31	53	67	38

8.	9.	10.	11.	12.	13.
$60.53	$48.07	$02.36	$20.09	$68.50	$75.05
75	56	27	98	46	84

107. Written Problems. 1. What is the cost of 18 car-loads of melons at $350 for each car-load?

2. How much are 1,238 acres worth, at $43?

3. In each foot there are 12 inches. How many inches are there in 1,672 feet?

4. What is the entire cost of 25 pieces of cloth, each containing 49 yards, at $.67 a yard?

5. What is the amount of 15 months' rent at $41.66?

6. An army consisted of 95 regiments, each containing 1,146 men. How many men were there in the army?

A railroad's daily receipts were $22,963 on the average *week of 6 days.* What were its total receipts?

8. What is the value of 35 horses at $189 each ?

9. How many hours are there in the last four months of each year ? In the last five months ?

10. If a man's car fare is $.35 daily, how much will it be for a year of 365 days ?

11. If there are 35,934 farms in a State, averaging 75 acres to the farm at $28 an acre, what is the entire value of the farms of the State ?

12. What is the entire value of the wheat crop of a State raising 44,325,268 bushels of wheat at $.95 a bushel ?

13. When Indian Territory had 53,648 sheep valued at $2 a head, what was their entire value ?

14. What is the entire value of 535,679 cows at $18 ?

108. To find the product when the multiplier is expressed by any number of figures.

1. Find the product of 7,256 multiplied by 437.

7,256
437
50 792
217 68
2 902 4
3,170,872

Explanation.—1. Write the multiplier under the multiplicand, placing units under units, tens under tens, etc.

2. Multiplying the multiplicand by the 7 in units' place in the multiplier, the product is 50,792 units. Write the first or right-hand figure of this product under the units.

3. Multiplying the multiplicand by the 3 in tens' place in the multiplier, the product is 21,768 tens. Write the first or right-hand figure of this product under the tens.

4. Multiplying by the 4 in hundreds' place, the product is 29,024 hundreds. The first or right-hand figure of this product should be written under the hundreds.

5. *Adding the partial* products together gives the entire product. *Answer, 3,170,872.*

2. Find the product of 5,023 multiplied by 908.

Explanation.—1. Write the multiplier under the multiplicand, placing units under units, etc.

2. Multiplying by the 8 in units' place gives 40,184 units for the first partial product.

3. As there are 0 tens in the multiplier, and as multiplying by 0 gives 0 for a product, the second partial product is omitted.

4. Multiplying by the 9 in hundreds' place gives 45,207 hundreds for the third partial product. Write 7, the units' figure of this partial product, under the hundreds.

5. The sum of the partial products gives : Answer. 4,560,884.

3.	4.	5.	6.	7.
21,635	8,407	9,071	8,872	42,075
3,207	325	563	403	6,321

8.	9.	10.	11.	12.
30,642	9,025	7,049	5,281	25,091
1,575	638	376	967	4,032

13.	14.	15.	16.	17.
28,704	6,503	8,872	3,978	38,608
5,306	976	759	649	4,083

18.	19.	20.	21.	22.
42,098	19,207	50,908	67,841	93,085
6,305	4,029	8,095	1,752	9,786

23.	24.	25.	26.	27.
7,049	42,968	59,098	73,735	80,976
	704	20,634	36,007	45,057

109. Rule for Multiplication.—1. Write the multiplier under the multiplicand, placing units under units, tens under tens, etc.

2. Beginning at the right, multiply the multiplicand by the number of units in each order of the multiplier in succession. Write the figure of the lowest order in each partial product under the figure of the multiplier used in producing that partial product.

3. Add the partial products and the sum is the entire product.

Proof.—*If the product is divided by the multiplier, the quotient equals the multiplicand; if the product is divided by the multiplicand, the quotient equals the multiplier.*

110. Find the products:

1.	2.	3.	4.	5.
7,848	12,806	21,753	19,685	35,079
195	1,903	2,047	1,781	3,685
6.	**7.**	**8.**	**9.**	**10.**
25,087	42,831	50,876	47,063	63,047
4,791	3,709	9,098	8,088	9,903
11.	**12.**	**13.**	**14.**	**15.**
100,976	200,649	275,009	306,402	275,083
20,384	45,078	90,087	48,075	50,009

111. To multiply when there are ciphers at the right of one or of both factors.

1. Find the product of \$75.80 multiplied by 10.

\$75.80
10
\$758.00; **or,** \$758.00

Explanation.—1. Writing the numbers, units under units, etc., and multiplying according to rule, the product is \$758, which is the same *as the multiplicand with* the dollar point one place to the right

2. Find the product of \$6.83 multiplied by 100.

Explanation.—1. Writing the numbers, units under cents, etc., and multiplying by the number of units in each order of the multiplier, the product is found to be \$683, the same as the multiplicand with the dollar point moved two places to the right.

\$6.83
1 00
683
\$683.00; **or,** \$685.

3. Find the product of 145 multiplied by 1,000.

Explanation.—1. Writing the numbers, units under units, etc., and multiplying by the number of units in each order of the multiplier, the answer is found to be 145,000, which is the same as the multiplicand with three ciphers annexed.

145
1 000
145
145,000; **or,** 145,000.

4. Find the product of 460 multiplied by 1,200.

Explanation.—1. Writing the numbers, units under units, etc., and multiplying by the number of units in each order of the multiplier, the product is 552,000, which is the same as the product of 46 multiplied by 12 with three ciphers annexed.

460
1 200
92 0 | 46 0
460 | 12 00
552,000; **or,** 552,000.

112. Rule for Multiplying by 10, 100, etc.—I. If the multiplier is 1 with ciphers annexed, the product is found by annexing as many ciphers to the multiplicand as there are ciphers at the right of the multiplier; or, by moving the dollar or decimal point as many places to the right as there are ciphers at the right of the multiplier.

II. If the multiplier is more than 1 with ciphers annexed, multiply disregarding the ciphers at the right of the multiplicand and multiplier, and to the product annex as many *ciphers as* there are at the right of the multiplicand and the *ltiplier together.*

113. Find the products:

1. **10** ×	*2.* **100** ×	*3.* **100** ×	*4.* **550** ×	*5.* **1090** ×
$1,563	13,287	$63.50	1,020	$40.60
$20,381	9,063	$197.00	18,600	$225.00
$6,092	4,751	$48.60	9,340	$98.70
$4,278	12,079	$500.00	5,500	$360.00
$5,095	3,427	$125.70	12,080	$1,058.00

114. Oral Problems. 1. What will 5 acres of land cost at $25 an acre? At $24? At $26?

2. How much will 10 pianos cost at $250 each?

3. How far will a ship sail in 5 hours, if it sails 19 miles an hour? If it sails 21 miles an hour?

4. How far will a train travel in 9 hours at the rate of 21 miles an hour?

5. At $12 a thousand, what will 12,000 bricks cost?

6. At 2 cents an egg, what will 4 dozen eggs cost?

7. What will be the cost of 85 shares of stock at $100 a share? At $101?

8. How much will 75 shares of stock cost at $101 a share?

9. How many miles will a train travel in 10 hours at the rate of 49 miles an hour?

10. How far can a man travel in 8 days, at the rate of 49 miles a day?

11. If 8 men can dig a cellar in 9 days, in what time can one man dig the cellar?

12. If Charles makes $.36 a day and spends $.16, how much will he save in 5 days?

13. Henry earns $6.50 a week and Thomas $1.50 a week. How much will they together earn in 11 weeks?

14. How far apart will two men travelling in opposite directions be, at the end of 8 hours, if one travels 7 miles each *hour, and the other* 5 miles?

115. Written Problems. 1. What is the cost of 150 miles of railroad at $34,575 per mile ?

2. If a mile of telegraph lines cost $5,000, what will 127 miles cost ?

3. If each ton of coal weighs 2,240 pounds, how much will 3,250 tons of coal weigh ?

4. If a steamer burns every day, for 7 days, 350 tons of coal, weighing 2,000 pounds each, how many pounds of coal in all does it burn ?

5. What is the amount of money paid an army of 240,360 men for a year's services, if on the average they receive $32.50 a month ?

6. At $1,975 each, what will 324 building lots cost ?

7. What is the entire value of 25,000 farms containing an average of 67 acres at $98 an acre ?

8. A merchant had in stock 1,250 suits of clothing, of $27.50 average value. What was the entire stock worth ?

9. What will the work of 1,560 men for 150 days cost, at $2.25 each, per day ?

10. How many miles does the earth move in 50 days in revolving around the sun at the rate of 19 miles a second ?

11. If a tenant pays $33 rent per month, how much rent does he pay in 5 years ?

12. At $16.25 a month, what is the rent of a house per year ?

13. How many acres are there in 100 square miles, each square mile containing 640 acres ?

14. What cost 100 dozen eggs, if each egg costs 2 cents ?

15. What is the cost of 100 square miles, each containing 640 acres at $10 ?

16. How much will 100 boxes of eggs cost, each box containing 120 eggs at $.01 ?

17. If a merchant gains $12.50 on each overcoat, how much will he gain on 120 overcoats ?

MISCELLANEOUS PROBLEMS.

116. Oral Problems. 1. What is the entire cost of 6 pounds of coffee at $.30, and 6 pounds of sugar at $.05 ?

2. Thomas gave $1 in payment for 4 pounds of butter at $.24 a pound. What change was due him ?

3. A lady had $10 ; she spent $4.50 for a pair of shoes ; $1.50 for a calico dress, and $1 for a pair of gloves. How much money did she have left ?

4. Mrs. Andrews is 56 years old, and her daughter 26 years old. How much older is the mother than the daughter ?

5. Frank walked 23 miles one day and 24 another day. How far did he walk in the two days ?

6. Mrs. Carnes spent $.37 for ribbon and $.98 for muslin. How much did she spend in all ?

7. Edgar earns $18 a week and spends $12 a week. How much does he save in 8 weeks ?

8. A piece of work can be done by 14 men in 3 days. In how many days can one man do the work ?

9. A lady spent $.71 and then had $.97 left. How much did she have at first ?

10. A merchant bought cloth for $1.68 a yard and sold it at a loss of $.49. What was the selling price ?

11. A wagon cost $125. A horse cost $98 more than the wagon. How much did the horse cost ?

12. How much does a merchant make by selling at $.13 a yard, 12 yards of muslin that cost $.12 a yard ?

13. If one pipe discharges into a cistern 15 gallons of water in a minute, while a second pipe discharges from it 12 gallons in a minute, how many gallons of the water will there be in the cistern at the end of 20 minutes ?

14. Ellen bought 2 pounds of butter at $.34 a pound, and a pound of cheese for $.12. She gave in payment $1. How *much change was due her ?*

117. Written Problems. 1. Mr. Mason's expenses for one year were: $2,100, rent; $1,750, food; $1,860, furniture; $216, fuel; $1,500, horses; and $3,300, help. What was the amount of his expenses?

2. The sun is how many times as large as the moon, if it is 1,384,500 times as large as the earth, and the earth is 45 times as large as the moon?

3. Five men entered into partnership investing respectively $6,500; $7,050; $5,981; $4,879; and $7,007. How much money altogether did they invest?

4. If a steamer starts on a voyage carrying 4,000,000 pounds of coal, how many pounds will it have left after burning 325 tons of 2,000 pounds each for 5 days?

5. Mr. Harris paid on a debt the following sums: $2,575; $1,983; $2,065; and $3,400. He then owed $17,293. How much did he owe at first?

6. How many miles has a train still to travel to make 575 miles, after travelling a mile a minute for 5 hours and 30 minutes?

7. Mr. Franklin bought 196 yards of cloth at $3.50 a yard, and 250 yards at $3.75 a yard. He sold all for $2,000. How much did he gain?

8. Mrs. Banks owns a house worth $21,900; a farm, $16,350; cattle, $7,835; and stock, $105,000. What is the entire value of the property?

9. How much does Mr. Wheeler make by buying 1,500 bales of cotton at $175 a bale, and selling it at $205 a bale?

10. A lady bought 45 yards of muslin at $.18 a yard; 12 yards of calico at $.08 a yard; and she gave the merchant in payment two $5 bills. How much change is due her?

11. A merchant bought 500 barrels of potatoes at $.85 a *barrel. He sold* 275 barrels at $1.15 a barrel and the remain*der at $1.25* a barrel. How much did he gain?

12. Mr. Benjamin earns $4,500 a year and he spends $2,850 a year. How much does he save in 15 years?

13. What is the entire weight of five loads of coal weighing respectively, 1,987, 1,893, 2,005, 2,104, and 2,155 pounds?

14. A farmer paid $5,000 for a farm; $2,700 for stock; $3,287 for a building. He sold all for $10,000. How much did he lose?

15. Mr. Hall in buying his house for $7,286 and a stable with horses for $1,500, paid for them with a $10,000 bill. How much money is due him?

16. A gentleman bought a saddle horse for $225 and two carriage horses for $175 each. He sold the three horses for $700. Did he gain or lose, and how much?

17. What is the result if seven hundred thousand times 75 is subtracted from seven million?

18. Mr. Benson bought 500 barrels of potatoes at $.80 per barrel; 350 barrels of apples at $1.75 per barrel, and 125 barrels of flour at $4.50 per barrel. How much did they cost him altogether?

19. Charles earned $750 a year. He spent $250 for board; $150 for clothing, and $98 in pleasure. He saved the remainder of his salary. How much did he save?

20. If Mr. Mason buys cloth at $.95 per yard and sells it at $1.12 per yard, how much will he make on 100 yards?

21. What is the entire value of 15 car-loads of wheat, each car containing 400 bushels at $.87 a bushel?

22. A real estate broker sold 1,500 acres of land at $22 per acre. The land cost him $25,000. What was his profit?

23. If 100 horses are bought at $75 a head and sold for $10,000, what is the profit?

24. Mr. Johnson bought 1,300 barrels of potatoes at $.85 a barrel. He sold 800 at $1.50 a barrel, and the remainder at $1.20 a barrel. How much did he gain?

25. What is the entire cost of 15 tons of coal at $5.25 a ton and 4 cords of wood at $2.50 a cord ?

26. A merchant bought grain for $9,500 ; live stock for $12,225 ; and land for $23,060. He sold at a gain of $125 on the grain, $3,050 on the live stock, and $14,500 on the land. What was the amount of his sales ?

27. A broker bought potatoes for $1,500 ; apples for $975, and flour for $2,250. He sold, gaining $275 on the potatoes, and $125 on the apples, and losing $500 on the flour. How much money in all did he receive ?

28. If a coal merchant buys 350 tons of coal at $3.50 per ton, and sells 80 tons at $4, 45 tons at $2.75, and the remainder at $4.50, how much does he gain ?

29. A merchant bought 125 barrels of potatoes at $.90 and 360 barrels at $1.05. He sold all for $500. How much did he gain ?

30. Two trains starting at the same time and place travel in the same direction at the rate of respectively 55 miles and 20 miles an hour. How far apart will they be at the end of the first day ? How far, if in opposite directions ?

31. A tenant paid $14.75 a month for rent. He spent $15 for repairs and $12 for fixtures. What did the tenement cost him for the year ?

32. A merchant bought 100 barrels of flour at $4.25 a barrel, and 150 barrels at $4.50. He sold them all for $1,355. How much was his gain ?

33. A farmer exchanged a farm containing 1,400 acres of land, valued at $40 an acre, for 30 city lots valued at $2,000 each. Did he gain or lose, and how much ?

34. A merchant had $10,000. He bought 600 barrels of flour at $4.50 a barrel, and 1,700 bags of wheat, each containing [illegible] bushels, at $1.50 a bushel. How much money did he have after paying the bill ?

DIVISION.

118. 1. How many hats at \$6 each can be bought for \$72?

2. If 5 horses cost \$450, what does one horse cost?

3. How many 4's are there in 480? In 4,800?

4. In 70 days how many weeks are there? In 700 days?

5. If a steamship travels 180 miles in 9 hours, how many miles per hour does it travel?

6. Edgar bought 12 slates for \$.60. What did one cost?

119. To **divide** one number by another is to find how many times the one number contains the other.

120. Division is a short method of finding how many times one number can be subtracted from another; or, how many times the one number is contained in the other number.

Thus: How many guns, at \$9 each, can be bought for \$27?

\$27 — \$9 — \$9 — \$9; or $\frac{\$9)\$27}{3}$.

121. The **dividend** is the number *to be divided.*

122. The **divisor** is the number by which to divide.

123. The **quotient** is the result obtained by dividing.

124. An **integer** is a *whole* number; that is, a number made up of *undivided* units.

Divisor. Dividend. \$9)\$27

Quotient 3 times.

125. If a unit is separated or divided into parts, each part, or any number of parts, is a **fraction.**

126. The signs of division are: ÷,), and ——. The signs are read *contains* or *divided by,* the number at the left of the first, at the right of the second, or above the third sign *containing* or being *divided by* the number on the other side of the sign.

Thus: 81 ÷ 9; 9)81; $\frac{81}{9}$ are read 81 *divided by* 9, or 81 *contains* 9; the second form is sometimes read 9 *in* 81; and the second and *the third forms are also* read one *ninth of* 81.

127. Copy, and commit to memory:

4	4	4	4	4	4	4	4	4	4
1	2	3	4	5	6	7	8	9	10
4)4	4)8	4)12	4)16	4)20	4)24	4)28	4)32	4)36	4)40
1	2	3	4	5	6	7	8	9	10

128. Construct similar tables for the other numbers to and including 12, and extend each to quotient of 12.

129. SLATE AND SIGHT EXERCISES. **Find quotients:**

2)4	2)6	2)8	2)10	2)12	2)14	2)16	2)18	2)20
3)6	3)9	3)12	3)15	3)18	3)21	3)24	3)27	3)30
4)8	4)12	4)16	4)20	4)24	4)28	4)32	4)36	4)40
5)10	5)15	5)20	5)25	5)30	5)35	5)40	5)45	5)50
6)12	6)18	6)24	6)30	6)36	6)42	6)48	6)54	6)60
7)14	7)21	7)28	7)35	7)42	7)49	7)56	7)63	7)70
8)16	8)24	8)32	8)40	8)48	8)56	8)64	8)72	8)80
9)18	9)27	9)36	9)45	9)54	9)63	9)72	9)81	9)90
10)20	10)30	10)40	10)50	10)60	10)70	10)80	10)90	10)100
11)22	11)33	11)44	11)55	11)66	11)77	11)88	11)99	11)110
12)24	12)36	12)48	12)60	12)72	12)84	12)96	12)108	12)120

12)132 12)144 2)22 2)24 3)33 3)36 etc

130. Oral Problems. 1. Mr. Frank earned $49 in 7 days. How much did he earn per day ?

2. If James buys oranges at 7 cents each, how many can he buy for $.49 ?

3. A merchant sold 7 yards of cloth for $4.90. How much did he receive per yard for the cloth ?

4. If a thousand bricks are sold for $7, how many can be bought for $4,900 ?

5. A man can build a wall in 350 days. In what time can 5 men build the wall ?

6. If $2,700 pays for 9 city lots, what is each lot worth ?

7. What number multiplied by 8 equals 64,000 ?

8. In 9 barrels of oil there are 270 gallons. How many gallons are there in each barrel ?

9. If there are desks for 480 pupils in 12 rooms, there are desks for how many pupils in each room ?

131. Illustrate the PRINCIPLES OF DIVISION by using the three following forms:

$4)$24	4)$24	4)24
6	$6	6

I. *The dividend is a concrete or an abstract number.*

II. *The divisor is an abstract number, unless the dividend is a concrete number ; and, then, the divisor is an abstract number, or a concrete number the same in kind as the dividend.*

III. *The quotient is concrete, if the dividend is concrete and the divisor abstract ; otherwise, the quotient is abstract.*

IV. *Multiplying or dividing the dividend by a number, respectively multiplies or divides the quotient by the same number.*

V. *Multiplying or dividing the divisor by a number, respectively divides or multiplies the quotient by the same number.*

VI. *Multiplying or dividing both dividend and divisor by the same number produces no effect on the quotient.*

132. Read quotients at sight:

2	8	14	4	16	10	2	18	12	24	6	22	20
3	12	6	24	3	21	30	9	27	36	15	33	18
4	20	28	12	48	16	44	4	40	24	32	8	36
5	15	40	25	55	5	60	30	50	10	20	45	35
6	24	54	18	48	30	66	42	60	6	72	36	12
7	14	42	63	7	84	77	49	21	56	28	70	35
8	32	64	24	80	56	40	88	72	16	96	48	8
9	27	63	18	81	108	54	90	9	36	99	72	45
10	50	30	120	80	110	20	100	60	40	90	10	70
11	77	55	99	33	121	66	22	110	88	44	132	11
12	48	96	12	72	120	144	36	132	60	24	108	84

Each number in the first column at the left is a divisor for all the numbers on a line with it at its right.

133. Construct problems, and find the number of items bought for each of the following sums:

Per Pound.	Per Barrel.	Per Yard.
1. \$.54 @ 6¢.	1. \$81 @ \$9.	1. \$1.10 @ 10¢.
2. \$.96 @ 12¢.	2. \$108 @ \$12.	2. \$1.32 @ 11¢.
3. \$.72 @ 9¢.	3. \$64 @ \$8.	3. \$1.20 @ 12¢.
4. \$.54 @ 9¢.	4. \$121 @ \$11.	4. \$1.44 @ 12¢.

134. Find the price per item in each of the following:

Quarts.	Yards.	Books.
1. 9 for \$.63.	1. 10 for \$1.00.	1. 8 for \$.96.
2. 11 " \$1.10.	2. 9 " \$.81.	2. 12 " \$.84.
3. 8 " \$.72.	3. 7 " \$84.	3. 11 " \$1.21.
4. 12 " \$1.32.	4. 11 " \$99.	4. 12 " \$.72.

135. Oral Problems. 1. How many yards are there in 2,700 feet, with 3 feet in each yard?

2. In 4,800 inches, how many feet are there?

3. At $5 a barrel, how many barrels of flour can be bought for $250? For $2,500? For $25,000?

4. If a steamer travels 20 miles per hour, in how many hours will it travel 80 miles?

5. How much cloth at $4 per yard can be bought for $1,600? For $2,000?

6. At 9 cents per pound, how many pounds of sugar can be bought for $7.20? For $72?

7. Edgar saved $.12 a day. In how many days can he save $12? In how many days can he save $24?

8. How many times does a wheel 11 feet in circumference turn, in rolling 2,200 feet?

9. In what time will one railroad train, traveling 42 miles an hour, overtake another traveling at the rate of 30 miles an hour, but 108 miles in advance?

10. In what time will two trains 500 miles apart meet, if they travel toward each other at the respective rates of 60 and 40 miles per hour?

11. How many hundred lemons at 2 cents for each lemon can be bought for $24?

12. Mr. Richards buys flour at $4 per barrel, and sells it at $6 per barrel. How many barrels does he sell to make $200? To make $500?

13. A gentleman buys 12 houses for $96,000. How much does he pay per house?

14. At $600 per year, how much is a man's rent per month?

15. If a man's expenses are $132 in 12 days, how much are they per day?

16. A merchant makes $1.44 on 12 yards of cloth. How much *does he make on each* yard?

17. How many bins containing 12 tons each can be filled from 132 tons of coal?

18. If a horse trots 121 miles in 11 hours, how many miles per hour does he trot?

19. If 10 drawing books can be bought for $1.10, how much does each book cost?

136. Read at sight the number of times:

in	in	in	in	in	in
4)24	**5**)25	**6**)30	**7**)42	**8**)24	**9**)18
36	50	42	35	56	54
44	20	24	49	80	99
16	40	36	63	32	36
28	15	48	28	72	90
20	35	72	56	48	72
32	45	18	77	96	27
12	55	66	70	88	81
40	30	54	21	40	63
48	60	60	84	64	45

137. Read the quotients and remainders obtained by dividing the numbers in each of the following lines by the divisor at its left, and by each succeeding divisor to 12.

in										
5)	50	51	52	53	54	55	56	57	58	59
6)	60	61	62	63	64	65	66	67	68	69
7)	70	71	72	73	74	75	76	77	78	79
8)	80	81	82	83	84	85	86	87	88	89
9)	90	91	92	93	94	95	96	97	98	99
10)	100	101	102	103	104	105	106	107	108	109
11)	110	111	112	113	114	115	116	117	118	119
12)	120	121	122	123	124	125	126	127	128	129

thus, 10, 10 and 1 over, 10 and 2 over, 11, 11 and 1

SHORT DIVISION.

138. To find the quotient when the divisor is not greater than 12.

1. Find the quotient of $68.58 divided by 9.

9)$68.58
$7.62

Explanation.—1. Write the divisor at the left of the dividend with a curved line between them; and draw a straight line under the dividend.

2. 9 is contained in 68 (dollars) 7 times, with 5 (dollars) remainder. Write 7 under the units of dollars, with a dollar sign before it.
3. The 5 dollars remainder with 5 dimes make 55 dimes; 9 is contained in 55 (dimes) 6 times, 1 (dime) remainder. Write 6 under dimes.
4. The 1 dime remainder with 8 cents makes 18 cents. 9 is contained in 18 (cents), 2 times. Write 2 under the cents. Answer, 7 dollars, 6 dimes, 2 cents; or $7.62.

NOTE.—In practice, simply: 9 in 68, **7**; in 55, **6**; in 18, **2**.

2. Find the quotient of $7308 divided by $12.

$12)$7308
609

Explanation.—1. Write the divisor at the left, etc.

2. $12 is contained in 73 (hundred) dollars 6 times times with 1 (hundred) remainder. Write 6 under the hundreds.
3. The 1 hundred dollars remainder with 0 tens of dollars make 10 tens of dollars. $12 is not contained in 10 (tens of) dollars. Write 0 under the tens of dollars.
4. The 10 tens of dollars with 8 dollars make 108 dollars. $12 is contained 9 times in $108. Write 9 under the units of dollars. Answer, 6 hundreds, 0 tens, 9 units; or 609.

3.	4.	5.	6.	7.
8)$95.60	9)$12.42	7)$46.55	12)$99.72	9)$10.44
8.	**9.**	**10.**	**11.**	**12.**
12)$40.75	10)$27.90	5)$88.65	6)$37.42	8)$70.20

NOTE.—Any remainder in the answer may be indicated thus, Ex. 8. $3.39 +.

139. Find the quotients:

5)9,876	**6**)10,536	**7**)500,960	**8**)1,000,000
3,045	4,078	38,075	875,875
12,960	12,802	426,097	1,200,600
4,388	3,979	80,763	400,560
9)300,840	**10**)1,200,643	**11**)1,686,423	**12**)5,505,600
25,079	578,093	370,075	392,725
106,809	1,470,902	1,206,430	1,000,860
45,090	300,036	587,096	470,325

LONG DIVISION.

140. Inductive Oral Problems. 1. How many city lots at $690 can be bought for $2,100 ?

The divisors and the dividends are considered each as the nearest number of tens, hundreds, or thousands, etc., and then the quotient is found as in *short division*. Disregard the remainders.

2. If Edith has $15, how many books at $1.95 each can she buy ? How many at $2.05 ?

3. Mr. Charles has $24,100. How many houses costing $5,200 each will his money purchase ?

4. Frank had $.60 with which to buy drawing books. The books cost $.18 each. How many could he buy ?

5. At $6.75 a pair, how many pairs of shoes can be bought for $22 ? For $15 ?

6. If readers are worth $.48 each, how many can be bought for $2.50 ?

7. For how many passengers at $8.50 each will $90 pay ?

8. How many tons of coal at $5.75 can be bought for $50 ? For $60 ? For $18 ?

9. At $.75 a dozen, how many dozen rose buds can be bought for $5.00 ? For $3.25 ?

10. If each acre of land costs $110, how many acres can be bought for $4,400 ?

11. In a tract of land there are 6,100 acres. How many farms of 500 acres each can be made out of the tract ?

12. At $195 each, how many horses can be bought with $12,000 ?

141. Read the quotients, but not the remainders, in the following :

20)41	**32**)51	**41**)42	**39**)41	**18**)40
63	65	82	80	60
89	60	81	67	42
31	63	75	91	32
79	97	83	78	79
99	95	98	95	90
105)740	**210**)640	**407**)408	**507**)614	
550	610	829	963	
945	850	814	1460	
930	890	811	2020	
606)608	**425**)430	**396**)346	**295**)276	
1212	849	490	605	
1015	852	889	822	
1815	837	1125	970	

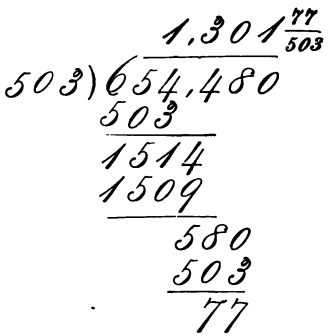
Thus: consider 18 as 2 tens and then divide 4, 6, 4, 3, 8, and 9 tens successively by 2 tens. The divisor thus considered is called a ***trial divisor.*** In dividing by 407, close inspection shows that while of 829 and 814, each contains the divisor 2 times, 811 contains it but once.

142. Read at sight the quotients only :

How many times.	How many times.	How many times.
1. $.50 in $1.70 ?	*5.* $2.10 in $15.00 ?	*9.* $1,010 in $2,500 ?
2. $.40 in $.90 ?	*6.* $1.95 in $14.00 ?	*10.* $3,975 in $16,000 ?
3. $.40 in $1.00 ?	*7.* $2.05 in $16.00 ?	*11.* $4,907 in $25,000 ?
4. $.30 in $1.*60* ?	*8.* $4.50 in $10.00 ?	*12.* $5,001 in $25,000 ?

143. To find the quotient when the divisor is expressed by any number of figures.

1. Find the quotient of 654,480 divided by 503.



Explanation. — 1. Write the divisor at the left of the dividend with a curved line between them. Draw above the dividend a straight line above which to write the quotient.

2. 503 is contained in 654 (thousands) 1 time with 151 remainder. Write 1 in the quotient above the thousands and 151 as a remainder under the thousands.

3. The 151 thousands with the 4 hundreds of the dividend make 1514 hundreds. 503 is contained in 1514 (hundreds) 3 times, with a remainder of 5. Write 3 in the quotient over the hundreds.

4. The 5 hundreds with the 8 tens of the dividend make 58 tens. 503 is not contained in 58 (tens). Write 0 in the quotient over the tens.

5. The 58 tens remainder with the 0 units of the dividend make 580 units. 503 is contained in 580 (units) 1 time with 77 units remainder. Write 1 in the quotient over the units ; and write 77 at the right of the quotient and the divisor under it, with a horizontal line between them. Answer, 1 thousand, 3 hundreds, 0 tens, and 1 unit, with a remainder of 77 ; or, $1,301\frac{77}{503}$.

Ex. 1, p. 51.

762
9)6858
63
55
54
18
18

NOTE 1.—If any partial product is greater than the partial dividend, the quotient is too great, and a smaller number for the quotient must be tried.

NOTE 2.—The examples on pp. 51 and 52 may be worked in the same form. See Ex. 1, p. 51, as worked above.

3.	4.	5.
57)6,983	87)9,748	95)16,845

144. Rule for Division.—1. Write the divisor at the left of the dividend, with a curved line between them. Draw a horizontal line under the dividend, if the divisor is not greater than 12; or above the dividend, if the divisor is greater than 12.

2. Divide in succession, the number of units in the various orders of the dividend, beginning at the left, by the number of units in the divisor; and, beginning with the first significant result, write the result of each division on the other side of the horizontal line, in the quotient, above or below the order of the dividend divided.

3. Multiply the divisor by the number of units in the order of the quotient just obtained, the order of the product being the same as the order of the dividend just divided.

4. Subtract the products of the divisor by the number of units in the successive orders of the quotient from the corresponding orders of the dividend.

5. Add any remainder thus obtained to the next lower order of the dividend by annexing, at the right of the remainder, the figure of this order before it is divided.

6. Write the final remainder, if any, at the right of the quotient and the divisor under it, with a horizontal line between them.

Proof.—Multiply the divisor and the quotient together and to the product add the remainder. The result should equal the dividend.

145. Find the quotients:

1. **97,** in	*2.* **1,109** in	*3.* **1,247** in	*4.* **3,605** in	*5.* **10,770** in
19,069	10,084	22,748	107,110	206,023
10,704	12,750	90,070	563,412	407,756
3,087	14,065	11,011	700,707	39,069
16,050	*5,900*	2,020	90,900	100,!

146. To divide when there are ciphers at the right of the divisor.

1. Find the quotient of $3,680 divided by 10.

Explanation.—1. Writing the numbers as in short division, the answer is found to be $368, the same as the dividend with the dollar point one place further to the left.

1st.	2d.
10)$3,680	
$368 ;	**or,** $368 | 0

2. Find the quotient of $4,856 divided by 100.

Explanation. — 1. Writing the numbers as in long division, the answer is found to be $48.56, the same as the dividend with the dollar point removed two places to the left.

2. **Or,** remove the dollar point two places to the left; that is, as many places to the left, as there are ciphers in the divisor.

1st.	2d.
$48.56	
100)$4,856.00 ;	**or,** $48 | 56.
4 00	
856	
800	
560	
500	
600	
600	

3. Find the quotient of $652.80 divided by 1,200.

Explanation.—1. Since the significant figures are not greater than 12, the numbers are written as in short division.

1,200)$6\|52.80
$.544

2. Marking off the two ciphers at the right of the divisor, as shown above, divides the divisor by 100; moving the dollar point two places to the left divides the dividend by 100.

3. As the divisor and the dividend have thus both been divided by 100, the work is reduced without changing the result. Prin. VI. The problem now is to divide $6.528 by 12. Answer, $.544.

4.	5.	6.	7.
40)3600	90)7,200	120)14,400	800)7,200
8.	**9.**	**10.**	**11.**
100)$4.650	10)$350.40	1,000)$6,090.	100)$40.50
	13.	**14.**	**15.**
	$40)$365,000	$700)$490,000	$1,200)$1,728,000

147. Rules for dividing by 10, 100, etc.—I. If the divisor is 1 with ciphers annexed, the quotient is found by marking off as many figures at the right of the dividend as there are ciphers at the right of the divisor; or, by moving the dollar point as many places to the left as there are ciphers at the right of the divisor.

II. If the divisor is more than 1 with ciphers annexed, proceed as above, and then divide by the number of units in the significant figures of the divisor.

148. Find the quotients:

560)172,500	**25,060**)100,000	**14,600**)9,060,400
49,060	506,800	807,500
300,700	44,070	1,050,000
87,050	300,900	407,070
100,000	88,000	5,000,000

149. Oral Problems. 1. How many weeks are there in 84 days? In 63 days? In 49 days?

2. How many acres of land at $25 an acre can be bought for $250. For $275? For $325?

3. What is the cost price per yard, if by selling 9 yards for $.90, a profit of $.09 is made?

4. If 12 men can build a wall in 10 days, in how many days can 10 men build the wall?

5. How many times does a wheel 12 feet in circumference turn in moving 2,400 feet?

6. A man earns $45 and spends $30 in a week. In what time will his savings amount to $75?

7. In what time will two men, 100 miles distant, meet, if they travel toward each other, one at the rate of 15 miles an hour, and the other at the rate of 10 miles an hour?

8. If a merchant makes $.18 by selling 9 yards of ribbon at $.12 a *yard, how much* did the ribbon cost him a yard?

9. If Mr. Nathan's wages are $18 a week, and his expenses $12, how long will it take him to save $66 ?

10. If 6 pecks of apples cost $1.20, what will 8 pecks of apples cost ? 10 pecks ? 12 pecks ?

11. How many pounds of butter at 30 cents a pound cost as much as 10 dozen eggs at 3 cents an egg ?

12. If a man earns $6 a day, how long will it take him to earn enough to pay for 18 cows costing $20 each ?

13. In 480 sheets of paper, how many quires of 24 sheets each are there ?

14. How many sheets of paper will be required to make a book of 360 pages, if one sheet makes 12 pages ?

15. In how many hours could a train, moving at the rate of 25 miles an hour, travel 25,000 miles, the distance around the earth ?

16. If there were 25 hours in a day, how many days would there be in 1,000 hours ?

17. A grocer exchanged 12 barrels of flour for 15 tons of coal at $4 a ton. What was the price per barrel of the flour ?

18. If apples cost 3 cents each, and oranges 5 cents each, buying an equal number of both, how many apples and oranges can be bought for $.96 ?

19. If apples cost 2 cents each and oranges 5 cents each, how many oranges are equal in value to 50 apples ?

20. Henry bought 20 newspapers at 2 cents each. He sold 15 at 3 cents each, and 5 at 1 cent each. In how many days at the same rate will he make $.50 ?

150. Written Problems. 1. If the receipts of a railroad company are $20,900 in 100 days, how much are they per day ?

2. In how many days will a ship sail 14,000 miles, if it sails 250 miles each day ?

3. If an army of 46,080 men is divided into 480 equal regi*ments, how many* men are there in each regiment ?

4. A gentleman left an estate valued at $1,050,000 to be divided into 4 equal parts. One of these parts was to be divided equally among 20 charities. How much did each charity receive?

5. In how many minutes will a ray of light travelling at the rate of 186,000 miles per second reach the earth, coming from the sun, a distance of 93,000,000 miles?

6. In how many seconds will thunder travelling at the rate of 1,110 feet a second, travel a distance of 6,290 yards?

7. Mr. Johnson travelled 70,200 yards in a carriage with a wheel 9 feet in circumference. How many turns did the wheel make?

8. If the earth in revolving around the sun moves 574,472,976 miles in a year of 365 days, how far does it move per hour?

9. A wholesale grocer's sales amounted to $1,505,690 in a year. What was the average per day, counting 310 business days to the year?

10. If there are 625 square miles in the island of Zanzibar, and 394,240 square miles in Egypt, how many times as large as Zanzibar is Egypt?

11. Mecca contains a population of 40,000 people, and Calcutta 871,504. Calcutta contains how many times as large a population as Mecca?

12. The Yenisei river is 3,400 miles long, and the Saghalien, 514 miles. How many rivers as long as the Saghalien would it take to make one equal in length to the Yenisei?

13. How many times as large is the value of the diamond export from Africa in one year, $13,712,350, as of pineapples, $2,110, in the same time?

14. In 1880, the capital in Pennsylvania invested in glass ware was $3,978,406, and in New Jersey, $310,000. How many times the capital in New Jersey was invested in industry *in Pennsylvania?*

151. Copy, and complete the following bills:

I. Bill of items bought at different times.

NEW YORK, *May* 12, 1889.

NEW YORK BAKING COMPANY,

To HENRY ARMSTRONG, DR.

1889					
Feb. 1	To 40 bbl.	Flour, ...	@	$4.00	$
" 5	" 15 "	Sugar, ...	@	7.50	
Mch. 3	" 75 gal.	Molasses, . . .	@	.15	
" 27	" 9 bbl.	Potatoes, . . .	@	.90	
April 14	" 5 bu.	Apples, ...	@	.75	
					$

Received payment,

HENRY ARMSTRONG.

II. Receipted bill for goods bought of a company.

BOSTON, *April* 9, 1889.

MESSRS. BROWN BROS.,

Bought of WORTHINGTON MILLS CO.

[Limited.]

1889					
Jan. 16	1,000 yd.	Sheeting, . . .	@	$.15	$
" 29	375 "	Gingham, . . .	@	.17	
Feb. 7	250 "	Muslin, . . .	@	.09	
Mch. 22	50 gross	Handkerchiefs, .	@	21.00	
					$

Paid July 12, 1889.

WORTHINGTON MILLS CO.

per C. M. Johnson,

Pres't.

MISCELLANEOUS PROBLEMS.

152. Oral Problems. 1. What is the entire cost of a pair of boots, $8; a pair of trowsers costing $4 more than the boots; a frock coat costing $4 more than the trowsers, and a spring overcoat costing $4 more than the frock coat?

2. How many gallons of milk, 4 quarts each, will be required to fill 8 pails, each holding 6 quarts?

3. A book was sold for $.63, which was $.12 less than its cost. For how much should it have been sold to gain $.12?

4. If 9 yards of lace cost $1.80, what will 12 yards cost?

5. If 90 men can do a piece of work in 100 days, in what time can 1,000 men do the work?

6. What will 15,000 bricks cost if 1,000 costs $10?

7. How much profit is made by selling 8 houses that cost $10,000 each for $92,000?

8. Jane has $.16; Mary $.15 more than Jane, and Anna as much as Mary and Jane together. How many cents has Anna?

9. How much will 50 eggs cost at $.24 a dozen?

10. If 100 dozen eggs cost $24, what does each egg cost?

11. A dozen oranges bought at the rate of 6 for $.18 were sold for $.40. What was the profit?

12. A boy bought a melon for $.35, and on paying for it he received $.25 change. He should have received $.15 more change. How much money did he offer in payment?

13. If 5 barrels of flour cost $35, what will 70 barrels cost?

14. How high is a house whose three stories are respectively 20, 19, and 18 feet high?

15. At 20 miles an hour, in what time will a train travel 220 miles?

16. Mr. Thomson bought 15 barrels of flour at $5 a barrel. At what price must he sell them to make $30?

17. If 20 men can dig a ditch in 5 days, in what time can 25 *men dig the ditch?*

18. Edith is 14 years old; her sister Emma is 16 years older than Edith. How old is their father, who is 28 years older than Emma?

19. If 2 barrels of potatoes cost $1.80, how many barrels can be bought for $4.50?

20. When will two horses starting from the same point at 9 o'clock A.M. be 150 miles apart, if they travel 8 and 7 miles an hour in opposite directions?

21. If two horses start at the same place and trot in the same direction, one at the rate of 7 miles an hour, and the other at the rate of 9 miles an hour, when will the one horse be 12 miles in advance of the other?

22. If a steamer can travel 20 miles an hour in still water, how long will it require to travel 100 miles down a river whose water flows 5 miles an hour?

23. How much time will a steamer that travels at the rate of 15 miles an hour in still water require to travel 200 miles up a river whose water flows 5 miles an hour?

24. A farmer bought 6 cows for $100. He sold so as to gain $20 on three cows together and to lose $5 on each of the three remaining cows. What was the amount of his sales?

153. Written Problems. 1. If the receipts of a street railroad are $25,600 every 5 days, how much will they be in 5 months of 30 days each?

2. In how many days will two ships starting at the same point and sailing in opposite directions be 2,560 miles apart, if one ship sails 20 miles each hour, and the other 15 miles?

3. A coal dealer bought 350 tons of coal, weighing 2,240 pounds each, at $3.50 a ton. He sold the coal at $4.25 a ton, each ton weighing 2,000 pounds. What was his profit?

4. Mr. Worth owes a debt of $4,200. His salary is $1,500 a year, and his expenses $75 a month. In what time can he pay the debt?

5. If a house rents for $900 a year, how much is the rent for 7 months? For 9 months?

6. If a man earns $2,900 a year and spends $125 every month, in what time can he pay a debt of $2,500 and buy a farm for $300?

7. In how many days will two steamers 1,050 miles distant meet if they travel toward each other at the rate of respectively 15 and 20 miles an hour?

8. In what time will the steamers meet, if they travel in the same direction?

9. The divisor was 165; the quotient, 25,060; the remainder, 47. What was the dividend?

DENOMINATE INTEGERS.

154. A compound or denominate number is a number composed of units of different names or denominations; but the same, in kind.

7 yd. 2 ft. 9 in.; 4 bu. 3 pk. 7 qt.; 5 mi. 30 rd. 5 yd. 2 ft. 10 in. are *compound numbers.*

155. Money is any currency of different denominations authorized by law as a medium of exchange.

Money consists of *coins* and *bills.* Coins are pieces of precious metals stamped with their values, as authorized by law. Bills are either *certificates* or *notes.* Certificates represent values in coin, or precious metal in bulk, called *bullion;* as, silver certificates. Notes accepted as money are promises to pay made by a nation, state, or bank. Checks, drafts, and notes of individuals, although constituting the greater part of the currency of the world, are not money.

156. United States Money is a decimal currency; that is, ten units of one order or denomination make one unit of *the next higher order* or denomination.

157. The *coins* in general use in the United States are the following: gold, $10 (eagle), $5, $2.50; silver, $1, $.50, $.25, $.10; nickel, 5 ¢, 3 ¢; bronze, 2 ¢, 1 ¢. The *bills* in general use in the United States are the following: Silver certificates, national bank bills, and United States notes (greenbacks), for $1, $2, $5, $10, $20, $100, $1,000, and $10,000.

158. Copy, and commit to memory the following table:

UNITED STATES MONEY.

10 MILLS (m.)	make	1 CENT, ¢, or ct.
10 CENTS (¢, cts.)	"	1 DIME, d.
10 DIMES	"	1 DOLLAR, $.
10 DOLLARS	"	1 EAGLE, E.

Expressions of United States money may be treated as simple or as compound numbers; either as composed of units of *one denomination:* the dollar, the dime, or the cent, etc.; or as composed of units of several denominations. They are generally treated as simple numbers.

159. English Money is not a decimal currency; nor, is its scale of units constant.

160. Copy, and commit to memory the following table:

ENGLISH MONEY.

4 FARTHINGS (far.)	make	1 PENNY, d.
12 PENCE	"	1 SHILLING, s.
20 SHILLINGS	"	1 POUND, OR SOVEREIGN, £.
21 SHILLINGS	"	1 GUINEA.

The florin = 2 shillings; the crown = 5 shillings; £1 = $4.8665.

161. The coins in general use are the following: gold, sovereign and half sovereign; silver, crown, half-crown, florin, shilling, and six-pence; copper, penny, half-penny, and farthing. The bills in general use are bank of England notes, the lowest denomination being £5.

hanges are frequently made thus: £1 = $5; crown own = $.625; florin = $.50; shilling = $.25; six- penny = 2 ¢; half-penny (ha' penny) = ½ ¢.

162. French Money is a decimal currency.

163. Copy, and commit to memory the following table:

FRENCH MONEY.

10 MILLIMES (m.)	make	1 CENTIME,	ct.
10 CENTIMES	"	1 DECIME,	dc.
10 DECIMES	"	1 FRANC,	fr.

20 francs = 1 Louis ; 1 fr. = $.193.

164. The coins in general use are the following : gold, 20 francs, 10 francs, 5 francs ; silver, 5, 2, and 1 franc ; also, 50 and 25 centimes ; bronze, 10, 5, 2, and 1 centime.

In small sums, exchanges are frequently made, thus : 25 fr. = $5 ; 5 fr. = $1 ; 1 fr. = $.20 ; 50 ct. = $.10 ; 25 ct. = $.05.

165. The unit in **German Money** is the **mark,** equal to 23.85 cents in U. S. Money.

The principal coins are the 20-mark gold piece (about $5), and the silver mark (about $.25).

REDUCTION DESCENDING.

166. Reduce or change the following sums of United States money:

to dimes.	to dimes.	to cents.	to cents.	to mills.
1. $5.00	*5.* $5.50	*9.* $5.00	*13.* $5.50	*17.* $5.00
2. $50.00	*6.* $37.40	*10.* $50.00	*14.* $37.40	*18.* $37.40
3. $45.00	*7.* $45.60	*11.* $45.00	*15.* $45.66	*19.* $45.66
4. $97.00	*8.* $99.10	*12.* $97.00	*16.* $99.12	*20.* $99.12

Since 1 of any order in U. S. money, as in simple (*decimal*) numbers, equals 10 of the next lower order, the answers may be found by multiplying by 10, 100, etc. ; or, by moving the dollar mark (*decimal point*) 1, 2, etc., places to the right.

167. Reduction descending is changing a denominate number *to units of a lower* denomination.

168. Oral Problems. 1. Charles has two English gold coins, or sovereigns. To how many shillings, silver coin, are they equal ?

As there are 20 shillings in each sovereign, there are 20 times as many shillings as sovereigns in any sum of money.

2. Frances has a £5 Bank of England note. To how many shillings is it equal ?

3. Edgar bought a bicycle for £5. 5s. How many shillings did it cost him ?

4. Mr. Brown paid £18. 8s. to cross the Atlantic Ocean. How many shillings did his passage cost him ?

5. Henry bought a cane for 5s. 6d. How many pence did it cost him ?

5s. = 60d.; 60d. and 6d. = 66d.

6. James stopped at a hotel paying 7s. 10d. per day. How many pence per day was the bill ?

7. At four pence half-penny (2 far.), how many farthings will a bottle of mucilage cost ?

8. In £5. 5s. 5d., how many pence are there ?

9. How many pence will a hat for £1. 10s. 6d. cost ?

169. Rule for Reduction Descending.—1. Multiply the number of units of the highest denomination in the multiplicand by the number that will reduce it to the next lower denomination ; and to the product add the number of units, if any, of this lower denomination in the multiplicand.

2. Multiply the result by the number that will reduce it to the next lower denomination, adding the number of units of this [illegible]omination in the multiplicand, if any, to the [illegible]

[illegible]n like manner, until the required denomination is reached.

170. Reduce the following sums of English money:

Illustration.

£.	s.	d.	far.	
	5.	5.	.	3.
	20			
	105	. . .		(20 × 5) + 5 s.
	12			
	1,260	. . .		d.
	4			
Ans.	5,043	. . .		(4 × 1,260) + 3 far.

	to shillings. £. s.	to pence. £. s. d.	to farthings. £. s. d. far.
1.	50. 5.	6. 37. 10. 6.	11. 5. 5. 0. 3.
2.	125. 10.	7. 25. 5. 9.	12. 10. 16. 8. 2.
3.	86. 9.	8. 42. 0. 3.	13. 4. 0. 5. 1.
4.	255. 8.	9. 50. 18. 9.	14. 15. 15. 10. 2.
5.	760. 6.	10. 63. 19. 11.	15. 25. 12. 9. 3.

REDUCTION ASCENDING.

171. Reduce or change the following sums of United States money:

to dimes.	to dimes.	to dollars.	to dollars.	to cents.
1. 50 ¢.	3. 50 m.	5. 50 d.	7. 50 ¢.	9. 50 m.
2. 55 ¢.	4. 55 m.	6. 55 d.	8. 55 ¢.	10. 55 m.

Since 10 of any order in United States money, as in simple (*decimal*) numbers, make 1 of the next higher order, the answers may be obtained by dividing by 10, 100, etc.; or by moving the dollar mark (*decimal point*) 1, 2, etc., places to the left.

172. Reduction ascending is changing a denominate number to units of a higher denomination.

173. Oral Problems. 1. Thomas paid 60 shillings for an overcoat. How many pounds did it cost him?

The number of pounds is one-twentieth of the number of shillings.

2. John's board and lodging cost him 120 shillings per month. How many pounds was this?

3. Matthew earned 55 shillings in a week. How many pounds did he earn? 20)55s. Ans.: £2. 15s.

4. A straw hat cost 63 pence. How many shillings did it cost? 12)63d. Ans.: 5s. 3d.

5. If a silk hat costs 262 pence, how many pounds, shillings, and pence does it cost? 12)262d. Ans.: 21s. 10d.; or £1. 1s. 10d.

6. Mr. Johnson's rent is 908 pence per month. What is his rent in *pounds*, shillings, and *pence*? 12)908d. 20)75s. 8d. Ans.: £

174. Rule for Reduction Ascending.—1. Divide the dividend of any denomination by the number of units that will reduce it to the next higher denomination. Write the remainder as units of the same denomination as the dividend of which it formed a part.

2. Divide the quotient by the number that will reduce it to the next higher denomination, and write the remainder as before.

3. Proceed in like manner, until the required denomination is obtained, writing the last quotient and the preceding remainders as the answer.

175. Reduce the following sums of English money:

Illustration.	to pence.	to shillings.	to shillings.	to pounds.
4)1,000 far.	*1.* 100 far.	*5.* 500 d.	*9.* 1,000 far.	*13.* 1,000 far.
12)250 d. + 0 far.	*2.* 150 "	*6.* 150 "	*10.* 1,565 "	*14.* 660 d.
20)20 s. + 10 d.	*3.* 500 "	*7.* 675 "	*11.* 2,080 "	*15.* 660 s.
1 £ + 0 s.	*4.* 820 "	*8.* 1,500 "	*12.* 1,860 "	*16.* 1,606 s.
Ans. £1. 0 s. 10 d.				

ADDITION AND SUBTRACTION.

176. 1. A grocer sold 4 pounds 8 ounces of sugar to one customer, and 3 pounds 10 ounces to another customer. How much sugar did he sell to both customers?

7 pounds 18 ounces; or, as 16 ounces equal 1 pound, 8 pounds 2 ounces. Ans.

2. A jeweller bought two silver spoons; one weighing 2 ounces 15 pennyweight; and the other, 3 ounces 10 pennyweight. How much did both weigh?

5 ounces 25 pennyweight; or, as 20 pennyweight equal 1 ounce, 6 ou· ·yweight. Ans.

·ry had 5 pounds 10 ounces of flowers of sul· ·ounds 8 ounces. How much was left?

Weight.

177. Avoirdupois Weight is used in weighing all substances except drugs at retail, and except gold, silver, and precious stones.

178. Copy, and commit to memory the following table:

Avoirdupois Weight.

16 ounces (oz.)	make	1 pound, . . .	lb.
25 pounds	"	1 quarter, . . .	qr.
4 quarters, or 100 pounds	"	1 hundredw't, .	cwt.
2,000 pounds	"	1 ton, ...	T.

A *long* ton, 2,240 lbs., is used in buying and selling coal at wholesale; and in estimating duties in the custom-houses.

179. Find the sums:

1. What is the sum of 3 T. 5 cwt. 80 lb.; 2 T. 17 cwt. 75 lb.; and, 5 T. 15 cwt. 50 lb.?

T.	cwt.	lb.	
3	5	80	
2	17	75	
5	15	50	
11	39	205	
11	19	5	Ans.

Explanation.—1. Write the number so that units of the same denomination and order stand in a column, and draw under the numbers two horizontal lines.

2. The sum of the pounds is 205. Write 205 in the column of pounds between the two horizontal lines.

3. Reduce 205 pounds to hundredweight by dividing by 100; that is, by imagining the decimal point removed two places to the left; 205 lb. thus becomes 2 cwt. 5 lb. Write 5 in the column of pounds under the lower horizontal line; and add the 2 hundredweight with the hundredweight in the column of hundredweight.

4. The sum of the hundredweight is 39. Write 39 in the column of hundredweight between the two horizontal lines.

5. Reduce the 39 hundredweight by dividing by 20. 39 cwt. thus becomes 1 T. 19 cwt. Write 19 in the column of hundredweight under the second horizontal line; and add the 1 T. with the tons.

6. The sum of the tons is 11, which write in the column of tons *between and below the* horizontal lines. Ans.: 11 T. 19 cwt. 5 lb.

2.			3.			4.			
T.	cwt.	lb.	T.	cwt.	lb.	£.	s.	d.	far.
5	12	90	12	9	45	100	10	10	1
23	19	65	40	89	30	95	18	11	0
17	11	88	28	52	85	225	5	2	3
20	8	72	35	65	70	68	19	0	2

180. Rule for Addition.—1. Write the numbers so that units of the same denomination and order form separate columns.

2. Find the sum of the units of the lowest denomination, and reduce this sum to units of the next higher denomination; writing the remainder, if any, under the column of the lowest denomination; and adding the units of the next higher denominations thus found with the units in the column of that denomination.

3. Proceed, in similar manner, with the units of the different denominations in their order, until the entire sum has been found. The sum of the units of the highest denomination and the remainders of the lower denominations form the answer.

Proof.—*Add each column in the opposite direction and find the same answer.*

181. Troy Weight is used in weighing gold, silver, and precious stones.

182. Apothecaries' Weight is used by retail druggists.

183. Copy, and commit to memory the following tables:

Troy Weight.				Apothecaries' Weight.			
24 grains	make	1 pennyw't, .	pwt.	20 grains .	make .	1 scruple, .	℈.
20 pwt. .	"	1 ounce, . .	oz.	3 scruples .	" .	1 dram, .	ʒ.
12 ounces	"	1 pound, . .	lb.	8 drams . .	" .	1 ounce, .	℥.
				12 ounces .	" .	1 pound, .	lb.

Apothecaries' grain, ounce, and pound are the same; pothecaries' pound contain 5,760 gr.; the Avoirdupois

184. Find the remainders:

1. What is the difference between 5 lb. 3 oz. 9 pwt. 15 gr. of silver, and 3 lb. 8 oz. 11 pwt. 20 gr. ?

lb.	oz.	pwt.	gr.
5	3	9	15
3	8	11	20
1	6	17	19

Explanation.—1. Write the subtrahend under the minuend, so that units of the same denomination and the same order stand in the same column. Draw a horizontal line under the subtrahend.

2. As 20 gr. cannot be taken from 15 gr., 1 pwt. is taken from the 9 pwt., leaving 8 pwt.; the 1 pwt. is changed to gr, making 24 gr., and added to the 15 gr., making 39 gr. 20 gr. from 39 gr. leaves 19 gr. Write 19 under the column of grains.

3. As 11 pwt. cannot be taken from 8 pwt., 1 oz. is taken from the 3 oz., leaving 2 oz.; the 1 oz. is changed to pwt., making 20 pwt., and added to the 8 oz., making 28 oz. 11 pwt. from 28 pwt. leaves 17 pwt. Write 17 under the column of pwt.

4. As 8 oz. cannot be taken from 3 oz., 1 lb. is taken from the 5 lb., leaving 4 lb.; the 1 lb. is changed to oz., making 12 oz., and added to the 2 oz., making 14 oz. 8 oz. from 14 oz. leaves 6 oz. Write 6 under the column of ounces.

5. 3 lb. from 4 lb. leaves 1 lb. Write 1 under the column of pounds.

Ans.: 1 lb. 6 oz. 17 pwt. 19 gr.

2.

lb.	ʒ.	ʒ.	ɔ.	gr.
8	6	5	1	11
3	11	7	2	18
4	6	5	1	13

3.

lb.	oz.	pwt.	gr.
10	9	6	9
5	10	19	21

4.

lb.	ʒ.	ʒ.	ɔ.	gr.
13	1	5	2	4
7	9	6	2	20

5.

lb.	ʒ.	ʒ.	ɔ.	gr.
14	9	6	2	18
12	8	7	1	19

6.

£.	s.	d.	far.
75	11	10	3
69	15	11	3

7.

T.	cwt.	qr.	lb.	oz.
40	17	2	20	11
35	18	3	21	12

8. How much heavier is 18 lb. 0 oz. 9 pwt. 10 gr. of gold, than 15 lb. *5 oz. 12 pwt. 5* gr. of silver ?

185. Rule for Subtraction.—1. Write the subtrahend under the minuend, so that units of the same denomination and units of the same order form separate columns.

2. Beginning with the lowest denomination, subtract the number of units in each denomination of the subtrahend from the number of units of the same denomination in the minuend, and write the result underneath the column.

3. If the number of units of any denomination of the subtrahend is greater than the number of the same denomination in the minuend, to the number in the minuend add one unit taken and reduced from the next higher denomination in the minuend; and remember when subtracting in the next higher order, that the number of that denomination in the minuend has been diminished.

4. If there are no units in the next higher denomination in the minuend, take one from the first higher denomination containing units, and proceed by continued reduction to find the requisite units.

5. The different remainders together form the answer.

Proof. *The remainder added to the subtrahend equals the minuend.*

186. Find the sums:

1.					2.				3.			4.			
lb.	℥.	ʒ.	℈.	gr.	lb.	oz.	pwt.	gr.	T.	cwt.	lb.	£.	s.	d.	far.
25	10	7	2	18	5	0	15	21	15	15	15	100	10	10	0
11	6	5	0	10	11	11	0	19	3	19	99	55	15	11	3
13	9	0	1	12	6	4	12	4	12	0	46	106	19	9	2

5.				6.			7.				8.			
£.	s.	d.	far.	T.	cwt.	lb.	lb.	oz.	pwt.	gr.	lb.	℥.	ʒ.	℈.	gr.
200	12	4	1	20	19	18	8	8	18	18	3	2	7	2	19
30	0	[illegible]		46	14	98	6	0	16	0	12	11	0	1	10
				13	0	50	14	11	0	23	5	10	6	2	0

187. Find the remainders:

1.				2.				3.			4.				
£.	s.	d.	far.	lb.	oz.	pwt.	gr.	T.	cwt.	lb.	℔.	ʒ.	ʒ.	ɔ.	gr.
100	10	10	2	40	10	10	10	19	9	19	4	1	0	2	10
49	19	11	3	15	11	11	21	11	19	29	2	11	6	2	12

5.			6.					7.				8.			
T.	cwt.	lb.	℔.	ʒ.	ʒ.	ɔ.	gr.	lb.	oz.	pwt.	gr.	£.	s.	d.	far.
25	18	67	10	0	6	2	16	5	0	15	15	200	0	0	0
17	19	76	7	11	7	1	19	3	10	16	16	99	19	11	3

MISCELLANEOUS PROBLEMS.

188. Oral Problems. 1. How many shillings are there in £34? In £49? In £107?

2. How many yards are there in 54 ft.?

3. If a Reader costs 1s. 6d., and a slate 3d., what change should be received on paying for them with a florin (2s.)?

4. A coal dealer bought coal by the long ton and sold it by the short ton (of 2,000 lbs.). How many pounds did he save?

5. If every 4 pounds are worth 1 cent, how much does a coal dealer save in saving 240 pounds?

6. If 10 gr. of silver are worth $2\frac{1}{2}$ cents, how much less than $.50 is a silver trinket weighing 100 gr. worth?

7. A druggist had 1 ʒ of quinine and sold 10 gr. How many gr. did he have left?

8. How much of $5 will be left, after paying for beef weighing a quarter (of a cwt.) at $.20 a pound? At $.19?

9. If 4 gr. of silver are worth 1 cent, what is the value of a silver spoon weighing 1 oz.?

10. A jeweller having a small bar of gold weighing 1 lb. removed 15 pwt. How many pwt. were left in the bar?

11. A gentleman travelling abroad gave a $5 gold piece for a £1 *gold piece, or sovereign.* How many cents did he lose?

12. How much would a traveller pay for the exchange, if he exchanges ten $5 gold pieces for 10 sovereigns, the difference in value between the two coins being $.135 ?

13. If a franc were worth $.20, how much would be lost in exchanging a $5 gold piece for a Louis ?

14. A franc being worth $.193, what is the difference in value between a half Louis and a $2.50 gold coin ?

15. What is the cost of 12 powders each containing 1 ʒ at 10 ¢ a ʒ ? At 15 ¢ a ʒ ?

16. A druggist had 1 ℔. of arsenic and sold 1 ʒ. How much did he have left ?

17. What is the united weight of 3 silver spoons, weighing respectively 10 pwt. 1 oz.; and, 1 oz. 10 pwt. ?

18. What is the cost of 3 silver spoons together weighing 3 oz., if 4 gr. of silver are worth $.01 ?

19. A grocer bought 1 cwt. of sugar and sold from it 5 packages each containing 15 oz. How much of the sugar did he have left ?

20. If 8 steaks, each weighing 1 lb. 4 oz., are cut from a hundredweight of beef, how much of the beef is left ?

21. In shopping, in London, a lady purchased three articles costing 5s. 6d., 2s. 3d., and 1s. 3d.; she paid for them with a half sovereign. How much change should she receive ?

189. Written Problems. 1. Mr. Andrews bought a farm for $16,325 ; he made two payments on it of respectively $8,250 and $4,500. How much does he owe on the farm ?

2. A merchant bought 61 rolls of carpet, each containing 72 yards, at $1.75 a yard. He gave in payment a $10,000 check ; how much money should be returned to him ?

3. A coal merchant bought 350 long tons of coal at $3.75 a ton ; he sold all at $4.50 a short ton. What was his gain ?

4. If a silversmith buys 15 lb. of silver at $.05 per pwt., *and sells it at the* rate of 40 gr. for 12 ¢, what is his profit ?

5. A druggist bought 100 lb. of copperas at $.03 per lb.; he sold it all in 8 ⅗ packages at $.10 each. How much money did he make?

6. Mr. Baker made $5,000 by selling 7,500 barrels of flour at $6.25 a barrel. How much per barrel did the flour cost?

7. How many dozen silver teaspoons, each spoon weighing 1 oz., can be bought for $86.40, at $.06 per pwt.?

8. Find the cost of a hundredweight of sugar mixed in equal portions from sugar at 5, 7, 8 and 10 cents per pound.

9. What is the average cost per horse, if 4 horses cost respectively $375, $425, $512, and $688?

10. A paid $25 per acre for his farm of 327 acres; and B paid $19 per acre for his farm of 417 acres. What is the difference in the value of the two farms?

11. What is the cost of the following amounts of coal at $4.62 per T.: 88 T. 1,900 lb.; 43 T. 1,775 lb.; 125 T. 1,225 lb.; 57 T. 1,100 lb.?

12. A gentleman left $2,200 to his wife, $150 to a servant, and the rest of his property in equal shares to his 7 children. The entire estate was worth $13,025. What was each child's share worth?

13. A jeweller owing $2,500 gave in part payment of his debt 100 doz. spoons, each spoon weighing 2 oz., at $.04 per pwt. How much did he still owe?

14. Change nine hundred seventy-four $5 gold coins to English sovereigns at $4.87 each; and with the money pay 5 creditors equal amounts. How many pounds does each creditor receive?

15. What is the difference in grains between 90 pounds in avoirdupois and 90 pounds in Troy weight?

16. From 11 pounds of Troy weight, subtract 11 ounces in *apothecaries' weight.* What is the remainder in grains?

MULTIPLICATION AND DIVISION.

190. 1. If each of 5 jars contain 1 gallon 1 quart of vinegar, how much do the 5 jars together contain ?

5 × 1 quart = 5 quarts = 1 gallon 1 quart ; 5 × 1 gallon = 5 gallons. Ans.: 6 gallons 1 quart.

2. A can of milk was emptied into 6 pans, each holding 3 quarts 1 pint. How much milk did the can contain ?

6 × 1 pint = 6 pints = 3 quarts ; 6 × 3 quarts = 18 quarts. Ans.: 21 quarts.

3. Edward had on his wagon 4 sacks, each containing 2 bushels 3 pecks of wheat. How much wheat did he have ?

4 × 3 pecks = 12 pecks = 3 bushels ; 4 × 2 bushels = 8 bushels. Ans.: 11 bushels.

4. A basket holding 2 quarts 1 pint of berries being filled 8 times, held as many berries as a pail. How many berries did the pail hold ?

8 × 1 pint = 8 pints = 4 quarts ; 8 × 2 quarts = 16 quarts. Ans.: 20 quarts, or 2 pecks 4 quarts.

191. Liquid measure is used for measuring liquids ; as milk, vinegar, etc.

192. Dry measure is used for measuring vegetables, fruits, grains, seeds, etc.

193. Copy, and commit to memory the following tables:

Capacity.

Liquid Measure.				Dry Measure.			
4	gills (gi.) make	1	pint, . . . pt.	2	pints (pt.) make	1	quart, . . . qt.
2	pints . . "	1	quart, . . qt.	8	quarts . . "	1	peck, . . . pk.
4	quarts . "	1	gallon, . . gal.	4	pecks . . "	1	bushel, . . bu.
31½	gallons . "	1	barrel, . . bl.	A *Small Measure* equals 2 quarts. A barrel equals about 2¾ bushels.			
63	gallons . "	1	hogshead, hhd.				

...on contains 231 cu. in.; the bushel, 2,150.42 cu. in. The bar... the hogshead are different for different substances and in ... States.

194. Find the products:

1. What is the product of 4 bu. 3 pk. 5 qt. multiplied by 46?

bu.	pk.	qt.
4	3	5
		46
225	2	6 Ans.

41	28	8)230
184	138	28 + 6
225	4)166	
	41 + 2	

Explanation.—1. Write the multiplier under the lowest denomination of the multiplicand; and draw two horizontal lines under the multiplier.

2. Multiplying the 5 qt. by 46 gives a product of 230 qt. Write 230 in the column of quarts under the lower line.
3. Reduce the 230 qt. to pecks by dividing by 8 (as there are 8 quarts in a peck), obtaining 28 pk. 6 qt. Write the 6 qt. in the column of quarts between the two lines; and write the 28 pk. in the column of pecks below the lower line to be added to the product of the pecks.
4. Multiplying the 3 pk. by 46 gives a product of 138 pk. Write 138 in the column of pecks under the lower line. Adding the 28 pk. obtained from the products of quarts gives 166 pk.
5. Reduce the 166 pk. to bu. by dividing by 4 (as there are 4 pecks in a bushel), obtaining 41 bu. 2 pk. Write the 2 pk. in the column of pecks between the lines; and write the 41 bu. in the column of bushels below the lower line to be added to the product of the bushels.
6. Multiplying the 4 bu. by 46 gives a product of 184 bu. Write 184 in the column of bu. below the lower line. Adding the 41 bu. obtained from the products of the pecks gives 225 bu. Write 225 in the column of bushels, between the lines. Ans. 225 bu. 2 pk. 6 qt.

2.

bu.	pk.	qt.
9	3	7
		25

3.

gal.	qt.	pt.
21	2	1
		33

4.

bu.	pk.	qt.
15	1	6
		53

5.

gal.	qt.	pt.
40	0	2
		69

6.

lb.	oz.	pwt.
25	9	18
		75

7.

T.	cwt.	lb.
33	19	75
		83

8.

lb.	ʒ.	ʒ.
46	11	7
		36

9.

pk.	qt.	pt.
3	7	1
		50

195. Rule for Multiplication.—1. Write the multiplier at the right under the lowest denomination of the multiplicand, and draw two horizontal lines under the multiplier.

2. Beginning at the right, multiply in succession the numbers of the denominations of the multiplicand by the multiplier, writing the products under the lower line.

3. Reduce each product, if possible, to units of the next higher denomination, adding the units of the higher denomination thus obtained to the product of that denomination; and writing the remainder, if any, between the lines in the column of the denomination from which it was obtained.

4. Write between the lines in the column of the highest denomination the result obtained by adding the product of the highest denomination to the units of the same denomination reduced from the products of the lower denominations. This result, together with the remainders of the products of the lower denominations, forms the answer.

Longitude and Time.

196. Time is a portion of duration, and it is measured by the revolution of the earth around the sun (a year); and by the rotation of the earth on its axis (a day).

197. The *solar* year and day being variable quantities, the calendar years and days and the clocks are adjusted so as to keep *mean* (or *average*) *solar* time.

198. The year is divided into months, called calendar months, whose names and number of days are as follows:

1. January, . . .	31 days.	*7.* July, ...	31 days
2. February, . . .	28 "	*8.* August, ...	31 "
3. March, ...	31 "	*9.* September, ...	30 "
4. April, ...	30 "	*10.* October, ...	31 "
5. May, ...	31 "	*11.* November, ...	30 "
6. June, ...	30 "	*12.* December, ...	31 "

In leap years, February has 29 days.

199. The number of days in each month may be easily remembered by the following couplets:

Thirty days hath September,	All the rest have thirty-one,
April, June, and November.	Save February, etc.

200. Copy, and commit to memory the following:

30 days	make	1 month, . .	mo.
12 months	"	1 year, . . .	yr.
365 days	"	1 year, . . .	yr.
366 days	"	1 leap year.	
100 years	"	1 century.	

Every fourth year is called a leap year, and contains 366 days. If the number of the year is divisible by 4, it represents a leap year. No number, however, ending in two ciphers represents a leap year, unless it is divisible by 400.

201. The earth making one complete rotation on its axis in twenty-four hours causes the sun to appear each day to describe around the earth the circumference of a circle. This circumference is divided into 360 equal parts, called **degrees (°) of longitude.**

202. Longitude is angular distance measured around the earth from east to west, or from west to east.

203. As the sun brings the noon-day toward the west, the time of day is always morning to the west of the sun; afternoon to the east of the sun; and noon on the north and south line (meridian) immediately under the sun.

204. Longitude and **time** are compared as follows:

Longitude.	Time.
60 seconds (″) make 1 minute (′).	60 seconds (sec.) make 1 minute, m.
60 minutes . . " 1 degree (°).	60 minutes . . " 1 hour, hr.
360 degrees . . " 1 circle, c.	24 hours . . . " 1 day, da.
30° = 1 sign, S.	7 da. = 1 wk.; 4 wk. = 1 mo.; 52 wk. = 1 yr.

205. The sun appears to move 360° in 24 hours, or 15° in 1 hr.; 15′ in 1 min.; 15″ in 1 sec.; hence, the

206. To Reduce Time to Longitude: Multiply by 15.

207. To Reduce Longitude to Time: Divide by

208. Find the quotients:

1. What is the quotient of 12° 40′ 32″ divided by 15?

Explanation.—1. Write the divisor at the left of the dividend, with a curved line between them; and draw two horizontal lines under the dividend.

15)	12°	40′	32″
	0	50	$42\frac{2}{15}$ Ans.
		720	600
		40	32
		15)760	15)632
		50+10	42+2

2. As the units of the first denomination of the dividend do not contain 15, reduce them to the next lower denomination, by multiplying by 60, as 1 degree equals 60 minutes, writing the product 720 in the column of that denomination under the lower line.
3. Add the 720 minutes to the 40 units of this denomination in the dividend, obtaining 760 minutes. Divide the 760 minutes by 15; write the quotient, 50, in the column of minutes between the lines; and reduce the remainder to units of the next lower denomination, writing 600, the number of those units, in the column of the next lower denomination under the lower line.
4. Add the 600 seconds to the 32 seconds in the dividend, obtaining 632 seconds. Divide the 632 seconds by 15, and write the quotient in the column of seconds between the lines.

Ans.: 50 min. $42\frac{2}{15}$ sec.

209. Rule for Division.—1. Write the divisor at the left of the dividend with a curved line between them and two horizontal lines below the dividend.

2. Beginning with the highest denomination, divide the denominations in succession by the divisor, writing the quotient in the column of the same denomination, between the horizontal lines, and reducing the remainder, if any, to units of the next lower denomination.

3. Add the units of the lower denomination thus obtained to the units of this denomination in the dividend and then divide. The successive quotients together with the last re- answer.

210. Find the products:

1.			2.			3.			4.		
°	′	″	hr.	min.	sec.	bu.	pk.	qt.	gal.	qt.	pt.
12	25	10	2	4	45	40	3	5	100	3	1
		12			25			38			50

5.			6.			7.				8.		
T.	cwt.	lb.	lb.	oz.	pwt.	℔	℥	ʒ	℈	£.	s.	d.
45	19	50	12	10	15	38	8	5	2	10	10	10
		75			63				27			93

211. Find the quotients:

1.				2.				3.				4.			
	°	′	″		hr.	min.	sec.		bu.	pk.	qt.		gal.	qt.	pt.
12)27	50	30		15)20	30	45		25)80	3	6		10)50	2	1	

5.				6.				7.					8.		
	T.	cwt.	lb.		lb.	oz.	pwt.		℔.	℥	ʒ	℈		£	s.	d.
40)28	18	80		12)25	10	10		10)13	9	7	2		19)90	9	9

Miscellaneous Tables.

212. Copy, and commit to memory the following tables:

Paper.			Counting.		
24 sheets	make	1 quire.	12 things	make	1 dozen, doz.
20 quires	"	1 ream.	12 dozen	"	1 gross, gro.
2 reams	"	1 bundle.	12 gross	"	1 great gross.
5 bundles	"	1 bale.	20 things	"	1 score.

213. In making books, large sheets of paper are folded, as follows:

No. of folds.	No. of leaves.	No. of pages.	Size of book.
1	2	4	folio.
2	4	8	quarto, 4to.
3	8	16	octavo, 8vo.
4	12	24	duodecimo, 12mo.
4	16	32	16mo.

MISCELLANEOUS PROBLEMS.

214. Oral Problems. 1. How many 3 pt. bottles can be filled from 12 gallons of wine?

2. How many 2 gr. pills can be made from 4 ʒ of quinine?

3. How much does a grocer charge for 3 lb. 12 oz. of spice at $.05 an ounce?

4. What does a druggist charge for 6 lb. 3 oz. of herbs at $.03 an ounce? At $.04?

5. If 4 lb. 6 oz. of candy is divided equally among 7 girls, how much does each girl receive?

6. How many tons of coal at $5 a ton will pay for 100 bushels of apples at $.20 a bushel?

7. What will 5 pints of milk cost, if 8 quarts cost $.64?

8. If 3 lb. 12 oz. of beef cost $.60, how much will 1 lb. 6 oz. cost?

9. At 2 ¢ per oz., what will 100 lb. of coffee cost?

10. Charles bought an Arithmetic for $.56; a Reader, $.22; and a Speller, $.15. How much change should he receive on paying for them with 9 dimes and a 5 ¢ piece?

11. A grocer had a bushel of potatoes and sold from it 1 pk. 6 qt. and 2 pk. 2 qt. How much did he have left?

215. Written Problems. 1. Having money to invest, Mr. Franklin bought 4 farms valued at $3,750 each; 125 head of cattle at $17.50 each; and 450 shares R. R. stock at $78 per share. He then had $1,975 left. How much money did he have at first?

2. A man bought 376 acres of land at $14 an acre, and sold it all for $5,675. Did he gain or lose, and how much?

3. From a barrel of oil, the following quantities were re-

l. 2 qt. 1 pt.; 13 gal. 3 qt.; and 9 gal. 1 qt.

s left?

4. What will 15 doz. cans of oil cost, if each can contains 2 qt. 2 pt. at $.03 a pint ?

5. How much flour at $4.50 a barrel should be given for a dozen sacks of wheat, each sack containing 2 bu. 2 pt. at $1.20 per bu.? At $.30 per pk.?

6. If each box of berries can be bought for 6 d., how many boxes of berries can be bought for £10. 10s. 6d ?

7. How much milk can be poured into four cans holding respectively 1 gal. 3 qt. 1 pt.; 2 gal. 0 qt. 1 pt.; 1 gal. 1 qt. 1 pt.; and 2 gal. 2 qt. 1 pt.?

8. A druggist sold 4 ℔. 3 ℥ 2 ʒ, and 2 ℔. 7 ℥ 5 ʒ of a drug at $.05 per ʒ. How much did he sell it all for ?

9. From 10 lb. 7 oz. 1 pwt. take 7 lb. 10 oz. 10 pwt.

MEASUREMENT.

216. 1. How many feet high are you ?

2. In walking, what is the length of an average step ?

3. How many inches long is your slate ? How wide ?

4. How long is your desk ? How wide ? How many feet ? How many yards ?

5. How long is the school-room ? How wide ? How high ?

6. How many feet are there in the width of the school-house lot ?

Length.

217. A **line** is that which has only length.

Long Measure.

12 inches (in.)	make	1 foot,	ft.	320 rods	make	1 mile,	mi.
3 feet	"	1 yard,	yd.	5,280 feet, / 1,760 yards,	"	1 mile.	
5½ yards, / 16½ feet,	"	1 rod,	rd.	3 miles	"	1 league.	

A furlong (fur.) equals *one-eighth* of a mile.

218. Find the number of:

inches in			feet in		
1. 9 ft.	4. 4 yd.	7. 3 yd. 1 ft.	10. 16 rd. 0 yd. 2 ft.		
2. 7 ft.	5. 3 rd.	8. 2 rd. 4 yd.	11. 3 mi. 30 rd. 1 ft.		
3. 12 ft.	6. 2 mi.	9. 5 mi. 5 yd.	12. 5 mi. 5 yd. 0 ft.		
yards in			**yards in**		**miles in**
13. 375 rd.	16. 400 in.		19. 25 mi.		22. 300,000 in.
14. 116 rd.	17. 320 ft.		20. 200 ft.		23. 75,000 ft.
15. 500 mi.	18. 625 in.		21. 150 rd.		24. 11,000 rd.

The preceding examples illustrate *reduction ascending* and *reduction descending*.

219. Surveyors' Measure is used in surveying, or in measuring land. The measuring line is Gunther's chain, 66 feet long, and containing 100 links.

7.92 in. make 1 link; 100 links make 1 chain; 80 chains make 1 mile.

Surface.

220. A **surface** has length and breadth only, as the *surface* of this page, of the floor, etc.

Square Measure.

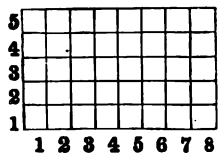
144	square inches (sq. in.)	make	1	square foot,	sq. ft.
9	square feet	"	1	square yard,	sq. yd.
30¼	square yards	"	1	square rod, or perch,	sq. rd. P.
40	square rods	"	1	rood,	R.
4	roods	"	1	acre,	A.
640	acres	"	1	square mile,	sq. mi.

625 square links = 1 square rod. | 10 square ch. = 1 A.
16 square rods = 1 square ch. | 160 square rods = 1 A.

221. A **rectangle** is a flat surface with four straight sides and four equal angles or corners.

222. A **square** is a rectangle with four equal sides and ıgles. A square inch is a square each of whose ch in length.

223. To find the area, or number of square units in a rectangle, multiply the number of units in the length by the number in the breadth. Thus, if a slate is 8 inches long and 5 inches wide, its surface contains 5 × 8 or 40 square inches.



224. Written Problems. 1. What is the area, or number of square units, in a floor, 28 feet long and 18 feet wide?

2. If a pavement is 25 feet long and 16 feet wide, how many square feet of surface has it?

3. How many square feet are there in a passage-way 40 feet long and 4 feet wide? How many square yards?

4. A rectangular field was 1,000 rods long and 125 rods wide. How many acres did the field contain?

5. In a territory having the figure of a square, each of the sides being 350 miles long, how many acres are there?

Volume.

225. A **solid** has length, breadth, and thickness.

Cubic Measure.

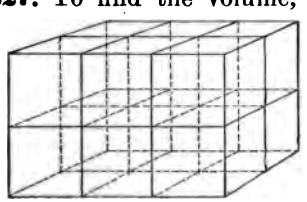
1,728 cubic inches (cu. in.) . .	make . .	1 cubic foot, .	cu. ft.
27 cubic feet	" . .	1 cubic yard, .	cu. yd.
128 cubic feet	" . .	1 cord, ...	C.

A pile of wood 8 feet long, 4 feet high, and 4 feet wide is a cord. One foot in length of this pile, or 16 cubic feet, is a cord foot.

226. A **cube** is a solid bounded by six equal squares.

A cubic foot is a cube each of whose edges is 1 foot long.

227. To find the volume, or number of cubic units, in any cube, or in any solid bounded by six rectangles, multiply together the number of units in the length, the breadth, and the thickness expressed in units of *the same denomination.*



228. Written Problems. 1. How many cubic inches are there in a cubical block of wood, each of the edges being 80 inches in length ?

2. What is the difference between 10 cubic inches and a cube each of whose edges is 10 inches long ?

3. A block of ice is 8 feet long, 6 feet wide, and 3 feet thick. How many cubic inches does the block contain ?

MISCELLANEOUS PROBLEMS.

229. Oral Problems. 1. How many cubic inches of air are there in a room 20 feet long, 20 feet wide, and 10 feet high ? 25 feet long, 15 feet wide, and 12 feet high ?

2. How many ounces are there in a tank of water 5 feet deep, 5 feet long, and 5 feet wide ?

1,000 oz. of water = 1 cu. ft.

3. How many ounces would there be in a bar of iron 1 foot thick, 1 foot wide, and 2 feet long, if each cubic foot weighed 8,000 ounces ?

4. In a block of stone, how many cubic feet are there if it is 1 foot wide, 1 foot thick, and 1 yard long ?

5. As 80 chains (surveyor's measure) make a mile, how many feet long is the chain ?

6. How many acres are there in a square tract of land, each side of the tract being 10 miles long ?

7. To contain 640,000 acres, how many miles long shall a tract of land be, if it is 10 miles wide ?

8. If a room 25 feet long by 20 feet wide contains 5,000 cu. ft., how high is the ceiling ?

9. A flag pole was 3 yd. 2 feet 8 in. long. How many inches long was the pole ?

10. A telegraph pole stood 50 feet in the air, 5 ft. 9 in. in *yr. and* 4 ft. 3 in. in the ground. How many feet long ? How many inches long ?

11. How many 3 pt. bottles can be filled with 15 gallons of milk? With 20 gallons?

12. How many doz. silver spoons, each spoon weighing 2 oz., can be made from 12 pounds of silver?

13. If from a ton of coal 4 boxes each containing 2 cwt. 1 qr. of coal are taken, how many hundredweight will be left?

14. A mixture of metal weighed 10 lb.; it contained 4 lb. 9 oz. of lead, and 2 lb. 7 oz. of iron; the remainder of the mixture was zinc. How much did the zinc weigh?

15. A box containing 2 bushels of strawberries was bought for $1.60 a bushel. The berries were put into quart baskets, and sold at 10 cts. a quart. What was the entire gain? What was the gain per quart?

16. How many English shillings should be given for $25 in gold, if the exchange is made on the basis of a sovereign for a $5 gold piece?

230. Written Problems. 1. Mr. Franklin paid each year $15, car fare; $5, postage; and $25, office rent. How much did these items cost him in 2 years?

2. A trader gave 48 horses for 144 cows at $43. What value per horse did he receive?

3. If a grocer buys 12 barrels of sugar at $16, and 17 barrels at $13, how much would he gain by selling the entire quantity at $18 a barrel?

4. What would 3 loads of hay weighing respectively 1 T. 2 cwt.; 19 cwt.; and 1 T. 1 cwt. cost, at $.60 per cwt.?

5. What would be the value of a bar of silver 4 feet long, 2 feet wide, and 1 foot thick, if a cubic foot weighed 10,000 Troy ounces, at $4.75 a pound?

6. At $80 per Troy pound, what will be the value of a bar of gold, 3 feet long, 2 feet wide, 1 foot thick, if each cubic *foot weighed 20,000 ounces?*

7. The ages of 5 men are respectively: 40, 33, 67, 24, and 36 years. What is their average age?

8. Mr. Richardson began business with a store worth $6,200; fixtures, $425; and stock, $29,732. The first year he gained $3,271. How much was his property then worth?

9. How many silver pitchers each weighing 3 lb. 5 oz. 10 pwt. can be made from 51 lb. 10 oz. 10 pwt. of silver?

10. Into how many farms containing 50 acres each, can a tract of land containing 100 square miles be divided?

11. If a wall contains 8,000 cubic feet of stone, and it is 2 feet thick by 100 feet deep, how many feet high is it?

12. Mr. Johnson made $3,000 by selling 7,000 barrels of flour at $4.77 a barrel. How much did each barrel cost him?

13. How many loaves of bread, each made with 8 ounces of flour, can be made from a barrel of 196 pounds of flour?

14. A manufacturer paid $23 for 5 loads of coal, each load containing 25 bushels, weighing 1 cwt. per bushel. How much did the coal cost him per ton?

15. What is the average price per pound of 4 kinds of coffee at respectively 26 ¢, 28 ¢, 30 ¢, and 32 ¢?

16. How much will 125 pounds of tea be worth, if it is a mixture of equal portions of tea of three different values: $.60, $.75, and $.90 per pound?

17. If 9 barrels of oil, each containing 30 gallons, are poured into cans, each containing 1 gal. 1 pt., for how much will all the oil be sold at 14 cents a can?

18. If from 100 lb. of sulphur a druggist sells 100 packages, each containing 3 ℥ 3 ʒ 2 ℈, how much of the sulphur will he have left?

19. From a farm of 250 acres three farms were sold containing respectively: 25 A. 2 R. 34 P.; 48 A. 2 R. 20 P.; *and 60 A. 0 R.* 38 P. How much of the farm remained

MULTIPLES AND DIVISORS.

231. A **factor** or *maker* of a number is any one of the numbers which multiplied together give that number as a product.

The *factors* of 6 are 3 and 2 ; the *factors* of 10 are 5 and 2.

232. A **multiple** of a number is a product obtained by *multiplying* that number by any integer or whole number.

The *multiples* of 2 are : 2, 4, 6, 8, 10, 12, 14, etc. ; of 3 : 3, 6, 9, 12, etc. ; of 4 : 4, 8, 12, 16, etc. ; of 5 : 5, 10, 15, 20, etc.

233. If a number is used twice as a factor, the multiple or product is called the **square** of that number.

$2 \times 2 = 4$, the *square* of 2 ; $3 \times 3 = 9$, the *square* of 3 ; $4 \times 4 = 16$, the *square* of 4 ; $5 \times 5 = 25$, the *square* of 5.

234. If a number is used three times as a factor, the multiple or product is called the **cube** of that number.

$2 \times 2 \times 2 = 8$, the *cube* of 2 ; $3 \times 3 \times 3 = 27$, the *cube* of 3 ; $4 \times 4 \times 4 = 64$, the *cube* of 4 ; $5 \times 5 \times 5 = 125$, the *cube* of 5.

235. State in succession some multiples:

1. Of 2.	*3.* Of 4.	*5.* Of 6.	*7.* Of 8.	*9.* Of 10.	*11.* Of 12.
2. Of 3.	*4.* Of 5.	*6.* Of 7.	*8.* Of 9.	*10.* Of 11.	*12.* Of 13.

236. State in succession the squares of the first 12 numbers.

237. State in succession the cubes of the first 12 numbers.

238. Find the squares of:

1. 20.	*3.* 40.	*5.* 80.	*7.* 120.	*9.* 469.	*11.* 1,704.
2. 25.	*4.* 50.	*6.* 100.	*8.* 125.	*10.* 978.	*12.* 2,369.

239. Find the cubes of:

1. 20.	*3.* 40.	*5.* 100.	*7.* 253.	*9.* 471.	*11.* 978
2. 25.	*4.* 50.	*6.* 125.	*8.* 306.	10. 602.	12. 1

240. To **factor** is to separate a number into its factors.

241. Factor by inspection, copying and completing the following tables of factors:

10 =

2 X 5	5 X 2

12 =

2 X 6	6 X 2
3 X 4	4 X 3

15 =

3 X 5	5 X 3

16 =

2 X 8	8 X 2
4 X 4	4 X 4

18 =

20 =

242. Construct similar diagrams, and in them write the factors of 15, 22, 25; 24, 30, 40; 36, etc.

243. Numbers that cannot be separated into factors are called **prime numbers.**

Prime (*first*) numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, etc.

244. A prime number used as a factor is called a **prime factor.**

Which of the factors in Arts. 241 and 242 are *prime factors?*

245. Separate into prime factors by inspection:

1. 4.	*4.* 9.	*7.* 14.	*10.* 18.	*13.* 22.
2. 6.	*5.* 10.	*8.* 15.	*11.* 20.	*14.* 24.
3. 8.	*6.* 12.	*9.* 16.	*12.* 21.	*15.* 25.

Thus, Ex. 1: $4 = 2 \times 2$; Ex. 2: $6 = 2 \times 3$; Ex. 3: $2 \times 2 \times 2$, etc.

246. Numbers are **prime to each other** when none of their prime factors are the same; as, 10 and 21.

247. Removing a factor from a number divides the number by that factor.

Thus, $24 = 2 \times 2 \times 2 \times 3$; removing one of the 2's, the result is $2 \times$ or 12, which is the same as 24 divided by 2.

248. Show that removing a factor from either of the following numbers divides the number by that factor:

1. 6.	*3.* 12.	*5.* 20.	*7.* 30.	*9.* 300.
2. 8.	*4.* 16.	*6.* 28.	*8.* 45.	*10.* 800.

249. A **factor** of a number is a *divisor* of that number.

250. A **divisor** of a number is a *factor* of that number; it will exactly divide the number without remainder.

A *divisor* is an *exact divisor*, unless otherwise expressed.

251. If the same factor is found in two or more numbers, it is a **common factor**; thus, 2 is a common factor of 4 and 6.

252. A **common factor** of two or more numbers is a number that will exactly divide each of them without remainder.

253. A *common factor* is a **common divisor.**

2 is a *common divisor* of 4 and 6; and 3, of 9 and 12.

254. *Even numbers* (ending in 0, 2, 4, 6, or 8) are **multiples of 2**; that is, 2 is a divisor of every even number.

Units only are noted, as any number of tens is divisible by 2.

255. Numbers, the sum of whose digits is any number of times 3, are **multiples of 3**; that is, 3 is a divisor of any number the sum of whose digits is 3 or any number of times 3.

Thus, **42**: $4 + 2 = 6 =$ two 3's; $\mathbf{40} = 12 \times 3 + \mathbf{4}$; and $\mathbf{2} = 0 \times 3 + \mathbf{2}$.

256. Numbers ending in 0 or 5 are **multiples of 5**; that is, 5 is a divisor of every number ending in 0 or 5.

Units only are noted, as any number of tens is divisible by 5.

257. Numbers, the sum of whose digits is any number of times 9, are **multiples of 9**; that is, 9 is a divisor of any number the sum of whose digits is 9, or any number of times 9.

Thus, **657**: $6 + 5 + 7 = 18 =$ two 9's; $\mathbf{600} = 66 \times 9 + \mathbf{6}$; and $\mathbf{50} = 5 \times 9 + \mathbf{5}$; and $\mathbf{7} = 0 \times 9 + \mathbf{7}$.

258. As 4 is a divisor of any number of hundreds, if the number expressed by the last two digits of a number is divisible by 4, that number is divisible by 4; that is, it is a **multiple of 4.**

259. As 8 is a divisor of any number of thousands, if the number expressed by the last three digits of a number is divisible by 8, that number is divisible by 8; that is, it is a **multiple of 8.**

260. By combining the principles in Arts. 254, 255, 256, 257, 258, 259, show what numbers are multiples of 6, of 12, of 15, of 18, of 36, of 45, etc.

A number is divisible by 6, if it is even and the sum of its digits is divisible by 3; for then it is divisible by 2 and by 3, or by 6; etc.

CANCELLATION.

261. 1. What is the cost of 8 barrels of flour, if 6 barrels cost \$30?

If 6 cost \$30, 1 barrel cost $\frac{\$30}{6}$ and 8 barrels cost $8 \times \frac{\$30}{6}$ or $\frac{8 \times \$30}{6}$ $= \$40$.

2. How many tons of coal at \$6 will pay for 9 barrels of flour at \$4 a barrel?

If 1 barrel of flour cost \$4, 9 barrels cost $9 \times \$4$. If each ton of coal costs \$6, as many tons will be required as \$6 is contained times in $9 \times \$4$; that is $\frac{9 \times 4}{6}$, or 6.

262. The solution of a problem involving both multiplication and division may be indicated, continued products forming the dividend and divisor, separated by a horizontal line.

263. Since the dividend and divisor may both be divided by the same number without affecting the quotient (Art. VI.), any *common factor* may be removed before the operations indicated.

264. To remove a factor from both dividend and divisor is to cancel that factor.

265. Cancellation is removing the common factors from the dividend and the divisor.

The object is to lessen the labor by leaving smaller numbers to operate upon.

266. Rule for Cancellation.—1. Draw a horizontal or a vertical line, and indicate the work by writing the factors of the dividend and of the divisor on opposite sides of the line.

2. Strike out all the common factors, or remove them by successively dividing both dividend and divisor by them, and then perform the operations indicated.

267. Find the quotients of the following indicated divisions with and without cancellation:

1. $\dfrac{4\times 6\times \cancel{5}}{\cancel{10}_{2}\times 2} = 6.$

2. $\dfrac{10\times \cancel{3}^{1}}{5\times \cancel{6}_{2}\times 2} = \frac{1}{2}.$

3. $\dfrac{20\times 6\times 8}{5\times 3\times 4} =$

4. $\dfrac{9\times 10\times 7}{14\times 2\times 6} =$

5. $\dfrac{18\times 20\times 5}{10\times 4\times 6} =$

6. $\dfrac{25\times 4\times 2}{8\times 5} =$

7. $\dfrac{12\times 8\times 10}{6\times 5\times 4} =$

8. $\dfrac{300\times 6}{4\times 50} =$

NOTE.—Dividing a quantity by itself gives 1 for a quotient. This, however, is not expressed except where, as in Ex. 2, the other factors disappear by cancellation.

268. Work by cancellation problems in Art. 152 and 153.

269. Written Problems. 1. If 690 men can build a railroad in 48 days, in what time can 32 men build the road?

2. How much will 125 barrels of flour cost, if 75 barrels cost $450?

3. How much are 50 cans of milk, each containing 5 gallons, worth, if 2 cans containing 45 quarts each are worth $3?

4. If sound travels 5,450 feet in 5 seconds, how far will it travel in one minute ?

5. At $8 per barrel, how many barrels of flour must be given in exchange for 320 barrels of potatoes at $.90 a barrel ?

6. A farmer exchanged 325 bushels of wheat at $1.15 per bushel for 500 bushels of corn. What did the corn cost him per bushel ?

7. If 100 sacks of wheat, each containing 3 bushels, at $1.25 per bushel, are given in exchange for flour at $5 per barrel, how many barrels of flour are required ?

8. How much flour will 375 bushels of wheat yield, if 5 bushels yield 196 pounds of flour ?

9. If 100 pounds of flour cost $4, how much must be paid for 50 bags of flour containing 25 pounds each ?

10. A drover exchanged cattle at $24 a head for 72 horses at $175. How many head of cattle did he give ?

11. How many sacks of coffee, each containing 60 pounds at 28 cents a pound, must be given for 12 barrels of apples, each containing 2 bushels, at $.20 a peck ?

12. At $1.75 a day, how many days' labor must be given for 50 tons of coal at $4.90 a ton ?

13. How many dozen eggs, at 3 cents per egg, must be given in exchange for 60 tubs of butter, containing 48 pounds each, at 20 cents per pound ?

14. If each piece of cloth contains 42 yards at $.33 a yard, how many pieces must be given for 121 barrels of potatoes, each containing 2 bushels of potatoes at $.63 per bushel ?

15. A gentleman exchanged 15 city lots, each containing 20 feet front, at $90 per foot, for a farm at $120 per acre. How many acres did he receive ?

16. [illegible] any city lots each containing 25 feet front at $[illegible] [illegible]ust be given in exchange for 4 farms, each [illegible] acres, at $40 per acre ?

GREATEST COMMON DIVISOR.

270. In cancellation, with integers and with common fractions, it is frequently desirable to use the *greatest common divisor*, to reduce the problem to its lowest terms.

271. Illustrate the PRINCIPLES OF DIVISORS by using the three following forms:

3)12 **3)3, 6, 9, etc.** **3)18 and 12**

I. *A factor of a number is a divisor of that number.*

II. *A factor or divisor of a number is a factor or divisor of every multiple of that number.*

III. *A factor or divisor of two or more numbers is a factor or divisor of their sum.*

IV. *A factor or divisor of two numbers is a factor or divisor of their difference.*

V. *A factor or divisor of two numbers is a factor or divisor of their remainder after division.*

NOTE.—Prin. V. is true, as the remainder after division is but the difference between one of the given numbers and a multiple of the other number. Prins. II. and IV.

272. A **divisor** is a *factor;* a common **divisor** is a *common factor;* and, the **greatest common divisor** is the *greatest common factor;* that is, the *product of all the common factors.*

273. To find the greatest common divisor of numbers readily factored.

1. What is the greatest common divisor of 36, 40, and 48?

$36 = 2 \times 2 \times 3 \times 3$
$40 = 2 \times 2 \times 2 \times 5$
$48 = 2 \times 2 \times 2 \times 2 \times 3$
G. C. D. $= 2 \times 2$ or 4. Ans.

Explanation.—1. Write the numbers in a column, and at the right of each write its prime factors.

2. As 2 taken twice, or 4, is the only factor common to the three numbers, it is the *common factor, or the greatest* common divisor of the gi

Explanation.—1. Write the numbers in a line.

2. Divide by 2, any prime number that will exactly divide all the numbers. This gives one common factor or divisor.

3. Divide the quotients by 2, any prime number that will exactly divide all the quotients. This gives a second common factor or divisor.

4. As no other number will exactly divide all the quotients thus obtained, there is no other common factor or divisor.

5. 2 × 2, or 4, the product of all the common factors, is the greatest common factor, or the greatest common divisor.

Or,

2)36 40 48

2)18 20 24

9 10 8

274. Rules for Greatest Common Divisor.—I. Write the numbers in a column, and separating each into its prime factors, find the product of the factors common to all ; **or,**

II. 1. Write the numbers in a line.

2. Divide by any number that will exactly divide each of the numbers without a remainder.

3. In like manner, divide the quotients, and thus proceed, until no number can be found that will exactly divide all the quotients without a remainder.

4. The product of all the common divisors thus found is the greatest common divisor.

275. Find the greatest common divisors :

1. 18 ; 24.	*4.* 96 ; 108.	*7.* 55 ; 735.	*10.* 12 ; 24 ; 40.
2. 32 ; 48.	*5.* 125 ; 375.	*8.* 90 ; 540.	*11.* 25 ; 40 ; 60.
3. 50 ; 75.	*6.* 400 ; 625.	*9.* 81 ; 639.	*12.* 80 ; 90 ; 35.

276. Written Problems. 1. A farmer has two farms containing 2,431 and 770 acres of land. He wishes to divide the farms into village lots of equal size. What is the greatest number of acres he can have in each lot ?

2. A coal merchant has 5,400 and 8,424 tons of coal of two *kinds*. How shall he divide the coal so that the coal pockets [illegible] the largest possible equal quantities ?

3. A cloth merchant has three rolls of cloth containing 40, 45, and 60 yards. What is the greatest length of equal parts into which he can divide the cloth without remnants ?

4. If 100, 125, and 180 barrels of flour of different brands are to be divided into equal lots, what is the largest number of each kind that can be put into each lot ?

5. What is the largest common measure of three areas containing respectively 13,356 ; 14,148 ; and 15,912 acres ?

6. If two gentlemen have respectively $7,056 and $7,392, what is the largest price per city lot that they can pay, so that each shall spend all of his money ?

7. A commission merchant sells 204 ; 1,392 ; 1,248 ; and 2,016 barrels of flour to different purchasers. What is the largest number of barrels he can send on a truck, so that no parts of loads need be sent ?

8. What is the greatest common divisor of five hundred, two hundred fifty, and one hundred seventy-five ?

277. To find the greatest common divisor of numbers not readily factored.

1. Find the greatest common divisor of 840 and 600.

$$
\begin{array}{rl}
 & 1 \\
600)\overline{840} & \\
600 & 2 \\
\hline
240)\overline{600} & \\
480 & 2 \\
\hline
120)\overline{240} & \\
240 & \\
\hline
\end{array}
$$

Explanation.—1. Divide 840 by 600. As there is a remainder, 600 is not a divisor. By Prin. V. the greatest common divisor is a divisor, not only of 840 and 600, but of 240, the remainder.

2. Divide 600, the divisor, by 240, the remainder. As there is a remainder 240 is not a divisor. By Prin. V. the greatest common divisor is a divisor, not only of 600 and 240, but of 120, the remainder after division.

3. Divide 240, the divisor, by 120, the remainder. As there is no remainder 120 is an exact divisor, and by Prin. V. it is thus shown to be the *greatest common* divisor, not only of 120 and 240, but of *240 and 600, and also of* 600 and 840. Ans. 120.

278. Rule for Greatest Common Divisor. 1. Divide the greater number by the less; if there is no remainder, the less number is the greatest common divisor.

2. If there is a remainder, use it as a divisor, and use the preceding divisor as a dividend, and thus proceed, using each remainder as a divisor, until a divisor is found that will divide without a remainder. The first exact divisor is the greatest common divisor.

3. If there are more than two numbers, find the greatest common divisor of two of them first; and then, the greatest common divisor of this and one of the other numbers; and thus proceed, until the greatest common divisor of all is found.

279. Find the greatest common divisors:

1. 224; 336.	*5.* 900; 3,474.	*9.* 4,165; 686.
2. 625; 1,225.	*6.* 1,379; 2,401.	*10.* 12 321; 54,345.
3. 429; 715.	*7.* 7,007; 7,392.	*11.* 92; 1,104; 5,672.
4. 377; 1,131.	*8.* 2,793; 2,660.	*12.* 204; 1,190; 1,445.

280. Written Problems. 1. A lady has 20, 28, and 32 yards respectively of red, white, and blue ribbon. What is the length of the longest equal strips into which she can cut the ribbon?

2. A grocer has 60, 75, and 80 pounds of different kinds of tea. What is the weight of the largest equal portions into which he can divide the tea?

3. Three tanks contain respectively 28, 42, and 56 gallons of oil. What is the largest measure which, used an exact number of times in each tank, will remove the oil?

4. If four tracts of land containing respectively 2,680; 4,235; 3,760; and 2,975 acres are to be divided into village ... what is the size of the largest equal lots into which the ... can be exactly divided?

5. Three pieces of silk contain respectively 40, 60, and 80 yards. What is the largest dress pattern into which they can be divided without remnant?

6. How many yards each will the largest aprons of equal size contain, if cut from four pieces of calico containing respectively 28, 36, 40, and 46 yards?

7. A merchant sold to four customers 25, 50, 65, and 70 barrels of sugar. What is the number of barrels with which to make the largest equal truck loads?

LEAST COMMON MULTIPLE.

281. A **multiple** of a number is a *product* of the number by any integer; a **common multiple** of two or more numbers is a product obtained by using those numbers as factors with or without other factors.

282. The **least common multiple** of two or more numbers is the least number that can be divided by each of them without a remainder; as 12 of 4 and 6; and 40 of 8, 10, and 20.

283. In solving certain problems, involving integral numbers, and in using fractions, it is frequently necessary to find the least common multiple of two or more numbers.

284. Illustrate the PRINCIPLES OF MULTIPLES by using the two following forms:

24, multiple of 12; 36, a common multiple of 6 and 9.

I. *A multiple of a number is a multiple of every factor of that number.*

II. *A common multiple of two or more numbers is a multiple of every factor of those numbers.*

III. *The least common multiple of two or more numbers contains all the factors of those numbers, with each factor repeated only as frequently as the greatest number of times it is repeated in any one of the numbers.*

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285. To find the least common multiple of numbers readily factored.

1. Find the least common multiple of 36, 40, and 48.

Explanation. — 1. Write the numbers in a column.

2. Write the prime factors of each number at its right.

3. Find the product of all the factors, repeating each factor only as often as the greatest number of times it is repeated in any one of the numbers. Ans. 720.

$36 = 2 \times 2 \times 3 \times 3$

$40 = 2 \times 2 \times 2 \times 5$

$48 = 2 \times 2 \times 2 \times 2 \times 3$

L. C. M. $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$= 720$. Ans.

Explanation.—1. Write the numbers in a line.

2. Divide by any prime number that will exactly divide two or more of the numbers. A factor common to two or more of the numbers is thus found. Write the other numbers and the quotients in a line underneath.

3. In like manner, divide the quotients and other numbers, until no number can be found to divide two of them.

4. The product of the divisors, the final quotients, and the undivided numbers is the answer.

Or,

2)36	40	48
2)18	20	24
2) 9	10	12
3) 9	5	6
3	5	2

286. Rules for Least Common Multiple.—I. Write the numbers in a column, and separate each into its prime factors. The product of all the factors with no factor repeated more frequently than it is repeated the greatest number of times in any number is the least common multiple ; **or,**

II. 1. Write the numbers in a line, and divide by any prime number that will exactly divide two or more of them.

2. Write the quotients and the undivided numbers in a line beneath, and divide them by any prime number that will exactly divide two or more of them without a remainder.

3. Proceed in the same manner, until no number can be found to divide two or more of the numbers.

4. The product of the divisors, the final quotients, and *the undivided* numbers is the least common multiple.

287. Find the least common multiples:

1. 6; 10; 24.	*6.* 8; 10; 12; 14.	*11.* 100; 90; 78.
2. 5; 15; 12.	*7.* 7; 14; 15; 21.	*12.* 65; 42; 39.
3. 4; 18; 30.	*8.* 9; 12; 14; 18.	*13.* 17; 21; 83.
4. 8; 14; 40.	*9.* 10; 8; 25; 32.	*14.* 204; 175; 60.
5. 9; 15; 20.	*10.* 40; 25; 30; 12.	*15.* 325; 625; 700.

288. Written Problems. 1. What is the smallest sum of money that will buy an exact number of pounds of sugar valued at 7, 8, or 9 cents?

2. If ribbons cost 10, 15, and 18 cents per yard, what is the least sum of money that will buy an exact number of yards of each?

3. The circumferences of three wheels are 9, 12, and 16 feet. What is the shortest distance over which they can each turn an exact number of times?

4. A carpenter has planks 12, 15, and 20 feet long with which to lay a walk. What is the shortest walk that can be made by using each an exact number of times?

5. Three horses can race around a track in respectively 16, 22, and 24 seconds. If they start at the same time and place, in what time will they first meet again at the starting point?

6. A lady wishes to buy a piece of cambric, that can be cut into dress patterns of 10, 12 or 15 yards. What is the smallest quantity that she can buy?

7. What is the smallest number of melons that will exactly fill barrels that will contain either 14, 18, or 22 melons?

8. If a cistern is to hold an exact number of barrels of water containing respectively 25, 30, or 35 gallons, what is the least number of gallons that it shall contain?

9. If three trains travel at the rate of a mile in respectively 60, 75, and 90 seconds, what is the shortest time in which they *will each have* travelled an exact number of miles?

COMMON FRACTIONS.

289. 1. Jane bought a third of a yard of silk, and a quarter of a yard of velvet. Of which did she buy the more?

2. If cloth sells for $\$\frac{1}{2}$ per yard, how much will 10 yards of the cloth cost?

3. What will $\frac{3}{4}$ of a pound of sugar cost at 8 ¢ a pound? What will $1\frac{1}{2}$ pounds cost?

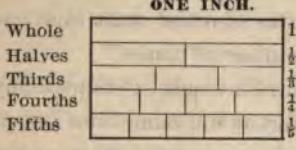
290. Show by the diagram that:

One-half, $\frac{1}{2}$, is *one* of the *two equal* parts of a unit;
One-third, $\frac{1}{3}$, is *one* of the *three equal* parts of a unit;
One-fourth, $\frac{1}{4}$, is *one* of the *four equal* parts of a unit;
One-fifth, $\frac{1}{5}$, is *one* of the *five equal* parts of a unit.

291. Count the number of $\frac{1}{2}$'s, $\frac{1}{3}$'s, $\frac{1}{4}$'s, $\frac{1}{5}$'s, in a unit.

292. Show by the diagram that:

ONE UNIT *equals:*



two-halves, $\frac{2}{2}$, two halves.

three-thirds, $\frac{3}{3}$, three thirds.

four-fourths, $\frac{4}{4}$, four fourths.

five-fifths, $\frac{5}{5}$, five fifths.

293. Write from dictation:

1. Two-thirds. 3. Two-fourths. 5. Four-fourths. 7. Three-fifths
2. Three-thirds. 4. Three-fourths. 6. Two-fifths. 8. Four-fifths

294. Copy, and explain:

$\frac{1}{2}$ $\frac{2}{2}$ | $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$ | $\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$ $\frac{6}{6}$

$\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$ | $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{5}{5}$ | $\frac{1}{7}$ $\frac{2}{7}$ $\frac{3}{7}$ $\frac{4}{7}$ $\frac{5}{7}$ $\frac{6}{7}$ $\frac{7}{7}$

295. A **fraction** is one or more of the *equal parts* of a unit or of a number.

Fraction comes from the Latin word *fractus,* broken; the unit is supposed to be divided or broken into parts.

296. A **fractional unit** is *one* of the equal parts into which a unit is divided.

Thus : $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, etc., are fractional units.

297. A **common fraction** is expressed by two numbers written one above the other with a line between them.

Thus : $\frac{3}{4}$, $\frac{2}{3}$, $\frac{7}{8}$, $\frac{9}{7}$, $\frac{6}{5}$ are *common fractions.*

298. The number below the line is called the **denominator,** as it *names,* or shows the size of the fractional units.

299. The number above the line is called the **numerator,** as it *numbers,* or tells how many of the equal parts are taken.

300. When the numerator of a fraction is less than the denominator, the fraction is called a **proper fraction.**

$\frac{7}{8}$, $\frac{5}{9}$, $\frac{9}{10}$ are *proper fractions.*

301. When the numerator of a fraction is equal to or greater than the denominator, the fraction is called an **improper fraction.**

$\frac{8}{7}$, $\frac{9}{5}$, $\frac{10}{9}$ are *improper fractions.*

Which of the fractions on page 102 are proper fractions, and which are improper fractions ?

302. A **compound fraction** is a fraction of a fraction.

Thus : $\frac{2}{3}$ of $\frac{1}{5}$, $\frac{5}{8}$ of $\frac{4}{7}$ are *compound fractions.*

303. A **complex fraction** is a fraction containing a fraction in its numerator, or in its denominator.

Thus : $\frac{2}{\frac{5}{8}}$, $\frac{\frac{3}{4}}{4}$, $\frac{\frac{3}{4}}{\frac{5}{6}}$, $\frac{2\frac{1}{2}}{7}$, $\frac{\frac{1}{2} \text{ of } \frac{1}{3}}{2}$, are *complex fractions.*

304. A whole number is an **integer.**

Integer is a Latin word, which means untouched, unbroken, undivided; the unit is supposed to be *whole.*

305. A number consisting of a whole number and a fraction *is called a* ***mixed number*** ; as, $4\frac{5}{7}$, $13\frac{1}{4}$, $10\frac{9}{11}$.

306. To read fractions, read the numerator first, and then the denominator, ending with th, or ths. Thus: $\frac{3}{4}$, three-fourths; $\frac{5}{6}$, five-sixths; $\frac{3}{8}$, three-eighths; $\frac{12}{25}$, twelve twenty-fifths; $\frac{9}{84}$, nine eighty-fourths.

NOTE.—When the denominator is 1, 2, or 3, read it as so many *ones*, *halves*, or *thirds;* when the denominator ends in 1, 2, or 3, then read it as so many *firsts*, *seconds*, or *thirds*, the rule above holding only for elevenths, twelfths, and thirteenths. Thus: $\frac{3}{21}$, three twenty-firsts; $\frac{7}{32}$, seven thirty-seconds.

307. Read the following fractions, and indicate the numerators and denominators, explaining the meaning of each:

$\frac{2}{3}$	$\frac{3}{4}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{7}{8}$
$\frac{3}{4}$	$\frac{4}{4}$	$\frac{4}{5}$	$\frac{5}{5}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{4}{7}$	$\frac{5}{7}$	$\frac{4}{8}$	$\frac{5}{8}$	$\frac{5}{9}$	$\frac{9}{10}$

Thus: $\frac{2}{3}$, two-thirds, has 3 for its denominator, and 2 for its numerator; the 3 means that the unit has been divided into 3 equal parts; the 2 means that 2 of these parts have been taken to form the fraction.

308. Write from dictation the fractions taken in any order from Art. 307.

REDUCTION.

309. To **reduce** a fraction is to change it without altering its value. The reduction of fractions depends upon the principles of division.

310. Illustrate the PRINCIPLES OF FRACTIONS by using the following forms:

$$\frac{4}{12} \qquad \frac{12}{4} \qquad \frac{6}{10} \qquad \frac{10}{6}$$

I. *Multiplying or dividing the numerator produces a like effect on the value of the fraction.*

II. *Multiplying or dividing the denominator produces an opposite effect on the value of the fraction.*

III. *Multiplying or dividing both numerator and denominator by the same number produces no effect on the value of the*

311. Reduction Descending. A fraction is reduced to lower terms by *dividing* both numerator and denominator by the same number. (Prin. III.)

$2)\frac{10}{12} = \frac{5}{6}$; $3)\frac{9}{9} = \frac{3}{3}$; $2)\frac{8}{20} = \frac{4}{10}$; $2)\frac{4}{10} = \frac{2}{5}$.

312. Reduce to lower terms:

8 $\vert$ $\frac{8}{24}$	$\frac{3}{9}$	$\frac{5}{10}$	$\frac{4}{6}$	$\frac{6}{8}$	$\frac{6}{9}$	$\frac{6}{12}$	$\frac{7}{14}$
$\frac{30}{10}$	$\frac{40}{8}$	$\frac{50}{25}$	$\frac{50}{10}$	$\frac{16}{24}$	$\frac{9}{36}$	$\frac{7}{21}$	$\frac{18}{45}$
$\frac{6}{20}$	$\frac{9}{15}$	$\frac{14}{35}$	$\frac{10}{40}$	$\frac{8}{22}$	$\frac{12}{36}$	$\frac{15}{36}$	$\frac{18}{28}$

313. A fraction is reduced to its **lowest** (smallest) **terms** by *dividing* both numerator and denominator by their *greatest common divisor.*

$4)\frac{8}{20} = \frac{2}{5}$; $9)\frac{45}{63} = \frac{5}{7}$; $12)\frac{36}{48} = \frac{3}{4}$.

314. Reduce to lowest terms:

1. $\frac{4}{8}$; $\frac{6}{12}$; $\frac{9}{27}$.
2. $\frac{5}{10}$; $\frac{8}{14}$; $\frac{12}{14}$.
3. $\frac{6}{9}$; $\frac{6}{15}$; $\frac{6}{24}$.
4. $\frac{8}{10}$; $\frac{9}{12}$; $\frac{10}{16}$.
5. $\frac{9}{18}$; $\frac{14}{20}$; $\frac{12}{16}$.
6. $\frac{7}{21}$; $\frac{7}{28}$; $\frac{7}{35}$.
7. $\frac{12}{16}$; $\frac{12}{18}$; $\frac{12}{24}$.
8. $\frac{9}{18}$; $\frac{9}{15}$; $\frac{9}{12}$.
9. $\frac{6}{18}$; $\frac{6}{15}$; $\frac{6}{12}$.
10. $\frac{8}{24}$; $\frac{12}{30}$; $\frac{20}{25}$.
11. $\frac{4}{30}$; $\frac{6}{30}$; $\frac{8}{30}$.
12. $\frac{6}{20}$; $\frac{8}{20}$; $\frac{10}{20}$.
13. $\frac{8}{10}$; $\frac{10}{10}$; $\frac{6}{10}$.
14. $\frac{8}{12}$; $\frac{10}{14}$; $\frac{12}{16}$.
15. $\frac{16}{24}$; $\frac{24}{32}$; $\frac{32}{36}$.

315. Explain the following series of fractions:

$\frac{16}{24} = \frac{8}{12} = \frac{4}{6} = \frac{2}{3}$
$\frac{24}{32} = \frac{12}{16} = \frac{6}{8} = \frac{3}{4}$
$\frac{20}{50} = \frac{10}{25} = \frac{4}{10} = \frac{2}{5}$
$\frac{24}{40} = \frac{12}{20} = \frac{6}{10} = \frac{3}{5}$
$\frac{32}{40} = \frac{16}{20} = \frac{8}{10} = \frac{4}{5}$

Thus: $\frac{12}{16} = \frac{6}{8}$: dividing both numerator and denominator of $\frac{12}{16}$ by 2 gives $\frac{6}{8}$.

$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15}$
$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20}$
$\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20} = \frac{10}{25}$
$\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20} = \frac{15}{25}$
$\frac{4}{5} = \frac{8}{10} = \frac{12}{15} = \frac{16}{20} = \frac{20}{25}$

Thus: $\frac{6}{9} = \frac{8}{12}$: multiplying both numerator and denominator by 3 gives $\frac{6}{9}$; and by 4 gives

316. State the reason for the answers in the following:

$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16}$, etc. | $\frac{1}{2} = \frac{3}{6} = \frac{9}{18}$, etc. | $\frac{1}{2} = \frac{4}{8} = \frac{16}{32}$, etc.

$\frac{1}{3} = \frac{2}{6} = \frac{4}{12} = \frac{8}{24}$, etc. | $\frac{1}{3} = \frac{3}{9} = \frac{9}{27}$, etc. | $\frac{1}{3} = \frac{4}{12} = \frac{16}{48}$, etc.

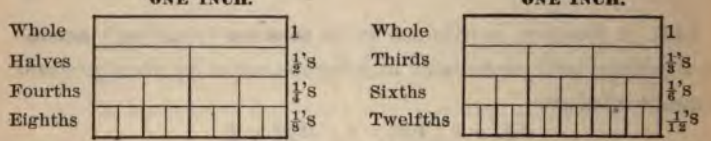
$\frac{1}{4} = \frac{2}{8} = \frac{4}{16} = \frac{8}{32}$, etc. | $\frac{1}{4} = \frac{3}{12} = \frac{9}{36}$, etc. | $\frac{1}{4} = \frac{4}{16} = \frac{16}{64}$, etc.

$\frac{1}{5} = \frac{2}{10} = \frac{4}{20} = \frac{8}{40}$, etc. | $\frac{1}{5} = \frac{3}{15} = \frac{9}{45}$, etc. | $\frac{1}{5} = \frac{4}{20} = \frac{16}{80}$, etc.

Thus: $\frac{4}{12} = \frac{8}{24} : 1 = \frac{24}{24}$; then $\frac{1}{12} = \frac{2}{24}$ and $\frac{4}{12} = \frac{8}{24}$.

$\frac{3}{9} = \frac{9}{27} : 1 = \frac{27}{27}$; then $\frac{1}{9} = \frac{3}{27}$ and $\frac{3}{9} = \frac{9}{27}$.

317. Reduction Ascending. A fraction may be reduced to higher terms by *multiplying* both numerator and denominator by the same number. (Prin. III.)



318. Show by the diagrams that:

$1 = \frac{2}{2} = \frac{4}{4} = \frac{8}{8}$.
$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.
$\frac{1}{4} = \frac{2}{8}$. $\frac{3}{4} = \frac{6}{8}$.

$1 = \frac{3}{3} = \frac{6}{6} = \frac{12}{12}$.
$\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$. $\frac{2}{3} = \frac{4}{6} = \frac{8}{12}$.
$\frac{1}{6} = \frac{2}{12}$. $\frac{3}{6} = \frac{6}{12}$. $\frac{5}{6} = \frac{10}{12}$.

319. Fractions are reduced to a **common denominator** by reducing them to fractions having a denominator that is a ***common multiple*** of the given denominators.

Thus, $\frac{4}{5}$ and $\frac{2}{3}$: a common multiple of the denominators is 15; $\frac{4}{5} = \frac{12}{15}$ and $\frac{2}{3} = \frac{10}{15}$.

320. Fractions are reduced to the **least common denominator** by reducing them to fractions with a denominator that is the ***least common multiple*** of the denominators.

$l\ \frac{7}{10}$: The least common multiple of the denominators is
$d\ \frac{7}{10} = \frac{28}{40}$.

321. Rule for Least Common Denominator.—1. Find the least common multiple of the denominators, and use it as the least common denominator.

2. Divide the least common denominator by the denominators of the different fractions in succession, and multiply the corresponding numerators by the respective quotients for new numerators.

3. Write each numerator thus obtained over the least common denominator.

322. Reduce to common denominators:

1. $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$.
2. $\frac{1}{3}$; $\frac{1}{2}$; $\frac{1}{6}$.
3. $\frac{1}{6}$; $\frac{1}{6}$; $\frac{1}{9}$.
4. $\frac{1}{2}$; $\frac{2}{8}$; $\frac{1}{6}$.
5. $\frac{3}{4}$; $\frac{2}{6}$; $\frac{3}{8}$.
6. $\frac{2}{3}$; $\frac{3}{4}$; $\frac{4}{5}$.
7. $\frac{2}{4}$; $\frac{2}{6}$; $\frac{2}{8}$.
8. $\frac{3}{4}$; $\frac{5}{6}$; $\frac{7}{8}$.
9. $\frac{6}{8}$; $\frac{5}{9}$; $\frac{4}{10}$.
10. $\frac{5}{6}$; $\frac{4}{8}$; $\frac{3}{12}$.
11. $\frac{7}{10}$; $\frac{8}{15}$; $\frac{2}{3}$.
12. $\frac{4}{5}$; $\frac{5}{8}$; $\frac{3}{4}$.

323. Reduce to least common denominators:

1. $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$.
2. $\frac{1}{3}$; $\frac{1}{2}$; $\frac{1}{6}$.
3. $\frac{1}{6}$; $\frac{1}{6}$; $\frac{1}{9}$.
4. $\frac{1}{2}$; $\frac{2}{6}$; $\frac{1}{6}$.
5. $\frac{3}{4}$; $\frac{2}{6}$; $\frac{3}{8}$.
6. $\frac{2}{3}$; $\frac{3}{4}$; $\frac{4}{5}$.
7. $\frac{2}{4}$; $\frac{2}{6}$; $\frac{2}{8}$.
8. $\frac{3}{4}$; $\frac{5}{6}$; $\frac{7}{8}$.
9. $\frac{6}{8}$; $\frac{5}{9}$; $\frac{4}{10}$.
10. $\frac{9}{10}$; $\frac{2}{3}$; $\frac{3}{4}$.
11. $\frac{7}{12}$; $\frac{1}{2}$; $\frac{4}{5}$.
12. $\frac{6}{9}$; $\frac{3}{8}$; $\frac{5}{12}$.

324. A fraction is reduced to a whole or mixed number by dividing the numerator by the denominator. The quotient is the whole number; and the remainder, if any, is written over the denominator, to form the fraction in the mixed number.

325. Reduce to whole or mixed numbers:

1. $\frac{8}{6}$; $\frac{10}{5}$; $\frac{12}{6}$.
2. $\frac{7}{7}$; $\frac{10}{7}$; $\frac{14}{7}$.
3. $\frac{9}{8}$; $\frac{16}{8}$; $\frac{20}{8}$.
4. $\frac{10}{9}$; $\frac{12}{9}$; $\frac{27}{9}$.
5. $\frac{10}{4}$; $\frac{12}{4}$; $\frac{20}{4}$.
6. $\frac{12}{6}$; $\frac{13}{6}$; $\frac{14}{6}$.
7. $\frac{30}{15}$; $\frac{25}{15}$; $\frac{45}{15}$.
8. $\frac{24}{12}$; $\frac{25}{12}$; $\frac{32}{12}$.
9. $\frac{40}{20}$; $\frac{50}{20}$; $\frac{60}{20}$.

326. A whole or mixed number is reduced to an improper fraction by multiplying the whole number by the denominator of the fraction, and adding the numerator, if any, to the product. *The product is* written over the denominator *the numerator of the* required fraction.

327. Reduce to improper fractions:

1. 5 to 4ths.
2. 7 to 5ths.
3. 6 to 3ds.
4. 9 to 10ths.
5. $3\frac{1}{8}$; $4\frac{1}{2}$; $5\frac{3}{4}$.
6. $8\frac{3}{4}$; $9\frac{2}{3}$; $7\frac{4}{5}$.
7. 19 to 6ths.
8. $71\frac{1}{2}$; $14\frac{3}{7}$.
9. 75 to 3ds.
10. $45\frac{2}{3}$; $108\frac{7}{9}$.
11. $164\frac{1}{4}$; $278\frac{4}{8}$.
12. $99\frac{1}{11}$; $433\frac{1}{3}$.

328. Read at sight the number of:

$\frac{1}{2}$'s	in	1	2	3	4	5	$1\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{2}$	$4\frac{1}{2}$	$5\frac{1}{2}$
$\frac{1}{3}$'s	"	1	2	3	4	5	$1\frac{1}{3}$	$1\frac{2}{3}$	$2\frac{1}{3}$	$2\frac{2}{3}$	$3\frac{1}{3}$
$\frac{1}{4}$'s	"	1	2	3	4	5	$1\frac{1}{4}$	$1\frac{3}{4}$	$2\frac{1}{4}$	$2\frac{3}{4}$	$3\frac{1}{4}$
$\frac{1}{5}$'s	"	1	2	3	4	5	$1\frac{1}{5}$	$1\frac{2}{5}$	$1\frac{3}{5}$	$1\frac{4}{5}$	$2\frac{1}{5}$
$\frac{1}{6}$'s	"	1	2	3	4	5	$1\frac{1}{6}$	$1\frac{5}{6}$	$2\frac{1}{6}$	$2\frac{5}{6}$	$3\frac{1}{6}$
$\frac{1}{7}$'s	"	1	2	3	4	5	$1\frac{1}{7}$	$1\frac{3}{7}$	$2\frac{1}{7}$	$2\frac{6}{7}$	$3\frac{5}{7}$

329. State the reasons for the answer in each of the preceding examples:

Thus: $\frac{1}{8}$'s in $2\frac{3}{4}$: In 1 unit there are $\frac{4}{4}$; in 2 units, 2 times $\frac{4}{4}$, or $\frac{8}{4}$, which with the $\frac{3}{4}$ make $\frac{11}{4}$.

330. Read at sight the number of units in:

$\frac{6}{2}$	$\frac{8}{2}$	$\frac{8}{4}$	$\frac{10}{5}$	$\frac{6}{3}$	$\frac{18}{2}$	$\frac{20}{4}$	$\frac{16}{8}$	$\frac{14}{7}$	$\frac{21}{7}$	$\frac{16}{4}$
$\frac{10}{2}$	$\frac{12}{3}$	$\frac{15}{5}$	$\frac{15}{3}$	$\frac{16}{2}$	$\frac{21}{5}$	$\frac{14}{3}$	$\frac{18}{6}$	$\frac{15}{2}$	$\frac{22}{2}$	$\frac{25}{12}$
$\frac{12}{2}$	$\frac{13}{3}$	$\frac{12}{4}$	$\frac{15}{6}$	$\frac{15}{3}$	$\frac{18}{5}$	$\frac{19}{2}$	$\frac{18}{3}$	$\frac{17}{6}$	$\frac{17}{9}$	$\frac{30}{12}$

331. State the reason for the answer in each of the preceding examples:

Thus, for $\frac{6}{2}$: $\frac{2}{2}$ equal 1 unit; $\frac{6}{2}$ equal as many units as 2 is contained times in 6. Answer, 3 units.

332. Copy, and commit to memory:

\$1.00 equals \$1.	\$.20 equals $\$\frac{1}{5}$.	\$.125 equals $\$\frac{1}{8}$.
\$ .50 " $\$\frac{1}{2}$.	\$.40 " $\$\frac{2}{5}$.	\$.375 " $\$\frac{3}{8}$.
\$.25 " $\$\frac{1}{4}$.	\$.60 " $\$\frac{3}{5}$.	\$.625 " $\$\frac{5}{8}$.
$\$\frac{3}{4}$.	\$.80 " $\$\frac{4}{5}$.	\$.875 " $\$\frac{7}{8}$.

\$.$08\frac{1}{3}$ equals $\$\frac{1}{12}$; \$.$16\frac{2}{3}$ equals $\$\frac{1}{6}$.

ADDITION.

333. 1. How many are 6 sheep and 5 goats ?

2. How many are 6 animals and 5 animals ?

3. How many are 3 eagles and 4 hawks ? Seven what ?

4. How many are 3 birds and 4 birds ?

5. With what common name can 3 trout, 4 salmon, and 5 pickerel be added ?

6. With what common name can 4 elms, 5 willows, and 7 oaks be added ?

7. How can halves, fourths, and eighths be added ?

8. How can halves, thirds, and sixths be added ?

9. How can halves, thirds, and fourths be added ?

10. How can thirds, fourths, and fifths be added ?

11. How can any fractions be added ?

334. Fractions can be added only when they have a common name, that is, a **common denominator**; and, if necessary, they are to be reduced to a common denominator before adding.

335. To add fractions with the same denominator.

1. Matthew spent \$$\frac{1}{5}$ and had \$$\frac{3}{5}$ left. How much did he have at first ?

2. John having walked $\frac{3}{8}$ of a mile, had $\frac{5}{8}$ of a mile further to walk. What was the entire distance ?

3. A merchant sold $1\frac{2}{3}$ yards from a piece of calico, and then he had $2\frac{1}{3}$ yards left. How many yards had he at first ?

4. Edith had saved \$$3\frac{1}{4}$, when her father gave her \$$2\frac{3}{4}$. How much money did she then have ?

5. Jane bought silk for \$$12\frac{9}{10}$, and linen for \$$2\frac{6}{10}$. How much did both cost her ?

6. After Frank had spent \$$5\frac{4}{5}$, he had \$$4\frac{1}{5}$ left. How much did he have at first ?

7. James subtracted $6\frac{3}{8}$ from a number and then had $2\frac{5}{8}$. *What was the number ?*

336. Find the sums:

1. $\frac{2}{3}+\frac{2}{3}$.
2. $\frac{3}{4}+\frac{3}{4}$.
3. $\frac{3}{5}+\frac{3}{5}$.
4. $\frac{2}{6}+\frac{4}{6}+\frac{3}{6}$.
5. $\frac{5}{6}+\frac{1}{6}+\frac{5}{6}$.
6. $\frac{3}{7}+\frac{1}{7}+\frac{4}{7}$.
7. $\frac{5}{8}+\frac{3}{8}+\frac{1}{8}$.
8. $\frac{7}{9}+\frac{3}{9}+\frac{2}{9}$.
9. $\frac{9}{10}+\frac{7}{10}+\frac{3}{10}$.
10. $1\frac{1}{12}+\frac{9}{12}+\frac{7}{12}$.
11. $1\frac{4}{15}+1\frac{2}{15}+\frac{8}{15}$.
12. $\frac{3}{20}+\frac{5}{20}+\frac{17}{20}$.

337. Copy, and complete:

1.	2.	3.	4.	5.	6.
$25\frac{1}{3}$	$36\frac{3}{4}$	$48\frac{4}{5}$	$90\frac{5}{6}$	70	$29\frac{11}{12}$
46	$15\frac{2}{4}$	20	71	$42\frac{9}{10}$	$41\frac{9}{12}$
$37\frac{2}{3}$	29	$37\frac{3}{5}$	$82\frac{5}{6}$	$37\frac{6}{10}$	$10\frac{2}{12}$
109	$81\frac{1}{4}$				

338. To add fractions with different denominators.

1. Edgar earned $\$2\frac{3}{4}$ one week, and $\$2\frac{1}{2}$ the next week. How much did he earn in both weeks?

2. Fannie studied $3\frac{5}{8}$ hours on Monday, and $2\frac{5}{8}$ hours on Tuesday. How much did she study in the two days?

3. Mr. Frank sold $4\frac{7}{8}$ yards of silk to one lady, and $4\frac{5}{8}$ yards to another lady. How many yards altogether did he sell?

339. Since to add fractions they must have a common denominator, if the fractions are to be changed they should be changed to fractions whose denominators are the least common multiples of the given denominators.

340. Find the sums in their lowest terms and explain:

$\frac{1}{2}+\frac{3}{4}+\frac{5}{8}=\frac{15}{8}=1\frac{7}{8}$	$\frac{1}{2}+\frac{1}{3}+\frac{3}{6}=$	$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=$
$\frac{1}{2}+\frac{2}{4}+\frac{6}{8}=$ =	$\frac{1}{2}+\frac{2}{3}+\frac{5}{6}=$	$\frac{1}{2}+\frac{2}{3}+\frac{1}{4}=$
$\frac{3}{2}+\frac{1}{4}+\frac{7}{8}=$ =	$\frac{1}{2}+\frac{4}{3}+\frac{1}{6}=$	$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}=$
$\frac{1}{2}+\frac{3}{4}+\frac{6}{8}=$ =	$\frac{3}{2}+\frac{2}{3}+\frac{7}{6}=$	$\frac{5}{2}+\frac{4}{3}+\frac{3}{4}=$

Thus: the least common multiple of the denominators 2, 4, and 8 is 8.

$^1=\frac{4}{8}$; $\frac{3}{4}=\frac{6}{8}$; then $\frac{4}{8}+\frac{6}{8}+\frac{5}{8}=\frac{15}{8}=1\frac{7}{8}$.

sive fractions are reduced, and the numerators added

341. Find in their lowest terms the sums in Art. 334.

342. Copy, and complete:

1.	2.	3.	4.	5.
1	2	2		
$\$45\frac{2}{3}$..8	$\$41\frac{1}{2}$..3	$\$89\frac{3}{4}$..	$\$98\frac{5}{6}$....	$\$19\frac{3}{4}$
$37\frac{1}{4}$..3 | 12	$63\frac{2}{3}$..4 | 6	$56\frac{1}{2}$..	$67\frac{3}{4}$....	$97\frac{3}{8}$
$68\frac{1}{2}$..6	$20\frac{5}{6}$..5	$47\frac{7}{8}$..	$85\frac{5}{12}$..	$46\frac{1}{2}$
$\$151\frac{5}{12}$. Ans.	$\$126$. Ans.	$		

343. Rule for Addition.—1. Write the whole numbers, units under units, etc., with the fractions in a separate column at the right.

2. Reduce the fractions, if necessary, to a common denominator.

3. Add the fractions, and reduce the sum, if possible, to a whole or mixed number, writing the remaining fraction, if any, underneath the column of fractions, and adding the units, if any, with the units in the column of units.

344. Find the sums:

1.	2.	3.	4.	5.
$\$2,000\frac{1}{6}$	$\$7,346$	$\$2,041\frac{5}{8}$	$\$1,410\frac{2}{3}$	$\$5,170\frac{4}{5}$
$1,658\frac{3}{4}$	$1,236\frac{7}{8}$	$1,586\frac{1}{4}$	1,629	2,962
5,276	$4,729\frac{5}{6}$	2,182	$7,777\frac{4}{5}$	$1,238\frac{3}{4}$
$1,437\frac{1}{2}$	$1,628\frac{1}{4}$	$7,600\frac{11}{12}$	$3,304\frac{1}{3}$	$4,076\frac{5}{6}$

6.	7.	8.	9.	10.
$\$475\frac{5}{6}$	$\$107\frac{1}{2}$	$\$509\frac{2}{5}$	$\$769\frac{7}{8}$	$\$666\frac{2}{3}$
$1,222\frac{3}{5}$	$2,083\frac{1}{4}$	$3,087\frac{2}{3}$	$1,091\frac{1}{4}$	$2,883\frac{1}{4}$
$2,071\frac{2}{3}$	$1,509\frac{2}{3}$	$1,604\frac{1}{5}$	$4,806\frac{1}{2}$	$791\frac{1}{6}$
$4,460\frac{4}{5}$	$3,068\frac{5}{6}$	$2,781\frac{3}{5}$	$975\frac{5}{8}$	$1,088\frac{3}{4}$

345. Oral Problems. 1. Henry paid $\$\frac{5}{8}$ for a pair of skates, and $\$\frac{3}{8}$ for a pair of mittens. How much money did he spend?

2. James spent $3\frac{1}{4}$ for a bonnet, and $\$1\frac{1}{2}$ for ribbon. What was the amount of her purchases?

3. Mary cut from a piece of ribbon $2\frac{7}{8}$ yards, and $\frac{1}{4}$ of a yard; and she then had $\frac{1}{2}$ of a yard left in the piece. How much did she have altogether?

4. A plot of land was divided into three parts, containing respectively $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{8}$ of an acre. How many acres of land were there in the plot?

5. How many bushels of apples are there in three barrels containing respectively $1\frac{3}{4}$, $1\frac{1}{2}$, and $1\frac{7}{8}$ bushels?

6. James spent $\$\frac{1}{2}$, and then had $\$4\frac{3}{8}$ left. How much money did he have at first?

7. How much will James and Henry together earn, if James earns $\$2\frac{1}{2}$, and Henry, $\$2\frac{3}{4}$?

346. Find the sums:

1.	2.	3.	4.
$\$3,380\frac{1}{2}$	$\$9,090\frac{1}{10}$	$\$2,273\frac{1}{2}$	$\$1,101\frac{11}{12}$
$9,090\frac{1}{2}$	$10,101\frac{1}{2}$	$2,272\frac{2}{10}$	$12,202\frac{1}{2}$
$3,384\frac{1}{2}$	$1,010\frac{1}{10}$	$13,076\frac{1}{2}$	$4,789\frac{1}{2}$
$10,101\frac{1}{2}$	$2,525\frac{1}{2}$	$4,040\frac{1}{2}$	$25,093\frac{1}{2}$
$4,040\frac{1}{2}$	$2,222\frac{1}{2}$	$2,020\frac{1}{10}$	$70,706\frac{1}{12}$

5.	6.	7.	8.
$7,484\frac{1}{2}$	$2,020\frac{1}{2}$	$54,040\frac{1}{2}$	$78,081\frac{1}{2}$
$4,040$	$1,010$	$2,020\frac{1}{2}$	$91,910$
$7,070\frac{1}{2}$	$3,535\frac{1}{2}$	$7,575\frac{1}{2}$	$26,065\frac{1}{2}$
$2,525$	$1,262\frac{1}{2}$	$4,040$	$14,478$
$5,050\frac{1}{2}$	$3,030\frac{1}{2}$	$3,030\frac{1}{2}$	$91,012\frac{1}{2}$

347. Oral Problems. 1. Thomas paid $\$\frac{1}{2}$ for writing paper, $\$\frac{1}{4}$ for a book, and $\$\frac{1}{8}$ for a box of pens. How much did he pay for all?

2. [illegible] a dollar's worth of coal. If it cost $\$\frac{1}{2}$ a ton, how much is selling it?

3. Henry had \$.25 and earned \$$\frac{1}{2}$. What part of a dollar did he then have?

4. If John spends \$$\frac{1}{5}$ for a base ball, and \$.50 for a bat, what fraction of a dollar does he spend?

5. A merchant bought cloth for \$$\frac{3}{8}$ per yard, and sold it so as to gain \$.12$\frac{1}{2}$ per yard. What was the selling price?

6. Mr. Thompson sold wheat at \$.87$\frac{1}{2}$ per bushel, and then lost \$$\frac{1}{8}$. What did the wheat cost him?

7. Mr. Shaw's rent was \$41$\frac{2}{3}$ per month, and his landlord increased the rent \$4$\frac{1}{6}$. What was the rent after the increase?

8. How many dollars will a pair of blankets, \$18$\frac{1}{2}$, and a quilt, \$12$\frac{3}{4}$, together cost?

9. If cloth is bought for \$$\frac{5}{6}$, and sold at an advance of \$.16$\frac{2}{3}$, what is the selling price?

10. James walked $\frac{1}{2}$ of a mile, ran $\frac{1}{4}$ of a mile, and rode $1\frac{1}{3}$ miles. How far did he go?

11. After $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of a yard had been cut from a piece of silk, $\frac{5}{6}$ of a yard were left. How many yards did the piece contain at first?

12. A pole stands $2\frac{1}{2}$ feet in the ground, $1\frac{1}{3}$ feet in the water, and $12\frac{1}{4}$ feet in the air. How long is the pole?

13. Jane spent \$2$\frac{1}{2}$ and \$2$\frac{1}{4}$, and then she had \$4$\frac{3}{4}$ left. How much money did she have at first?

14. What is the number from which if $5\frac{3}{8}$ be subtracted, the result is $5\frac{3}{4}$?

15. Henry is now $12\frac{5}{6}$ years old. How old will he be in $1\frac{3}{4}$ years?

348. Written Problems. 1. How many acres are there together in four fields containing $150\frac{3}{5}$, $225\frac{1}{4}$, and $1{,}060\frac{4}{5}$ acres?

2. Mr. Thompson spent in four months \$560$\frac{1}{2}$, \$387$\frac{3}{4}$, \$869$\frac{9}{10}$, and \$747$\frac{4}{5}$. How much altogether did he spend?

3. If Mr. Brown deposits \$1,260$\frac{3}{5}$, \$906$\frac{1}{4}$, \$810$\frac{1}{2}$, and \$1,008$\frac{3}{4}$, *how much* money has he in bank?

4. What is the amount of the following bills: \$100$\frac{7}{8}$, \$250$\frac{3}{4}$, \$409$\frac{1}{4}$, \$672$\frac{1}{2}$, and \$511$\frac{5}{8}$?

5. Mr. Jenkins lends \$1,925$\frac{1}{2}$ to one man, \$473.34 to another, \$1,536$\frac{3}{4}$ to a third, \$2,732.13 to a fourth, and, then, he has \$2,560$\frac{4}{5}$ left. How much money did he have at first?

6. A man bought a house, \$6,275$\frac{1}{2}$; a barn, \$1,522.75. He spent \$2,760$\frac{3}{8}$ in repairs, and then sold all at a profit of \$3,276$\frac{3}{4}$. What was his selling price?

7. A farmer sold from his farm at different times 30$\frac{7}{8}$ acres, 83$\frac{5}{8}$ acres, 101$\frac{3}{8}$ acres, and 97$\frac{1}{2}$ acres. He then had 470$\frac{1}{8}$ acres left. How many acres did his farm first contain?

8. A coal merchant had in different coal pockets the following numbers of tons of coal: 709$\frac{9}{10}$; 1,008$\frac{3}{8}$; 1,407$\frac{1}{4}$; 3,300$\frac{3}{4}$; and 2,550$\frac{7}{10}$. How much coal in all did he have?

9. Mr. Thompson bought one farm for \$3,690$\frac{1}{5}$; another for \$2,976$\frac{1}{2}$; and a third for \$4,085$\frac{3}{4}$. He then had \$6,093$\frac{4}{5}$ left. How much money did he have at first?

10. What was the amount of a grocer's receipts in five days, the amounts for the several days being: \$2,086$\frac{1}{4}$; \$1,979$\frac{3}{8}$; \$2,506$\frac{1}{2}$; \$1,878$\frac{7}{10}$; and \$2,960$\frac{3}{4}$?

11. A merchant's expenses for a week of six days were: \$4,302$\frac{9}{10}$; \$2,578$\frac{3}{8}$; \$1,789$\frac{3}{10}$; \$2,145$\frac{1}{4}$; \$1,881$\frac{1}{2}$; and \$2,090$\frac{1}{20}$. What was the entire amount for the week?

12. A gentleman spent the following sums of money: \$1,564$\frac{7}{8}$; \$2,037$\frac{3}{4}$; \$1,980$\frac{1}{2}$; \$1,263$\frac{1}{4}$; and \$536$\frac{3}{8}$. He then had \$2,540$\frac{1}{4}$. How much did he have at first?

13. A remainder is 40,607$\frac{5}{6}$, and the subtrahend, 19,009$\frac{3}{4}$. What is the minuend?

14. What is the amount of one thousand sixty-nine and one-half; two thousand six hundred ninety and six-ninths; fifteen thousand five hundred ten and one-sixth; twenty-f[illegible]d twenty-five and two-thirds; and eleven thou-*sand* one-half?

SUBTRACTION.

349. 1. Miles had \$5, and spent \$$1\frac{1}{2}$. How much money did he then have ?

2. Henry earned \$4, and spent all but \$$2\frac{1}{4}$. How much did he spend ?

3. If Kate saves \$$\frac{3}{5}$ more than Jane, and thus saves \$$9\frac{9}{10}$, how much does Jane save ?

4. How much more does a hat at \$$6\frac{3}{4}$ cost, than one at \$$5\frac{1}{2}$?

5. Henry earned \$$8\frac{7}{8}$ per week, and spent \$6.50 per week. How much did he have left each week ?

6. If Martha's dress cost \$$8\frac{1}{2}$, and her bonnet, \$$3\frac{1}{3}$, her dress cost how much more than her bonnet ?

350. Fractions can be subtracted only when they have a common name, that is, a **common denominator**; and, if necessary, they are to be reduced to a common denominator before subtracting.

351. To subtract numbers when the fraction in the subtrahend is less than the fraction in the minuend.

1. If John studies Algebra for $2\frac{3}{4}$ hours daily, and Greek $2\frac{3}{8}$ hours daily, how much the more time per day does he give to Algebra ?

2. James was $5\frac{1}{2}$ feet tall, and John $3\frac{5}{12}$ feet. James was how much taller than John ?

3. A post stands $2\frac{1}{3}$ feet in the ground, and $4\frac{1}{12}$ feet in the air. How many more feet stand in the air than in the ground ?

352. Find the differences :

1. $\frac{1}{2} - \frac{1}{3}$.	4. $\frac{1}{2} - \frac{2}{8}$.	7. $\frac{2}{3} - \frac{1}{2}$.	10. $\frac{3}{4} - \frac{3}{5}$.
2. $\frac{1}{2} - \frac{1}{4}$.	5. $\frac{1}{3} - \frac{2}{9}$.	8. $\frac{3}{4} - \frac{2}{3}$.	11. $\frac{5}{6} - \frac{2}{3}$.
3. $\frac{1}{2} - \frac{1}{5}$.	6. $\frac{1}{4} - \frac{2}{12}$.	9. $\frac{4}{5} - \frac{3}{4}$.	12. $\frac{7}{8} - \frac{2}{3}$.

353. Make *problems for the* numbers in Art. 352.

354. Copy, and complete:

1.	2.	3.	4.	5.	6.
$\$40\frac{2}{3}$ .. 16 | 24	$\$61\frac{2}{3}$	$\$36\frac{5}{6}$	$\$250\frac{3}{4}$	$\$99\frac{3}{8}$	$\$415\frac{1}{2}$
$33\frac{3}{8}$ .. 9 |	$17\frac{3}{8}$	$29\frac{3}{4}$	$117\frac{5}{8}$	$73\frac{5}{8}$	$216\frac{2}{7}$
$\$7\frac{7}{24}$					

355. To subtract numbers when the fraction in the subtrahend is greater than the fraction in the minuend.

1. If James gives \$10 in payment for a silver watch costing him $\$7\frac{3}{8}$, how much change is due him?

$$\$10 = \$9\tfrac{8}{8}\ ;\ \$9\tfrac{8}{8} - \$7\tfrac{3}{8} = \$2\tfrac{5}{8}.$$

2. When Henry was 15 years old, how old was his sister, who is $2\frac{3}{4}$ years younger than he is?

3. Mr. Benson bought flour for $\$4\frac{3}{4}$ per barrel, and sold it for $\$6\frac{1}{4}$. How much money did he gain per barrel?

$$\$6\tfrac{1}{4} = \$5\tfrac{5}{4}\ ;\ \$5\tfrac{5}{4} - \$4\tfrac{3}{4} = \$1\tfrac{2}{4} = \$1\tfrac{1}{2}.$$

4. If coats are bought for $\$17\frac{4}{5}$, and sold for $\$22\frac{1}{2}$, how much is the profit?

$\$22\frac{1}{2} = \$22\frac{5}{10}$; $\$17\frac{4}{5} = 17\frac{8}{10}$; $\$22\frac{5}{10} = \$21\frac{15}{10}$; $\$21\frac{15}{10} - \$17\frac{8}{10} = \$4\frac{7}{10}$.

5. A lady purchased a pair of gloves for $\$1\frac{4}{5}$, and a parasol for $\$8\frac{1}{4}$. The parasol cost her how much more than the pair of gloves?

6. If a river flows $6\frac{5}{8}$ miles per hour, and a horse trots $7\frac{1}{4}$ miles per hour, how much the faster does the horse trot?

356. Copy, and complete:

1.	2.	3.	4.	5.
3	0	5	17	59
$\$24\frac{1}{8}$ .. 9 | 8	$\$41\frac{1}{5}$ .. 12 | 10	$\$96\frac{2}{3}$ .. 20 | 12	$\$18\frac{1}{8}$ .. 24 | 20	$\$60\frac{1}{3}$
$16\frac{3}{4}$ .. 6 |	$15\frac{1}{2}$ .. 5 |	$37\frac{3}{4}$ .. 9 |	$11\frac{3}{4}$ .. 15 |	$25\frac{3}{5}$
$\$7\frac{3}{8}$	$\$25\frac{7}{10}$	$\$58\frac{11}{12}$	$\$7\frac{9}{20}$	\$

Ex. 1 : $\frac{3}{4}$ cannot be taken from $\frac{1}{8}$; 1 unit is taken from the 24 units, leaving 23 units ; it is changed to $\frac{1}{8}$'s, making $\frac{8}{8}$'s, which with the $\frac{1}{8}$ *makes* $\frac{9}{8}$; $\frac{3}{4}$ *or* $\frac{6}{8}$ from $\frac{9}{8}$ leaves $\frac{3}{8}$, and 16 from 23, 7. Answer, $\$7\frac{3}{8}$.

357. Rule for Subtraction.—1. Write the subtrahend under the minuend, units under units, etc., with fractions, if any, at the right.

2. Reduce the fractions, if necessary, to a common denominator.

3. If the fraction in the subtrahend is less than that in the minuend, subtract it from the fraction in the minuend, and write the result in its lowest terms underneath the column of fractions.

4. If the fraction in the minuend is the less, take one unit from the integral part of the minuend, reduce it to a fraction having the common denominator; add it to the fraction in the minuend, and then subtract, writing the result in its lowest terms underneath the column of fractions. Then proceed, as in subtraction of whole numbers.

358. Find the remainders:

1.	2.	3.	4.	5.	6.
$63\frac{1}{3}$	$90\frac{1}{2}$	$27\frac{1}{2}$	$48\frac{2}{3}$	$99\frac{3}{8}$	$200\frac{1}{4}$
$48\frac{1}{2}$	$15\frac{5}{8}$	$19\frac{4}{6}$	$37\frac{3}{4}$	$56\frac{4}{6}$	$49\frac{4}{6}$

7.	8.	9.	10.	11.	12.
$450\frac{1}{2}$	$327\frac{5}{6}$	$450\frac{2}{3}$	$809\frac{5}{6}$	999	$474\frac{2}{3}$
$299\frac{5}{6}$	$188\frac{7}{8}$	$309\frac{4}{6}$	$748\frac{9}{10}$	$189\frac{19}{20}$	$297\frac{7}{8}$

359. Oral Problems. 1. If a baker buys 9 barrels of flour and uses $7\frac{4}{6}$ barrels, how many barrels will he have left?

2. A barrel contained $2\frac{1}{2}$ bushels of potatoes. After $\frac{7}{8}$ of a bushel had been used, how many bushels of potatoes were left in the barrel?

3. If Mr. Jacobs buys hats at $\$2\frac{1}{3}$, and sells them at $\$4$ each, how much does he gain on each hat?

4. A merchant had $27\frac{1}{3}$ yards of cloth in a piece. How many yards did he have in the piece, after selling $\frac{1}{2}$ a yard?

360. Find the remainders:

1.	2.	3.	4.	5.
$\$1{,}268\frac{1}{4}$	$\$4{,}320\frac{3}{8}$	$\$9{,}708\frac{1}{6}$	$\$8{,}647\frac{2}{5}$	\$10,050
$979\frac{3}{8}$	$2{,}608\frac{5}{6}$	$6{,}059\frac{3}{4}$	$5{,}088\frac{9}{10}$	$8{,}063\frac{7}{10}$

6.	7.	8.	9.	10.
$\$250.45\frac{1}{2}$	\$178.06	\$203.40	$\$578.33\frac{3}{4}$	\$900.89
$169.07\frac{5}{8}$	$99.77\frac{1}{2}$	$189.72\frac{1}{4}$	349.05	$75.99\frac{5}{8}$

11.	12.	13.	14.	15.
$50{,}000\frac{1}{2}$	$40{,}001\frac{1}{9}$	$27{,}738\frac{1}{3}$	$39{,}099\frac{1}{4}$	$17{,}648\frac{1}{3}$
$4{,}989\frac{7}{8}$	$39{,}078\frac{1}{5}$	$14{,}969\frac{1}{4}$	$29{,}089\frac{1}{3}$	$9{,}080\frac{1}{2}$

361. What is the value of:

1. $\frac{1}{4} + \frac{1}{2} - \frac{3}{8}$?
2. $\frac{2}{3} - \frac{3}{6} + \frac{1}{4}$?
3. $\frac{1}{8} + \frac{3}{4} - \frac{1}{2}$?
4. $\frac{4}{7} - \frac{1}{2} + \frac{3}{7}$?
5. $\frac{1}{3} + \frac{8}{9} - \frac{2}{3}$?
6. $\frac{5}{6} - \frac{1}{2} + \frac{2}{3}$?
7. $2\frac{1}{2} - \frac{7}{8} + \frac{1}{4}$?
8. $4\frac{1}{4} - \frac{2}{3} + \frac{1}{2}$?
9. $3\frac{3}{5} - \frac{3}{4} + \frac{1}{5}$?

362. Oral Problems. 1. If James buys a book for $\$\frac{1}{10}$, and gives in payment $\$\frac{1}{4}$, how much change is due him?

2. Martha bought a slate for \$.11, and gave $\$\frac{1}{5}$ in payment. How much change should she receive?

3. A grocer bought sugar for $7\frac{1}{2}$ ¢ a pound, and sold it at $\$1\frac{1}{2}$ per pound. How much did he gain per pound?

4. If a yard of cloth costs $11\frac{1}{4}$ ¢, and is sold for $\$\frac{1}{8}$, what is the profit?

5. What is the loss per bushel, if potatoes are bought for \$.90 a bushel, and sold for $\$\frac{7}{8}$ a bushel?

6. At what price were goods bought which were sold for 55 cents per yard, at a profit of $\$\frac{1}{6}$?

7. Thomas can walk $4\frac{1}{8}$ miles an hour; he can run $6\frac{1}{2}$ miles an hour. How much farther per hour can he run?

8. If a pound of butter is bought for \$.28, and sold for
's the profit?

9. A pole stands $14\frac{5}{8}$ feet in the air and $2\frac{1}{2}$ feet in the ground. How many more feet are in the air than in the ground ?

10. Jane is $12\frac{1}{12}$ years old ; and Edith, $11\frac{1}{2}$ years old. Jane is how much older than Edith ?

11. A yard of cloth was bought for \$.33$\frac{1}{3}$, and sold for \$$\frac{3}{8}$. What part of a dollar was the gain ?

12. John's wages was increased from \$$7\frac{1}{2}$ to \$$9\frac{1}{4}$ per week. How much was the increase ?

363. Written Problems. 1. A house and lot were bought for \$5,750.50, and sold for \$$6,183\frac{1}{4}$. What was the gain ?

2. In a coal yard there were $20,460\frac{3}{20}$ tons of coal. How many tons were left after $9,090\frac{3}{4}$ tons had been sold ?

3. Mrs. Jones gave two \$10 bills in payment for a dress costing \$$15\frac{7}{8}$. How much change was due her ?

4. If a gold watch and chain cost \$$255\frac{3}{4}$, and a silver watch and chain cost \$$37\frac{9}{10}$, how much the more does the gold watch and chain cost ?

5. Arthur paid \$75 for his velocipede ; he sold it for \$$62\frac{4}{5}$. How much did he lose ?

6. If a merchant buys chairs at \$$23\frac{1}{4}$, and sells them at \$$25\frac{1}{5}$, how much does he gain ?

7. A city lot was bought for \$$66\frac{2}{3}$ per foot, and sold for \$$71\frac{1}{10}$ per foot. How much was the profit ?

8. If a farm containing 1,000 acres of land is divided into two parts, one of which contains $769\frac{3}{20}$ acres, how many acres does the other part contain ?

9. If carpet costing \$$1\frac{7}{8}$ per yard is sold for \$$2.33\frac{1}{3}$ a yard, what part of a dollar profit is made on each yard ?

10. A mass of metal in a bell was composed of $38,693\frac{1}{2}$ pounds of copper and zinc. The copper weighed $21,960\frac{3}{4}$ pounds. How much did the zinc weigh ?

11. Silk was bought at \$1.75 per yard, and sold at \$2$\frac{1}{8}$ per yard. What part of a dollar profit was made on each yard?

12. Mr. Mason had \$2,050$\frac{1}{2}$ in bank, and Mr. Brown \$1,979$\frac{19}{20}$. Mr. Mason had how much more money in bank than Mr. Brown had?

13. From one thousand fifty and five-sixths, subtract 907$\frac{6}{7}$. What is the remainder?

14. If seventeen hundred seven and seven-eighths is taken from two thousand two and four-fifths, how much is left?

MISCELLANEOUS PROBLEMS.

364. Oral Problems. 1. Martha bought a slate, \$.11; a pencil, \$.03; a sponge, \$.05. She gave \$$\frac{1}{5}$ in payment. How much change is due her?

2. Three girls spent \$2 in going to the park. Jane spent \$$\frac{3}{4}$; Sarah \$.50; and Edith the remainder. How much money did Edith spend?

3. Kate bought braid for \$$\frac{1}{4}$, and muslin for \$$\frac{1}{5}$; she gave in payment \$$\frac{1}{2}$. How much change should she receive?

4. Ned had \$$\frac{4}{5}$, and spent \$.42 for a book; \$.08 for a slate; and \$$\frac{1}{4}$ for a ball. What fraction of a dollar did he have left?

5. Mr. Brown had \$24. How much more must he have to buy a suit of clothes for \$44, and have \$6 left?

6. If John earns \$150 a year, and spends \$104 for board, \$16 for clothing, and \$10 for other expenses, how much money can he save in 5 years?

7. Jane had \$2, and spent \$$\frac{1}{4}$ for candy, \$$\frac{3}{4}$ for silk, and \$$\frac{4}{5}$ for lace. How much money did she have left?

8. Which costs the more, 9 yards of ribbon at 5 ¢, or 6 yards at 6 ¢; and how much more?

9. Henry bought four books, for which he paid \$1.50. *t he* paid \$$\frac{1}{4}$; for the second \$$\frac{1}{3}$; and for the *w* much did he pay for the fourth book?

10. How much change should Joseph receive on paying, with $\$\frac{3}{4}$, for 3 books at 12 ¢ each and 3 slates at 8 ¢ each?

11. If James earns \$20 per week, and spends \$12 per week, how long will it take him to save \$40, and buy a suit of clothes for \$56?

12. Sarah bought silk at four different prices: \$2, $\$1\frac{1}{4}$, $\$2\frac{1}{2}$, and $\$2\frac{1}{4}$. What was the average price per yard?

13. How much money will Mrs. Johnson have left if, having four \$10 bills, and two \$5 bills, she buys 24 yards of carpet at \$2 per yard?

14. If 7 slates cost \$.49, how much will 100 cost?

15. Ellen bought a melon, \$.15; a pound of coffee, $\$\frac{1}{4}$; and she gave in payment $\$\frac{1}{2}$. What fraction of a dollar should she receive in change?

16. Which will cost the more, and how much; 12 pounds of tea at $\$\frac{3}{4}$ per pound; or, 100 pounds of sugar at \$.12?

17. To gain $\$\frac{3}{8}$ on cloth costing $\$1\frac{1}{4}$, what is the selling price?

18. How much more than $\$\frac{7}{8}$ will 4 books at \$.25, and 5 pencils at \$.05 cost?

365. Written Problems. 1. A ship is owned by three men. One man owns $\frac{9}{20}$ of the ship; and another man $\frac{1}{3}$ of the ship. What part of the ship does the third man own?

2. Mr. Baldwin had $\$1{,}090\frac{1}{2}$ in bank. He deposited $\$527\frac{3}{4}$, and then drew his check for $\$1{,}265\frac{1}{8}$. What was his balance?

3. A gentleman had \$1,000 in bank. How much has he after paying out checks for $\$233\frac{1}{4}$, $\$350\frac{1}{2}$, and $\$175\frac{4}{8}$?

4. If Mr. Jenkins buys a gold watch and chain for $\$150\frac{4}{8}$, and a pair of bracelets for $\$90\frac{3}{4}$, and pays for them with a check for \$250, what money should be returned to him?

5. If a flour merchant buys 3,000 barrels of flour at \$4 per barrel, and *sells them* all for $\$15{,}760\frac{7}{10}$, what is his *profit?*

6. How many pounds of flour are left in 10 barrels containing 196 pounds each, after $1,473\frac{9}{16}$ pounds have been used?

7. If a lady owes a grocer $\$24\frac{7}{10}$, $\$31\frac{3}{4}$, and $\$25\frac{4}{5}$, how much does she owe after paying \$65.25 on account?

8. A merchant's receipts for two days are $\$1,877\frac{1}{4}$ and $\$2,036\frac{9}{10}$. What has he left after paying out $\$2,460\frac{3}{4}$?

9. A farmer had two fields containing respectively $137\frac{1}{2}\frac{1}{5}$ and $240\frac{1}{2}$ acres. How much must he buy to have 500 acres?

10. A coal dealer bought 300 tons of coal at \$3.75 per ton, and sold it all for $\$1,267\frac{3}{8}$. How much did he gain?

11. How many gallons will be left if $140\frac{1}{4}$ gallons of vinegar are taken from barrels containing the following numbers of gallons: $25\frac{1}{2}$, $29\frac{1}{4}$, $30\frac{3}{4}$, $31\frac{1}{2}$, $28\frac{3}{8}$, $29\frac{4}{5}$, and $30\frac{7}{8}$?

12. If a man buys a wagon for $\$40\frac{1}{3}$, and spends $\$10\frac{1}{6}$ in repairs, how much does he gain by selling it for \$55?

MULTIPLICATION.

366. 1. If one yard of cloth costs $\$\frac{4}{5}$, what will 10 yards cost?

2. At \$6 per barrel, what is $\frac{2}{3}$ of a barrel of flour worth?

3. What will 5 pairs of kid gloves cost at $\$1\frac{1}{4}$ per pair?

4. In a barrel there are $2\frac{1}{2}$ bushels of apples. How many bushels of apples are there in half of the barrel?

5. If Matthew earns $\$\frac{9}{10}$ in one day, how much does he earn in $\frac{2}{3}$ of a day?

367. To find a part of a fractional unit.

1. Edgar had $\frac{1}{2}$ of an orange, and he gave to his brother $\frac{1}{2}$ of what he had. What part of the orange did he give to his brother? $\frac{1}{2}$ of $\frac{1}{2}$ equals?

2. If coffee cost $\$\frac{1}{3}$ per pound, how much does $\frac{1}{2}$ of a pound cost? $\frac{1}{2}$ of $\frac{1}{3}$ equals?

is the cost of $\frac{1}{2}$ of a yard of cloth, if each yard $\frac{1}{2}$ of $\frac{1}{4}$ equals?

4. If John earns $\$\frac{1}{2}$ per hour, how much does he earn in $\frac{1}{3}$ of an hour? $\frac{1}{3}$ of $\frac{1}{2}$ equals ?

5. Chintz was bought at $\$\frac{1}{3}$ per yard, and sold so as to gain $\frac{1}{3}$ of the cost. What part of a dollar was the gain ?

368. To find a part of a fractional unit, multiply the denominator of the given fractional unit by the number of the parts into which the unit is divided.

369. To multiply a fraction by a fraction.

1. If a yard of ribbon costs $\$\frac{3}{8}$ per yard, how much will $\frac{4}{5}$ of a yard cost ?

$\frac{1}{5}$ of $\frac{1}{8} = \frac{1}{40}$; $\frac{4}{5}$ of $\frac{3}{8} = \frac{3}{40}$; $\frac{4}{5}$ of $\frac{3}{8} = \frac{12}{40} = \frac{3}{10}$. Ans. $\$\frac{3}{10}$.

2. A train of cars travelled at the rate of a mile in $\frac{4}{5}$ of a minute. In what time did it travel $\frac{2}{3}$ of a mile ?

$\frac{1}{3}$ of $\frac{4}{5} = \frac{4}{15}$; $\frac{1}{3}$ of $\frac{4}{5} = \frac{4}{15}$; $\frac{2}{3}$ of $\frac{4}{5} = \frac{8}{15}$. Ans. $\frac{8}{15}$ of a minute.

3. John picked $\frac{7}{8}$ of a bushel of berries, and sold $\frac{2}{3}$ of them. What part of a bushel did he sell ? $\frac{2}{3} \times \frac{7}{8} = \frac{14}{24} = \frac{7}{12}$.

4. If Jane saves in a week $\frac{5}{8}$ as much as her sister, who saves $\$\frac{4}{10}$, how much does she save ?

5. What fraction divided by $\frac{4}{5}$ gives $\frac{5}{8}$? $\frac{5}{8} \times \frac{5}{4} = \frac{1}{2}$.

370. Rules for Multiplication.—I. Multiply the numerators together for the numerator, and the denominators together for the denominator of the answer ; **or,**

II. After indicating the multiplication, cancel, if possible, and then multiply together the remaining factors of the numerator for the numerator, and those of the denominator for the denominator of the answer.

371. Oral Problems. 1. If one pound of sugar costs $\$\frac{1}{8}$, what will $\frac{2}{3}$ of a pound cost ?

2. What is the cost of $\frac{3}{4}$ of a pound of tea at $\$\frac{3}{5}$ a pound ?

3. If a yard of linen costs $\$\frac{1}{3}$, what costs $\frac{3}{4}$ of $\frac{1}{2}$ of a yard ?

4. What will $\frac{1}{2}$ of $\frac{2}{3}$ of a yard of cloth cost at $\$\frac{3}{4}$ per yard ?

5. *At* $\$\frac{1}{7}$ *per pound, what* is $\frac{5}{8}$ of a pound of butter worth ?

6. If a pound of rice costs $\$\frac{1}{11}$, what is the cost of $\frac{1}{16}$ of a pound?

7. Mr. Johnson owns $\frac{2}{3}$ of a ship. He sold $\frac{3}{4}$ of his share. What part of the ship did he sell?

372. Find the products:

Without Cancellation.		With Cancellation.	
1. $\frac{2}{3}$ of $\frac{4}{5}$.	6. $\frac{3}{4} \times \frac{5}{7}$.	11. $\frac{2}{3}$ of $\frac{6}{8}$.	16. $\frac{3}{4} \times \frac{4}{9}$.
2. $\frac{3}{4}$ of $\frac{3}{5}$.	7. $\frac{2}{3} \times \frac{4}{5}$.	12. $\frac{3}{4}$ of $\frac{2}{9}$.	17. $\frac{5}{7} \times \frac{3}{10}$.
3. $\frac{4}{5}$ of $\frac{3}{7}$.	8. $\frac{5}{7} \times \frac{2}{3}$.	13. $\frac{4}{5}$ of $\frac{8}{15}$.	18. $\frac{4}{5} \times \frac{3}{8}$.
4. $\frac{3}{8}$ of $\frac{2}{5}$.	9. $\frac{4}{9} \times \frac{5}{9}$.	14. $\frac{3}{8}$ of $\frac{4}{6}$.	19. $\frac{3}{10} \times \frac{5}{6}$.
5. $\frac{5}{8}$ of $\frac{5}{8}$.	10. $\frac{3}{5} \times \frac{4}{7}$.	15. $\frac{5}{6}$ of $\frac{3}{10}$.	20. $\frac{7}{12} \times \frac{2}{3}$.

NOTE.—By analysis, $\frac{3}{5}$ of $\frac{4}{5}$ is found to be the same as $\frac{3}{5} \times \frac{4}{5}$.

373. To multiply a fraction by an integer.

1. At the rate of $\frac{5}{6}$ of a mile per hour, how many miles can be travelled in 12 hours?

$12 \times \frac{5}{6} = \frac{60}{6} = 10$; or, $\overset{2}{\cancel{12}} \times \frac{5}{\cancel{6}} = 10$. Ans., 10 miles.

2. A farmer sold 8 village lots, each containing $\frac{3}{4}$ of an acre. How many acres in all did he sell?

3. What is the cost of 6 books, each book costing $\$\frac{3}{8}$?

$6 \times \frac{3}{8} = \frac{18}{8} = 2\frac{2}{8} = 2\frac{1}{4}$. Ans. $\$2\frac{1}{4}$.

4. In selling coal, a merchant made $\$\frac{5}{8}$ on each ton. How much did he make in selling 9 tons of the coal?

5. What fraction divided by 7 gives $\frac{1}{14}$?

374. Find the cost of the following:

Groceries.			Notions.		
	lb.	$		yd.	$		doz.	$
1.	10 @	$\frac{4}{5}$.	5. 25 @	$\frac{2}{3}$.	9. 24 @	$\frac{3}{8}$.	13. 12 @	$\frac{1}{8}$.
2.	9 @	$\frac{2}{3}$.	6. 40 @	$\frac{3}{4}$.	10. 36 @	$\frac{5}{6}$.	14. 18 @	$\frac{1}{4}$.
3.	8 @	$\frac{5}{8}$.	7. 45 @	$\frac{3}{5}$.	11. 28 @	$\frac{5}{7}$.	15. 28 @	$\frac{3}{7}$.
4.	7 @	$\frac{3}{4}$.	8. 50 @	$\frac{3}{8}$.	12. 90 @	$\frac{7}{10}$.	16. 63 @	$\frac{2}{3}$.

In multiplying, any whole number may be used as a fraction for its denominator.

375. Written Problems. 1. At $\$\frac{2}{3}$ per yard, what is the cost of 100 yards of cloth?

$$3\overline{)200}\quad 66\tfrac{2}{3}$$

2. Mr. Benjamin bought 500 bushels of apples at $\$\frac{3}{5}$ per bushel. What was the entire cost?

$$\begin{array}{r} 500 \\ 3 \\ \hline 5\overline{)1500} \end{array}\quad 300 \qquad \text{or,} \qquad 5\overline{)500}\quad \begin{array}{r} 3 \\ \hline 300 \end{array}$$

NOTE.—In practice, the integer is multiplied by the fraction.

3. How many bushels of strawberries are there in 250 baskets, each holding $\frac{3}{4}$ of a bushel?

$$\begin{array}{r} 250 \\ 3 \\ \hline 750 \end{array}\qquad 4\overline{)750}\quad 187\tfrac{2}{4}$$

4. A merchant gained $\$\frac{3}{8}$ on each of 750 yards of cloth. How much altogether did he gain?

5. If a grocer loses $\$\frac{7}{12}$ on each of 150 barrels of sugar, how much in all does he lose?

6. How much money will be received for 1,250 excursion tickets, at $\$\frac{3}{10}$ each?

7. If the water of a brook flows $\frac{1}{7}$ of a mile an hour, how far will it flow in 10 days?

376. Find the products:

	1.	2.	3.	4.	5.
$\frac{2}{5}\times$	$250.50	$438.75	$690.45	$239.98	$1008.60
	6.	**7.**	**8.**	**9.**	**10.**
$\frac{3}{4}\times$	$417.08	$690.95	$816.60	$754.40	$919.89
	11.	**12.**	**13.**	**14.**	**15.**
$\frac{4}{5}\times$	$225.47	$711.84	$655.09	$210.81	$313.03
	16.	**17.**	**18.**	**19.**	**20.**
$\frac{7}{8}\times$	$707.70	$800.08	$460.37	$404.04	$505.05

377. To multiply an integer by a fraction, or to find a required part of an integer.

1. A house and lot together cost $4,500. The lot cost $\frac{1}{3}$ of this sum. How much money did the lot cost? $\frac{1}{3}$ of $4500 = ?$

2. A gentleman left to his wife $\frac{3}{4}$ of his property, which was valued at $32,500. How much property did he leave *his wife?*

3. Mr. Brown owned $\frac{4}{5}$ of a business worth $75,000. What was the value of his share of the business?

4. There were 1,450 acres in a farm; $\frac{5}{8}$ of it was planted with wheat. How many acres of wheat were there?

378. Find the values of the following:

Without Cancellation.				With Cancellation.			
yd.	$	bu.	$	bl.	$	T.	$
1. $\frac{3}{4}$ @	5.	5. $\frac{2}{3}$ @	10.	9. $\frac{9}{10}$ @	5.	13. $\frac{8}{9}$ @	6.
2. $\frac{4}{5}$ @	4.	6. $\frac{8}{9}$ @	7.	10. $\frac{7}{5}$ @	10.	14. $\frac{5}{3}$ @	15.
3. $\frac{5}{6}$ @	3.	7. $\frac{3}{7}$ @	12.	11. $\frac{2}{3}$ @	9.	15. $\frac{3}{12}$ @	14.
4. $\frac{7}{8}$ @	6.	8. $\frac{5}{8}$ @	2.	12. $\frac{4}{7}$ @	14.	16. $\frac{2}{6}$ @	10.

379. Written Problems. 1. A pays $540 a year for rent; B pays $\frac{2}{3}$ as much. What is B's rent per year?

2. Mr. Johnson paid $1,250 per year for board and $\frac{3}{4}$ of this sum for help. How much per year did he pay for help?

3. Of a farm of 2,275 acres, $\frac{4}{5}$ was planted with corn. How many acres of corn were there?

4. A house that cost $12,848 was sold for $\frac{7}{8}$ of its cost. What was the selling price?

5. James earned $700 per year and spent $\frac{7}{10}$ of it. How much money did he spend?

6. A mass of metal weighed 3,492 pounds; $\frac{11}{12}$ of it was iron. What was the weight of the iron?

7. If silk plush is sold for $\frac{8}{7}$ of its cost, which was $3.50 per yard, what is the selling price?

8. A company failing for $1,000,000 paid $.83$\frac{1}{3}$ on the dollar. How much did they pay?

9. An army consisted of 150,424 soldiers, $\frac{7}{8}$ of them being under 50 years of age. How many of the soldiers were under 50 years of age?

a forest there were 2,526 trees, of which $\frac{5}{6}$ were
es. How many of the poplar trees were there?

11. Mr. Matthews paid a debt of $4,560.30. He paid $\frac{7}{10}$ of the amount in gold. How much gold did he pay?

12. If a dry-goods firm pays $25,500 a year rent, how much does it pay for 11 months?

380. To multiply a fraction by a fraction, or to find a required part of a fraction.

1. Henry earned $\$\frac{5}{6}$ per hour. How much did he earn in $\frac{3}{4}$ of an hour? $\frac{1}{4}$ of $\$\frac{1}{6} = \$\frac{1}{24}$; $\frac{1}{4}$ of $\$\frac{5}{6} = \$\frac{5}{24}$; $\frac{3}{4}$ of $\$\frac{5}{6} = \$\frac{15}{24}$.

NOTE.—The answer is the same whether **of** or × is used. $\frac{3}{4}$ **of** $\frac{5}{6} = \frac{15}{24}$; $\frac{3}{4} \times \frac{5}{6} = \frac{15}{24}$.

2. Frank has $\$\frac{9}{10}$ and John has $\frac{2}{3}$ times as much. How much money has John?

3. Jane spent $\$\frac{4}{5}$ for lace and $\frac{3}{8}$ of that amount for ribbon. How much did the ribbon cost her?

4. Thomas bought his sled for $\$\frac{5}{8}$ and sold it so as to lose $\frac{3}{5}$ of his money. How much did he lose?

5. If wheat costs $\$1\frac{1}{2}$ per bushel and is sold for $\frac{12}{11}$ of its cost price, what is the selling price per bushel?

6. If drawing books are bought for $\$.62\frac{1}{2}$ and sold so as to gain $\frac{3}{5}$ of the cost price, what is the profit?

7. A hat was bought for $\$1.12\frac{1}{2}$ and sold so that $\frac{1}{3}$ of the cost was lost. How much money was lost?

8. How much are 3 pecks of potatoes worth, if a bushel is worth $\$.66\frac{2}{3}$?

381. Written Problems. 1. If a horse trots $\frac{3}{8}$ of a mile in a minute, how far does he trot in $\frac{2}{3}$ of a minute?

2. James cut a stick $\frac{5}{12}$ of a foot long into two parts. How long was that part of the stick which was $\frac{4}{5}$ of the whole?

3. Matthew bought a knife for $\$.50$ and sold it at a gain of $\frac{3}{5}$ of the cost price. What was the gain?

4. Edgar's skates cost $\frac{4}{3}$ as much as his sled. His sled cost $\$.75$. *What did his* skates cost?

5. In one basket there was $\frac{2}{3}$ of a bushel of blackberries, in another $\frac{9}{10}$ as much. What part of a bushel was there in the second basket?

6. A number divided by $\frac{7}{8}$ gives a quotient of $\frac{2}{3}$. What is the number?

7. If a man can do $\frac{2}{3}$ of $1\frac{1}{2}$ of a piece of work in a week of 6 days, how much of the work can he do in one day?

382. Find the cost of the following with or without cancellation:

Notions.

yards.	$		dozen.	$
1. $\frac{5}{6}$ @	$\frac{3}{4}$.	5.	$\frac{1}{2}$ @	.80.
2. $\frac{2}{3}$ @	$\frac{1}{2}$.	6.	$\frac{3}{4}$ @	.66⅔.
3. $\frac{4}{3}$ @	$\frac{3}{8}$.	7.	$1\frac{7}{12}$ @	.37½.
4. $\frac{7}{8}$ @	$\frac{4}{5}$.	8.	$\frac{1}{6}$ @	.75.

Confections.

pounds.	$		pounds.	$
9. $\frac{3}{4}$ @	$\frac{4}{5}$.	13.	$\frac{1}{2}$ @	.66⅔.
10. $\frac{1}{4}$ @	$\frac{1}{5}$.	14.	$\frac{1}{4}$ @	$\frac{1}{4}$.
11. $\frac{10}{16}$ @	.40.	15.	$\frac{5}{8}$ @	$\frac{4}{10}$.
12. $\frac{3}{8}$ @	.75.	16.	$\frac{7}{16}$ @	.80.

Dry Goods.

yards.	$		yards.	$
17. $\frac{2}{3}$ @	.50.	21. $\frac{7}{8}$ @		$\frac{2}{3}$.
18. $\frac{5}{8}$ @	.66⅔.	22. $\frac{1}{4}$ @		$\frac{1}{5}$.
19. $\frac{3}{4}$ @	$\frac{1}{2}$.	23. $\frac{1}{3}$ @		.50.
20. $\frac{1}{2}$ @	$\frac{1}{4}$.	24. $\frac{1}{8}$ @		$\frac{4}{5}$.

Groceries.

pecks.	$		pounds.	$
25. $\frac{1}{2}$ @	$\frac{1}{5}$.	29.	$\frac{11}{16}$ @	.81⅔.
26. $\frac{1}{4}$ @	$\frac{1}{3}$.	30.	$\frac{3}{8}$ @	.50.
27. $\frac{3}{4}$ @	$\frac{2}{5}$.	31.	$\frac{3}{4}$ @	.80.
28. $\frac{5}{8}$ @	.70.	32.	$\frac{1}{2}$ @	$\frac{1}{8}$.

383. To multiply a mixed number by an integer.

1. How many yards together are there in 7 remnants each containing $1\frac{3}{8}$ yards?

$7 \times \frac{3}{8} = \frac{21}{8} = 2\frac{5}{8}$; $7 \times 1 = 7$; $7 + 2\frac{5}{8} = 9\frac{5}{8}$. Ans., $9\frac{5}{8}$ yd.

2. What is the cost of $9\frac{3}{8}$ yards of cloth at \$4 per yard?

$4 \times \frac{3}{8} = \frac{12}{8} = 1\frac{1}{2}$; $4 \times 9 = 36$; $36 + 1\frac{1}{2} = 37\frac{1}{2}$. Ans., \$37½.

NOTE.—In practice, the mixed number is usually multiplied by the integer, even when, in theory, the integer should be multiplied by the mixed number.

3. *If one* pound of sugar costs $7\frac{1}{4}$ cents, how much will 7 *pounds cost?*

4. At $6\frac{1}{4}$ cents, what is the cost of 15 yards of muslin? Of 25 yards?

5. What is the cost of 5 dozen cloak buttons at $\$1.16\frac{2}{3}$ each?

6. If one yard of lace is worth $\$1.33\frac{1}{3}$, how much is 40 yards of the lace worth?

7. At $\$4\frac{3}{4}$ a pair, what is the cost of 16 pairs of shoes?

384. Find by cancellation the cost of the following:

Fruits. bu. $	Drugs. lb. $	Carpets. yd. $	Notions. doz. $
1. $4\frac{1}{2}$ @ 2.	5. 2 @ $1\frac{2}{3}$.	9. 45 @ $1\frac{5}{6}$.	13. $5\frac{2}{3}$ @ 2.
2. $6\frac{3}{8}$ @ 3.	6. 3 @ $4\frac{5}{8}$.	10. 33 @ $2\frac{1}{3}$.	14. 7 @ $1\frac{5}{8}$.
3. $5\frac{2}{3}$ @ 4.	7. 5 @ $2\frac{1}{4}$.	11. 35 @ $1\frac{4}{5}$.	15. $6\frac{1}{2}$ @ 2.
4. $3\frac{5}{8}$ @ 2.	8. 9 @ $1\frac{3}{8}$.	12. 50 @ $2\frac{1}{2}$.	16. 8 @ $1\frac{3}{4}$.

NOTE.—The mixed numbers may be expressed as improper fractions.

385. Written Problems. 1. How much will 508 acres of land cost at $\$45\frac{1}{2}$ per acre?

2. If each clock costs $\$2\frac{1}{4}$, how much will 150 clocks cost?

3. What is the value of 275 sheep at $\$3\frac{1}{3}$?

$$\begin{array}{r} 508 \\ 45\frac{1}{2} \\ \hline 254 = \frac{1}{2} \\ 2540 \\ 2032 \\ \hline \$23,114 \end{array}$$

4. At the rate of $15\frac{3}{4}$ miles an hour, how far will a ship sail in 250 hours?

5. How much will it cost to clothe a regiment of 1,120 men at $\$31\frac{2}{3}$ each?

6. If sound travels 1,095$\frac{7}{3}$ feet in a second, how far will it travel in one minute?

7. At the rate of $45\frac{7}{8}$ miles per hour, how far will a train travel in two days?

$$\begin{array}{r} 250 \\ 15\frac{3}{4} \\ 62\frac{1}{2} = \frac{1}{4} \\ 3 \\ \hline 187\frac{1}{2} = \frac{3}{4} \\ 1250 \\ 250 \\ \hline 3,937\frac{1}{2} \text{ mi.} \end{array}$$

8. What is the value of 10 pieces of muslin, each containing 45 yards, at $\$.16\frac{2}{3}$ a yard?

9. A number divided by $14\frac{5}{6}$ gives a quotient of 258. What is the number?

10. The profit on each acre being $\$15\frac{5}{6}$, how much profit is made by selling 7,693 acres?

386. Find the products:

	1.	2.	3.	4.	5.
$2\frac{2}{5}$ ×	\$120.24	\$307.48	\$90.90	\$75.75	\$100.32
	6.	**7.**	**8.**	**9.**	**10.**
$4\frac{5}{8}$ ×	\$37.92	\$206.56	\$387.44	\$569.04	\$740.96
	11.	**12.**	**13.**	**14.**	**15.**
$3\frac{1}{6}$ ×	\$47.70	\$804.42	\$570.93	\$700.02	\$860.04
	16.	**17.**	**18.**	**19.**	**20.**
$5\frac{4}{5}$ ×	\$200.50	\$475.35	\$816.25	\$400.40	\$370.35

387. To multiply mixed numbers that are not large.

1. If John earns $\$2\frac{1}{2}$ per day, how much will he earn in $3\frac{1}{4}$ days? $\$2\frac{1}{2} = \$\frac{5}{2}$; $3\frac{1}{4} = \frac{13}{4}$; $\frac{13}{4} \times \frac{5}{2} = \frac{65}{8} = 8\frac{1}{8}$. Ans., $\$8\frac{1}{8}$.

2. How far does a river flow in $3\frac{3}{8}$ hours, if it flows $5\frac{3}{8}$ miles per hour?

3. A merchant sold $6\frac{2}{3}$ yards of carpet at $\$1\frac{4}{5}$ per yard. How much money should he receive in payment?

4. What is the cost of $3\frac{3}{4}$ bushels of pears at $\$1\frac{7}{10}$ per bushel?

5. Henry's expenses were $\$5\frac{1}{4}$ per week. How much were they in $6\frac{3}{7}$ weeks?

6. How much does a farmer receive for $4\frac{5}{7}$ bushels of wheat, if he receives $\$1\frac{1}{8}$ for each bushel?

NOTE.—When the numbers are not large, reduce them to improper fractions before multiplying.

388. Find the values of the following, with or without cancellation:

yards.	$	pounds.	$	bushels.	$
1. $2\frac{1}{4}$ @	$2\frac{2}{3}$.	*5.* $8\frac{3}{5}$ @	$1\frac{7}{8}$.	*9.* $10\frac{1}{4}$ @	$1\frac{1}{2}$.
2. $3\frac{1}{3}$ @	$2\frac{2}{5}$.	*6.* $9\frac{1}{4}$ @	$1\frac{1}{2}$.	*10.* $3\frac{7}{8}$ @	$2\frac{1}{3}$.
3. $2\frac{5}{8}$ @	$1\frac{11}{12}$.	*7.* $7\frac{3}{8}$ @	$2\frac{1}{4}$.	*11.* $8\frac{1}{6}$ @	$1\frac{4}{5}$.
4. $2\frac{2}{3}$ @	$4\frac{2}{5}$.	*8.* $5\frac{11}{16}$ @	$1\frac{3}{5}$.	*12.* $12\frac{1}{2}$ @	$2\frac{1}{5}$.

389. To multiply any two mixed numbers.

1. Mr. James purchased $25\frac{3}{8}$ cargoes of wood valued at $\$375\frac{3}{4}$ each. What was the entire value?

$\$375\frac{3}{4}$
$25\frac{3}{8}$
1 875
7 50
9,375 = 25 × $375.
250 = $\frac{3}{8}$ × $375.
$18\frac{3}{4}$ = $\frac{3}{8}$ × $25.
$\frac{3}{4}$ = $\frac{3}{8}$ × $\frac{3}{4}$.
$\$9,644\frac{1}{4}$

Explanation.—1. Multiply the integers together.
2. Multiply each integer by the fraction in the other factor.
3. Multiply the two fractions together.
4. Add all the partial products.

2. A manufacturer purchased 187 tons of coal at $\$3\frac{3}{8}$ per ton. How much did it cost him?

187
$3\frac{3}{8}$
$23\frac{3}{8}$ = $\frac{1}{8}$ of 187.
3
$70\frac{1}{8}$ = $\frac{3}{8}$ of 187.
561 = 3 × 187.
$631\frac{1}{8}$ = $3\frac{3}{8}$ × 187.

NOTE.—In practice, the larger number is taken for the multiplicand.

3. What is the cost of 425 yards of broadcloth at $\$2\frac{1}{8}$ per yard?

4. A merchant gained $\$1\frac{7}{8}$ on each barrel of flour. How much did he gain on 745 barrels of flour?

5. If a ferry company's expenses are $\$395\frac{4}{5}$ every day, how much are they for 28 days?

6. In a block of 17 brown-stone houses, each front cost $\$1,541\frac{2}{3}$. How much did the 17 fronts cost?

7. What number divided by 37 gives a quotient of $1,305\frac{2}{10}$?

8. How much must be paid for 12 round-trip tickets across the continent, at $\$187\frac{3}{4}$ each?

9. A speculator bought 25 city lots, costing $\$1,357\frac{1}{2}$ each. How much did the 25 lots cost him?

10. How much per year does a clerk earn, if he earns $\$81\frac{2}{3}$ per month?

11. A man pays $\$133\frac{1}{3}$ per month rent. How much rent does he pay per year?

12. What is the product of twelve and eleven-twelfths by $14\frac{2}{7}$?

13. What amount is paid in rent at $237\frac{1}{2}$ per month, for $2\frac{1}{2}$ years ?

14. If a railroad costs $\$45{,}675\frac{7}{8}$ per mile, how much will $22\frac{3}{4}$ miles of the road cost ?

15. What will it cost to construct $250\frac{2}{3}$ miles of telegraph at $\$5{,}970\frac{5}{6}$ per mile ?

16. How much does a steamship earn in $5\frac{7}{12}$ years, if it earns an average of $\$15{,}703\frac{7}{8}$ each year ?

390. Find the products :

1. $15 \times 207\frac{1}{2}$.	6. $500 \times 37\frac{3}{4}$.	11. $18\frac{2}{3} \times 405\frac{3}{8}$.	16. $204\frac{1}{4} \times 308\frac{1}{3}$.
2. $31 \times 483\frac{3}{5}$.	7. $296 \times 184\frac{4}{5}$.	12. $20\frac{9}{10} \times 763\frac{3}{8}$.	17. $571\frac{2}{3} \times 694\frac{3}{4}$.
3. $40 \times 306\frac{5}{8}$.	8. $781 \times 405\frac{5}{6}$.	13. $474\frac{4}{7} \times 1092\frac{5}{6}$.	18. $191\frac{3}{8} \times 708\frac{5}{6}$.
4. $27 \times 192\frac{5}{9}$.	9. $372 \times 25\frac{1}{3}$.	14. $38\frac{2}{5} \times 979\frac{1}{9}$.	19. $427\frac{3}{4} \times 891\frac{5}{8}$.
5. $19 \times 563\frac{1}{4}$.	10. $198 \times 507\frac{7}{9}$.	15. $507\frac{7}{8} \times 819\frac{3}{10}$.	20. $739\frac{1}{2} \times 648\frac{3}{7}$.

MISCELLANEOUS PROBLEMS.

391. Oral Problems. 1. What is the cost of a bushel of berries, at $\$.12\frac{1}{2}$ a quart ?

2. James had $1 and bought a whip for $.15, a bat for $.40, and a ball for $.20. What fraction of a dollar did he then have ?

3. What is the cost of $3\frac{1}{4}$ pounds of butter at $.24 a pound ?

4. If Jane gives a half-dollar in payment for $3\frac{1}{2}$ pounds of crushed sugar at $.12 a pound, what part of a dollar in change is due her ?

5. A merchant gave two $10 bills in payment for 45 baskets at $\$.33\frac{1}{3}$ a basket. How much change should he receive ?

6. What will 10 spools of silk cost, if 7 spools cost $.91 ?

7. If $3.90 pay for 13 bushels of oats, how much will pay *5 bushels ?*

8. Kate spent \$.30 for paper, \$.62½ for an inkstand, and \$.12½ for a rubber. How much in all did she spend ?

9. How much of \$5 will John have left after paying for 18 quarts of berries at \$.16⅔ a quart ?

10. Sarah bought a Reader for \$½, a Speller, \$¼, and a slate, \$1/10. She paid for them with 4 quarter-dollars. How much money is due her ?

11. How many dozen oranges, at 4 ¢ for each orange, can be bought for \$4 ? For \$3.04 ?

12. If 10 pounds of cake cost \$1.20, how much will 4¾ pounds cost ?

13. What will 40 dozen slates cost at \$.08⅓ for each slate ?

14. Fred bought 50 papers at 2½ cents each and then had 3 quarter-dollars left. How much money did he have at first ?

15. If 5 oranges cost \$.20, what will ¾ of a dozen cost ?

16. If 7 weeks' board cost \$49, how much will 10⅝ weeks' board cost ?

392. Written Problems. 1. Mr. Brown bought a house for \$4,786¾. He spent \$237⅜ for painting, \$1,569½ for repairs, and \$2,548 7/10 for furniture. How much did the house and furniture cost him ?

2. A lady bought a watch for \$237½ ; and a chain for \$96¾. She sold both for \$150. How much did she lose ?

3. What is the entire cost of 8½ bushels of rye at \$⅞ ; 7⅝ bushels of wheat at \$1.12½, and 9¼ bushels of oats at \$⅞ ?

4. If, in repairing a railroad, 96 tons of iron are used on each mile of the road, how many pounds will be used on 375⅝ miles ?

5. What is the value of $(97\frac{1}{3} + 109\frac{1}{6} - 50\frac{1}{4}) \times 81\ \frac{9}{10} - (746 - \frac{1}{2}$ of $327\frac{1}{4})$?

6. If a man's salary is \$1,800 per year, and his expenses \$735¾ for board, \$327¼ for clothing, and \$100 for other items, *how much can he save* in 5 years ?

7. A farmer had 1,000 bushels of wheat. How many bushels did he have left after selling $325\frac{1}{4}$, $206\frac{3}{8}$, and $167\frac{1}{2}$ bushels? How much was what he had left worth at $\$1.12\frac{1}{2}$ per bushel?

8. Mr. French, after paying for two houses $\$3,977\frac{1}{2}$, and for 45 shares of railroad stock at $\$95\frac{3}{4}$, had $\$15,462\frac{1}{8}$ left. How much money did he have at first?

9. Four men, A, B, C, and D, bought a ship for $350,000. A's share was $\$40,333\frac{1}{3}$; B's share was $\$10,946\frac{4}{5}$ greater than A's share, and C's share was $5,000 less than A's and B's shares together. How much was D's share?

10. Jonas Morgan bought of Herman Arnold in New York, on September 15, 1888, the following goods: $12\frac{1}{2}$ yd. of silk at $\$2\frac{1}{4}$; $24\frac{3}{8}$ yd. of flannel at $\$\frac{3}{8}$; and $45\frac{3}{4}$ yd. of sheeting at $\$.33\frac{1}{3}$. Make a receipted bill for the amount. How much money should Mr. Arnold receive?

DIVISION.

393. 1. If a father divides $\$\frac{1}{2}$ equally among his 5 boys, how much does each boy receive?

2. For $5, how much cloth can be bought at $\$\frac{3}{8}$ per yard?

3. At $\$\frac{3}{8}$ per yard, how much cloth can be bought for $\$\frac{3}{4}$?

4. If a man earns $\$2\frac{1}{4}$ in 3 hours, how much does he earn in one hour?

5. John ran 5 miles in $2\frac{1}{2}$ hours. How many miles per hour did he run?

6. What does each pound of tea cost, if $3\frac{1}{4}$ pounds cost $\$1\frac{1}{2}$?

394. To divide a fraction by an integer.

1. A farmer divided $\frac{2}{3}$ of an acre of land into 2 equal village lots. How much land is there in each lot?

$\frac{1}{3} \div 2 = \frac{1}{6}$; $\frac{2}{3} \div 2 = \frac{2}{6}$, or $\frac{1}{3}$.

2. If $\$\frac{3}{4}$ pays for 3 bushels of apples, what is the cost of *one bushel?*

$\frac{3}{4} \div 3 = \frac{3}{12}$, or $\frac{1}{4}$.

3. If a ton of coal costs \$5, how many tons can be bought for \$15? How much for \$$\frac{3}{5}$?

4. Charles saved \$$\frac{4}{5}$ in 4 days. What did he save in one day?

The answer will be the same whether the numerator be divided, or the denominator be multiplied by the integer, and then reduced. Divide the numerator if possible, as the result will be in lower terms. Always reduce the answers to their lowest terms.

5. What is the cost of one pound of sugar, if 7 pounds cost \$$\frac{7}{8}$?

6. If a sheet of paper, $\frac{1}{2}$ of a square foot in area, is divided into 4 equal parts, what is the area of each part?

7. Samuel spent \$.20 for 5 packs of torpedoes. What fraction of a dollar did each pack cost him?

8. If $\frac{2}{3}$ of a dozen of books cost \$$\frac{4}{5}$, how much does each book cost?

9. What number multiplied by 9 will give $\frac{3}{4}$?

395. To divide by a fraction.

1. How many pounds of tea at \$$\frac{2}{3}$ can be bought for \$5?

As many pounds of tea can be bought for \$5, as \$5 contains times \$$\frac{2}{3}$. In \$5 there are \$$\frac{15}{3}$. \$$\frac{15}{3}$ contains \$$\frac{2}{3}$ as many times as 15 contains 2, which is $7\frac{1}{2}$ times. Ans., $7\frac{1}{2}$ pounds.

2. How many straw hats, at \$$\frac{3}{4}$, can be bought for \$9?

For \$9 as many straw hats can be bought as \$$\frac{3}{4}$ is contained times in \$9. Find how many fourths there are in \$9 by multiplying by the denominator of the fraction. Then find how many times this product contains \$$\frac{3}{4}$, by dividing by the numerator of the fraction.

3. At \$$\frac{2}{3}$ per yard, how much cloth can be bought for \$$\frac{3}{4}$?

Reduce the fractions to a common denominator, and divide the numerator of the dividend by the numerator of the divisor. The result is most readily obtained by multiplying the numerator of the dividend by the denominator of the divisor; and the denominator of the dividend by the numerator of the divisor, and then dividing; *or, by inverting the divisor*, and multiplying.

4. How many pounds of coffee at $\$\frac{1}{8}$ per pound can be bought for $\$\frac{3}{4}$?

5. A yard of ribbon costs $\$\frac{3}{4}$. How much of the ribbon can be bought for $\$\frac{5}{8}$?

6. At $\$\frac{3}{8}$ per pound, how many pounds of creamery butter can be bought for \$15 ?

7. What number multiplied by $\frac{3}{4}$ gives $\frac{7}{10}$?

396. Rule for Division.—1. Invert the divisor and proceed as in multiplication.

2. If the dividend or the divisor is an integer, it may be treated as a fraction with 1 for the denominator.

397. Find the quotients, with or without cancellation :

1. $\frac{3}{4} \div 6$.	6. $\frac{2}{3} \div \frac{3}{4}$.	11. $\frac{7}{8} \div \frac{3}{4}$.	16. $7 \div \frac{7}{10}$.
2. $\frac{2}{3} \div 8$.	7. $\frac{5}{6} \div \frac{3}{8}$.	12. $\frac{4}{5} \div \frac{2}{15}$.	17. $3 \div \frac{3}{5}$.
3. $\frac{4}{5} \div 3$.	8. $\frac{4}{7} \div \frac{2}{3}$.	13. $\frac{5}{8} \div \frac{2}{3}$.	18. $4 \div \frac{3}{7}$.
4. $\frac{7}{8} \div 14$.	9. $\frac{7}{9} \div \frac{5}{3}$.	14. $\frac{7}{9} \div \frac{1}{2}$.	19. $9 \div \frac{2}{3}$.
5. $\frac{9}{10} \div 30$.	10. $\frac{3}{10} \div \frac{6}{5}$.	15. $\frac{1}{2} \div \frac{1}{4}$.	20. $5 \div \frac{4}{5}$.

398. Oral Problems. 1. A grocer sold 6 doz. eggs for $\$\frac{9}{10}$. What was the price per dozen ?

2. At $\$\frac{1}{3}$ per yd., how many yards of cloth can be bought for \$3 ?

3. If a man spends $\$\frac{2}{5}$ a day for car-fare, how long can he pay car-fare with \$12 ?

4. If a man earns $\$\frac{3}{4}$ in 6 hours, how much does he earn in one hour ?

5. How many times is $\frac{7}{8}$ contained in $3\frac{5}{8}$?

6. When pine-apples sell for $\$\frac{1}{5}$ each, how many can be bought for $\$\frac{8}{10}$?

7. What number multiplied by $\frac{5}{9}$ gives $\frac{1}{12}$?

8. *If a man* ploughs $\frac{8}{9}$ of a field in 6 days, how much can *he plough in* one day ?

9. When tea is $\$\frac{2}{5}$ a pound, how many pounds can be bought for $\$\frac{8}{9}$?

10. Into how many lots containing $\frac{2}{3}$ of an acre each can 8 acres be divided?

399. Written Problems. 1. If Henry earns $\$\frac{4}{5}$ a day, how long will it take him to earn \$72?

2. At $\$\frac{8}{10}$ a basket, how many baskets of currants can be bought for \$40?

3. If a man owns $\frac{12}{21}$ of a farm and his son $\frac{9}{21}$ of it, how many times the son's share is the father's?

4. How many pounds of beef at $\$\frac{1}{8}$ can be bought for $\$.62\frac{1}{2}$? For $\$.87\frac{1}{2}$?

5. If a man builds $\frac{12}{20}$ of a wall in 14 days, what part can he build in one day?

6. When oranges are worth $\$.37\frac{1}{2}$ cents a dozen, how many dozen can be bought for \$60?

7. How many times is $\frac{14}{15}$ contained in $\frac{42}{67}$?

8. How many pieces of wire, each containing $\frac{7}{9}$ ft., can be cut from 56 ft. of wire?

9. At $\$\frac{9}{10}$ per pound, what part of a pound of tea can be bought for $\$\frac{3}{5}$?

10. How much will John earn in one day, if he earns $\$\frac{8}{9}$ in 4 days?

11. At \$6 a yd., what part of a yd. of velvet can be bought for $\$\frac{24}{25}$?

12. What will one pound of sugar cost, if 14 pounds cost $\$.87\frac{1}{2}$?

13. How many loaves of bread, each weighing $\frac{4}{3}$ of a pound, can be made from a barrel of flour containing 196 pounds?

14. If there are 25 bushels of coal in each load, how many boxes, each holding $\frac{5}{8}$ of a bushel, will it fill?

15. A silversmith divided a silver wire $\frac{7}{8}$ inches long into 35 *equal parts*. *How long* was each part?

16. If a cistern will hold 500 barrels of water, in what time can it be filled by a pipe discharging into it $\frac{5}{8}$ of a barrel every minute ?

17. The circumference of one wheel of a wagon being $\frac{7}{4}$ of a yard in length, and that of the other wheel being $\frac{2}{3}$ of a yard, the circumference of the first wheel is how many times as long as that of the second ?

18. If a pail holds $\frac{1}{7\frac{1}{5}}$ of a hogshead, how many times can the pail be filled from $\frac{1}{2}\frac{2}{5}$ of a hogshead of wine ?

19. How long will it take Frank to do $\frac{2}{3\frac{1}{0}}$ of a piece of work, if he can do $\frac{3}{5}$ of it in an hour ?

20. In what time can a horse trot $\frac{3}{5}$ of a mile, if he can trot $\frac{4}{9}$ of a mile in a minute ?

21. If a merchant makes $\$\frac{3}{2\frac{1}{0}}$ on a yard of cloth, how many yards must he sell to make $36 ?

22. By selling 10 pounds of sugar, a grocer gained $\$.83\frac{1}{3}$. What fraction of a dollar did he gain on each pound ?

23. In what time will a horse, running $\frac{1}{3}\frac{5}{2}$ of a mile in a minute, run around a race track $\frac{7}{8}$ of a mile long ?

400. To divide with small mixed numbers.

1. For $\$2\frac{2}{5}$, how many yards of cloth can be bought at $\$2\frac{2}{5}$ a yard ?

Divide $\$2\frac{2}{5}$, or $\$\frac{1^2}{5}$, by $\$2\frac{2}{5}$, or $\$\frac{5}{5}$, the price of one yard.

2. How many weeks' board will $\$15\frac{3}{4}$ pay for, at $\$10\frac{1}{2}$ per week ?

Small mixed numbers are reduced to improper fractions, before dividing.

3. John earned $\$7\frac{1}{2}$ per week. How long did it take him to earn $\$5\frac{5}{8}$?

4. If $6\frac{3}{3}$ yards of cloth is required for each suit of clothes, how many suits can be made from 500 yards of cloth ?

5. For $90, how many bushels of wheat can be bought at *$1.12½ per* bushel ?

6. At $\$1.33\frac{1}{3}$ per yard, how many yards of silk can be bought for $\$14\frac{2}{3}$?

7. A farm, containing 200 acres of land, is divided into village lots containing $1\frac{1}{4}$ acres each. Into how many village lots was it divided?

8. A grocer made $\$5\frac{3}{8}$ by selling $5\frac{1}{3}$ pounds of tea. How much did he make on each pound?

9. How much is lost on each of $3\frac{2}{3}$ barrels of flour, if \$2.50 is lost altogether?

10. What number multiplied by $5\frac{1}{7}$ equals 27?

11. If each mile of a telegraph line costs $5\frac{3}{4}$ thousands of dollars, how many miles can be constructed for $94\frac{7}{8}$ thousands of dollars?

401. To divide with large mixed numbers.

1. How many casks, each containing $15\frac{3}{3}$ gallons of wine, can be filled from a tank containing 1,860 gallons of wine?

$15\frac{3}{3})1860$
$3 \quad 3$
$47 \;)5580(118\frac{34}{47}$
47
88
47
410
376
34

Explanation.—1. Divide 1860 by $15\frac{3}{3}$.

2. As one of the numbers is a mixed number, multiply both numbers by 3, the denominator of the fraction. (Art. 47, Prin. 131.)

3. Then divide, as in whole numbers, obtaining for the answer, $118\frac{34}{47}$.

2. What is the quotient of $3{,}203\frac{4}{5}$ divided by $126\frac{3}{4}$?

$126\frac{3}{4}) \quad 3203\frac{4}{5}$
$20 \quad 20$
$15 \quad 16$
$2520 \;\; 64060$
$2535) \;\; 64076(25\frac{701}{2535}$
5070
13376
12675
701

Explanation.—1. Multiply both numbers by 20, the common denominator of their fractions. (Art. 47, Prin. 131.)

2. Then divide as in whole numbers, obtaining for the answer, $25\frac{701}{2535}$.

3. How many bales of hay at $\$12\frac{7}{8}$ can be bought for \$500?

4. At $\$18\frac{3}{4}$ each, how many dress patterns of silk can be bought for \$250?

5. If a gross of buttons cost $\$3.37\frac{1}{2}$, what will each button cost?

6. What is the cost of each lead pencil, if $12\frac{1}{2}$ gross cost \$63?

7. A farm containing $175\frac{2}{3}$ acres of land is valued at \$7,487$\frac{2}{4}$. What is the value of each acre?

8. What is the cost of each cargo of coal, if $604\frac{4}{5}$ cargoes cost \$776,412?

402. Find the quotients:

1. $2,057\frac{4}{5} \div 8$.	*6.* $276\frac{2}{3} \div 109\frac{3}{4}$.	*11.* $1,598\frac{4}{7} \div 201\frac{3}{5}$.	*16.* $3,094 \div 17\frac{1}{2}$.
2. $1,962\frac{2}{3} \div 5$.	*7.* $487\frac{3}{8} \div 368\frac{5}{6}$.	*12.* $2,087\frac{2}{3} \div 360\frac{4}{5}$.	*17.* $4,879 \div 32\frac{3}{4}$.
3. $4,713\frac{1}{2} \div 6$.	*8.* $900\frac{3}{5} \div 400\frac{5}{7}$.	*13.* $7,001\frac{3}{4} \div 18\frac{5}{6}$.	*18.* $6,712 \div 29\frac{7}{8}$.
4. $5,091\frac{7}{8} \div 7$.	*9.* $640\frac{4}{5} \div 260\frac{2}{3}$.	*14.* $9,090\frac{5}{8} \div 20\frac{3}{5}$.	*19.* $5,304 \div 41\frac{5}{6}$.
5. $8,652\frac{5}{6} \div 9$.	*10.* $578\frac{3}{7} \div 306\frac{1}{2}$.	*15.* $18,018\frac{7}{8} \div 48\frac{3}{5}$.	*20.* $7,083 \div 50\frac{4}{5}$.

403. Find the values of the following:

1. $\frac{\frac{3}{4} \text{ of } 500}{17\frac{1}{2}}$.	*4.* $\frac{\frac{9}{10}+\frac{5}{100}+\frac{7}{1000}}{\frac{3}{10}+\frac{1}{100}+\frac{9}{1000}}$.	*7.* $(\frac{3}{4} \text{ of } \frac{2}{3} \times 78) \div (27\frac{3}{4}+14\frac{1}{2})$.	*10.* $\frac{275\frac{3}{4}}{406\frac{5}{8}}$.
2. $\frac{\frac{4}{5} \text{ of } \frac{7}{8}}{17\frac{1}{2}}$.	*5.* $\frac{2\frac{1}{2} \times 1\frac{3}{5}}{3\frac{3}{5}-2\frac{1}{2}}$.	*8.* $(\overline{\frac{9}{10} \text{ of } 81}-\overline{\frac{1}{5} \text{ of } 20}) \div 6$.	*11.* $\frac{390\frac{4}{5}}{581\frac{2}{3}}$.
3. $\frac{25\frac{3}{4}}{\frac{5}{7} \text{ of } 28}$.	*6.* $\frac{5\frac{3}{7}+4\frac{5}{8}}{7\frac{2}{5}+2\frac{2}{11}}$.	*9.* $(15-\frac{1}{2} \times \frac{3}{8} \text{ of } 10) \div \frac{1}{2} \text{ of } 41\frac{1}{2}$.	*12.* $\frac{794\frac{1}{4}}{809\frac{1}{2}}$.

NOTE.—Complex fractions are reduced by multiplying both numerator and denominator by the least common denominator of the fractions in the numerator and denominator. (Art. 310, Prin. III.)

If the numerator or the denominator is compound, it must be reduced to a simple number, or a mixed number, before applying this rule.

404. Written Problems. 1. How many pieces of cord, each containing $2\frac{3}{4}$ yards, can be cut from 22 yards?

2. If a man shovels $1\frac{3}{8}$ tons of coal in one hour, in how many hours can he lay away 24 tons?

3. For \$$9\frac{1}{3}$, how much silk can be bought at \$$1\frac{3}{4}$ a yard?

4. How much does a man earn in one day, if he earns \$$21\frac{1}{3}$ in $4\frac{1}{3}$ days?

5. If each jacket requires $3\frac{3}{8}$ yards of cloth, how many *jackets can* be made from $14\frac{3}{4}$ yards?

6. At $\$6\frac{1}{4}$ a week, how many weeks' board will $\$34\frac{3}{8}$ pay?

7. How many ties can be made from $26\frac{1}{4}$ yards of ribbon, if each tie requires $\frac{5}{8}$ of a yard?

8. A man divided his farm of $518\frac{7}{8}$ acres among his sons, giving each one $172\frac{1}{8}$ acres. How many sons had he?

9. How many barrels of flour, at $\$8\frac{3}{4}$ each, can be bought for $\$113\frac{3}{4}$?

10. A man bought $55\frac{1}{2}$ yards of carpet for $\$124\frac{7}{8}$; how much did he pay per yard?

11. How many yards of satin, at $\$2\frac{2}{3}$ a yard, can be obtained for $\$205\frac{1}{3}$?

12. If 51 baskets of pears cost $\$140\frac{1}{4}$, what does one basket cost?

13. If each dress requires $18\frac{3}{4}$ yards, how many dresses can be made from 225 yards?

14. If a man pay $\$18.50$ per week for board, for how many weeks' board can he pay with $\$1{,}008\frac{1}{4}$?

405. To find a number, a part of it being given.

1. 5 is $\frac{5}{6}$ of what number?

If 5 is $\frac{5}{6}$ of a number, $\frac{1}{6}$ is $\frac{1}{5}$ of 5 or 1, and $\frac{6}{6}$, or the number is 6 times 1, or 6.

2. 25 is $\frac{5}{7}$ of what number?

If 25 is $\frac{5}{7}$ of a number, $\frac{1}{7}$ is $\frac{1}{5}$ of 25, or 5, and $\frac{7}{7}$, is 7 times 5, or 35.

3. Henry is $7\frac{9}{12}$ years old. His age is $\frac{3}{14}$ of the age of his father. How old is his father?

4. Mr. Johnson has $\$480$ in his cash box, and this is $\frac{6}{7}$ of what he has in bank. How much has he in bank?

5. One river, 900 miles long, is $\frac{3}{10}$ the length of another river. How long is the other river?

406. To find the number of which a part is given, divide the given part by the numerator, and then multiply by the *denominator of the fraction.*

407. Find the number of which:

1. 16 is $\frac{4}{7}$.	*5.* $4\frac{4}{7}$ is $\frac{8}{9}$.	*9.* 148 is $\frac{5}{6}$.	*13.* $313\frac{1}{3}$ is $\frac{5}{27}$.
2. 15 is $\frac{3}{5}$.	*6.* $9\frac{1}{3}$ is $\frac{4}{13}$.	*10.* 235 is $\frac{5}{14}$.	*14.* $629\frac{2}{7}$ is $\frac{5}{6}$.
3. 12 is $\frac{6}{11}$.	*7.* $10\frac{2}{3}$ is $\frac{4}{15}$.	*11.* 288 is $\frac{24}{25}$.	*15.* $189\frac{1}{7}$ is $\frac{2}{5}$.
4. 38 is $\frac{2}{19}$.	*8.* $11\frac{1}{12}$ is $\frac{2}{3}$.	*12.* 450 is $\frac{9}{23}$.	*16.* $170\frac{2}{11}$ is $\frac{1}{35}$.

408. Written Problems. 1. What number is that to which, if $\frac{3}{8}$ of itself be added, the result is 552?

2. A farmer finds that the number of his sheep has increased $\frac{1}{8}$. If he has 749 now, how many had he before the increase?

3. A man failing in business could pay his creditor only $2,355. How much did he owe, if the creditor thereby lost $\frac{2}{5}$ of the debt?

4. A house and furniture cost $10,350, the furniture costing $\frac{2}{7}$ as much as the house. How much did the house cost?

5. $\frac{3}{19}$ of 570 is $\frac{6}{13}$ of what number?

6. After losing $\frac{2}{9}$ of his money, a man had $3,171 left. How much had he at first?

7. Mr. Lewis received $13,044 for his share of a farm, of which he owned $\frac{4}{7}$. For what sum was the farm sold?

8. A son earns $1,525, which is $\frac{5}{8}$ of what his father earns in the same time. How much does the father earn?

9. A merchant's profit for one year was $4,137, which was $\frac{7}{8}$ as much as his profit for the preceding year. How much did he gain the preceding year?

10. Paid $270 for a carriage, which was $\frac{3}{7}$ of 2 times the cost of the horse. What did the horse cost?

409. To find what part one number is of another.

1. 5 is what part of 8? 1 is $\frac{1}{8}$ of 8; 5 is $\frac{5}{8}$ of 8.

2. 25 is what part of 35? 1 is $\frac{1}{35}$ of 35; 25 is $\frac{25}{35}$ or $\frac{5}{7}$ of 35.

3. [illegible]'s age is 7 years; and Edith's age is $12\frac{1}{2}$ years. what part of Edith's age?

410. To find what part one number is of another, write the part as the numerator, and the number of which it is a part as the denominator of a fraction, and reduce the fraction to its lowest terms.

411. Find what part:

1. $\frac{4}{7}$ is of $\frac{3}{5}$.	*5.* $\frac{2}{5}$ is of 3.	*9.* $\frac{4}{5}$ is of $2\frac{2}{3}$.	*13.* $5\frac{1}{2}$ is of $3\frac{2}{3}$.
2. $\frac{6}{7}$ " $\frac{8}{9}$.	*6.* $\frac{3}{10}$ " 4.	*10.* $\frac{3}{9}$ " $3\frac{3}{5}$.	*14.* $2\frac{1}{12}$ " $2\frac{1}{2}$.
3. $\frac{2}{9}$ " $\frac{3}{8}$.	*7.* $\frac{5}{9}$ " 10.	*11.* $\frac{5}{3}$ " $8\frac{1}{3}$.	*15.* $4\frac{4}{6}$ " $1\frac{3}{6}$.
4. $\frac{4}{15}$ " $\frac{6}{9}$.	*8.* $\frac{7}{8}$ " 2.	*12.* $\frac{3}{12}$ " $4\frac{1}{2}$.	*16.* $6\frac{2}{3}$ " $8\frac{1}{3}$.

412. Written Problems. 1. A bought a house for $6,352, and sold it for $7,940. What part of the cost was the gain?

2. The wages of three laborers were $97.50. A worked 9 days, B worked 12 days, and C, 18 days. What was each man's pay?

3. What part of his money does a man lose, if he buys a lot for $10,092, and sells it for $8,410?

4. If 36 yards of carpet cost $90, at the same rate what is the cost of 48 yards?

5. Henry has $2,056, and John has $5,140. Henry's money is what part of John's money?

6. When 14 tons of coal cost $91, what is the cost of $\frac{4}{5}$ of a ton?

7. A man had $895, and spent $716. What part of his money had he left?

8. An agent sold a house for $3,000, charging $180 for his services. What part of the selling price did the owner receive?

9. A man having 320 acres sells 80 to one man, and $\frac{1}{4}$ of the remainder to another. What part of the whole has he left?

10. One field yields 510 bushels of corn, another $\frac{2}{3}$ as many, and a third $\frac{1}{2}$ as many as the second. The third field yields *how much compared with* the first?

MISCELLANEOUS PROBLEMS.

413. Oral Problems. 1. Lucy has 28 cents, which is $\frac{4}{7}$ as much as Henry has. How many cents has Henry?

2. If $\frac{1}{4}$ of a pound of candy costs $8\frac{3}{4}$ cents, what will one pound cost?

3. $\frac{3}{4}$ of a certain number is 18. What is the number?

4. How long will $\frac{1}{2}$ of a bushel of oats last a horse, if $2\frac{1}{2}$ bushels last him one week?

5. A man bought a coat for $\$12\frac{1}{2}$. What did he pay for his vest, if it cost $\frac{2}{5}$ as much as the coat?

6. How many tons of coal at $\$6\frac{1}{2}$ can be bought for $\$26$?

7. If a bonnet can be made from $\frac{7}{8}$ of a yard of velvet, how many bonnets can be made from 21 yards?

8. If Mrs. Wilson has $\$62\frac{1}{2}$, and spends $\$12\frac{1}{2}$, what part of her money does she spend?

9. William has 45 cents, which is $\frac{5}{9}$ as much as his sister has. How much money has his sister?

10. If $\frac{4}{5}$ of a street can be paved in 20 days, how many days will be required to complete the work?

11. By selling his horse for $\$96$ A gains $\frac{1}{5}$ of the cost. What did the horse cost him?

12. Mr. James bought $\frac{2}{3}$ of a farm, and then sold $\frac{1}{4}$ of his share. What part of the whole farm had he left?

13. If $\frac{3}{4}$ of a pound of coffee costs 24 cents, what is the cost of 3 pounds?

14. How many yards of lace can be bought for $\$15$, if $3\frac{1}{3}$ yards cost $\$10$?

15. Charles had 36 cents. He lost $\frac{2}{3}$ of it, and spent $\frac{1}{4}$ of it. How much had he left?

16. The sum of two fractions is $1\frac{7}{12}$, and one of them is $\frac{3}{4}$. What is the other fraction?

17. If a barrel of flour costs $\$6\frac{3}{4}$, how much will half

18. Mr. Wright sold a clock for $49, thereby losing $\frac{1}{8}$ of the cost. How much did he pay for the clock ?

19. 72 is $\frac{8}{9}$ of what number ?

20. Mr. Williams divided $3\frac{3}{4}$ among some boys, giving each boy $\frac{3}{4}$ of a dollar. How many boys were there ?

21. Samuel earns $1\frac{1}{2}$ a day, and Frank earns $2\frac{1}{4}$ a day. What part of Frank's wages is Samuel's ?

22. What will $\frac{2}{3}$ of 12 yards of ribbon cost, if 9 yards cost 54 cents ?

23. A man sold a cow for $24, and a calf for $\frac{3}{8}$ as much. How much more did the cow cost than the calf ?

24. A man bought 5 bushels of corn, and paid $3, which was $\frac{4}{5}$ of the cost. What was the cost per bushel ?

25. How many dozen eggs at $16\frac{2}{3}$ cents a dozen will pay for 3 pounds of butter at $33\frac{1}{3}$ cents a pound ?

414. Written Problems. 1. If a man earns $1\frac{3}{4}$ a day, how many days will he be in earning $75 ?

2. A merchant sells three pieces of silk, one containing $40\frac{1}{4}$ yards, another $33\frac{1}{3}$ yards, and the other $35\frac{5}{12}$ yards, at $1\frac{1}{4}$ per yard. What was the entire bill ?

3. What will 35 bushels of wheat cost at $1\frac{7}{8}$ per bushel ?

4. If 60 pounds of wheat make a bushel, how many bushels are there in 24,632 pounds ?

5. What will $2\frac{1}{2}$ pecks of wheat cost at $.08\frac{1}{3}$ a pound, counting 60 pounds to the bushel ?

6. How many pieces of twine, each containing $7\frac{1}{3}$ yards, can be cut from a ball of twine containing 84 yards ?

7. If a quire of paper cost $\frac{1}{5}$, what will $19\frac{1}{2}$ quires cost ?

8. A farmer had $125\frac{1}{2}$ acres in one farm, and $66\frac{1}{3}$ acres in another; he gave his son $87\frac{3}{4}$ acres. How many acres had he left ?

9. If a house and stable cost $13,000, and the stable ~~is~~ $\frac{1}{4}$ *as much as the house*, what was the cost of each ?

10. A building worth $12,500 was insured for $\frac{3}{5}$ of its value. How much did the man who owned $\frac{1}{3}$ of the building receive, when the building was destroyed by fire ?

11. A man hires four boys to pick cherries at $1\frac{1}{2}$ ¢ a quart. One picks $8\frac{1}{3}$ quarts ; the second, $4\frac{1}{4}$ quarts ; the third, $7\frac{3}{5}$ quarts ; and the fourth, $9\frac{3}{4}$ quarts. How much does he pay for all ?

12. How long will it take a carriage travelling at the rate of $7\frac{1}{2}$ miles an hour to travel $156\frac{3}{4}$ miles ?

13. A can do a piece of work in 5 days, and B, in 7 days. How long will it take both to do the work ?

14. The difference between two numbers is $21\frac{3}{7}$. The greater number is $203\frac{4}{5}\frac{6}{7}$. What is the other number ?

15. How many times will a wheel $10\frac{1}{2}$ feet in circumference turn, in passing over 3 miles ?

16. The product of two numbers is $\frac{4}{9}$, and one of the numbers is $\frac{7}{8}$. What is the other number ?

17. Divide $\frac{5}{7}$ of $\frac{14}{16}$ by $\frac{3}{4}$ of $\frac{2}{3}$.

18. If chickens sell for $.87$\frac{1}{2}$ each, how many can be bought for $7 ?

19. Divide $246\frac{3}{4}$ by 24. What is the quotient ?

20. If $\frac{3}{5}$ of $\frac{6}{7}$ of a barrel of flour costs 4\frac{4}{5}$, what will $\frac{1}{4}$ of $\frac{5}{9}$ of 36 barrels cost ?

21. 24 is $\frac{1}{3}$ of $\frac{4}{5}$ of what number ?

22. If $\frac{1}{3}$ of a piece of cloth, 63 yards in length, is cut into 7 equal parts, what fractional part of the whole will one of these parts be ?

23. From $41\frac{1}{6}$ take $27\frac{1}{3}$. What is the remainder ?

24. What number multiplied by $1\frac{5}{6}$ and divided by $2\frac{1}{4}$ equals $3\frac{1}{3}$?

25. A merchant sold two pieces of cloth, one containing
nd the other $24\frac{1}{2}$ yards, at 3\frac{3}{8}$ per yard. What
unt of the bill ?

26. A man paid $5,085 for his house, and $\frac{3}{5}$ as much for the lot. The cost of both amounted to $\frac{2}{3}$ of all his money. How much had he?

27. A man has a farm containing 800 acres. He sells $231\frac{2}{3}$ acres to one man, and $343\frac{1}{3}$ acres to another. What part of the whole has he left?

28. A man bought a horse and 9 cows at $\$26\frac{1}{3}$ each, for $392. What was the cost of the horse?

29. Mr. Martin buys 150 tons of hay. He sells 35 tons at $\$4\frac{3}{5}$ per ton, 40 tons at $\$5\frac{1}{4}$ per ton, and $\frac{2}{3}$ of the remainder at $\$5\frac{1}{2}$ per ton. How much money does he receive?

30. What is the cost of 18 pounds of beef at $\$.16\frac{2}{3}$ a pound, 16 quarts of syrup at $\$.37\frac{1}{2}$ a quart, and 32 quarts of berries at $\$.06\frac{1}{4}$ a quart?

31. What number is that which is $103\frac{1}{3}$ less than $16\frac{1}{4}$ times $18\frac{2}{3}$?

32. Reduce $\frac{5\frac{1}{2}}{8\frac{1}{4}}$ to its simplest form.

33. How long is a strip of wood which can be sawed into 5 pieces, each piece 1 yd. 2 ft. 7 in. long?

34. If 80 feet of timber is sawed into 24 pieces of equal length, how long is each piece?

35. If 1 sq. mi. 40 A. 2 R. 20 P. is divided into 25 equal farms, what is the area of each farm?

36. What is the value of 75 powders, each containing 1 ℥. 6 ℥. 2ʒ. 18 gr. of a drug at $\$.01$ per grain?

37. At $\$.06$ per quart, what are 50 cans of milk worth, each can containing 10 gal. 3 qt. 1 pt.?

38. If 29 men can do a piece of work in 58 days, in what time can 17 men do the work?

39. How many yards of cloth costing $\$.37\frac{1}{2}$ per yard, must be given in exchange for 15 tons of coal at $\$4.50$ per ton?

40. To the quotient of $27\frac{1}{2}$ divided by $\frac{3}{4}$ add $15\frac{1}{3}$.

DENOMINATE FRACTIONS.

415. Compound numbers are frequently called **denominate numbers.**

8 bu. 3 pk. 2 qt. 1 pt.; 5 lb. 3 3. 7 3. 2 ʒ, are *denominate* or *compound* numbers. They contain units of different names or denominations.

416. A **denominate fraction** is a fraction whose integral unit is a denominate number: as, $\frac{3}{5}$ A.; $\frac{4}{9}$ sq. rd.; $\frac{7}{8}$ yd.; .5 mi.; and $\frac{2}{3}$ T.

NOTE.—A denominate fraction may be a *common fraction* or a *decimal fraction*. The treatment of *decimal* denominate fractions is reserved for the chapter on decimal fractions.

REDUCTION.

417. To reduce a denominate fraction to lower denominations.

1. Reduce $\frac{5}{9}$ cubic yards to cubic inches.

Explanation.—1. Multiply the given fraction by 27, to reduce it to cubic feet.

2. Multiply the result by 1,728, to reduce it to cubic inches.

3. Indicating the operations, the work may be abbreviated by cancellation. Ans., 25,920 cu. in.

$$\frac{5}{\cancel{9}_{1}} \times \cancel{27}^{3} \times 1728$$

15

8640

1728

Ans., 25,920 cu. in.

2. Reduce $\frac{2}{3}$ of a bushel to lower denominations.

Explanation.—1. Multiply by 4 to reduce bushels to pecks, obtaining $\frac{8}{3}$, or $2\frac{2}{3}$ pk.

2. Multiply the fraction of a peck by 8, to reduce it to quarts, obtaining $\frac{16}{3}$, or $5\frac{1}{3}$ qt.

3. Multiply the fraction of a quart by 2, to reduce it to pints, obtaining $\frac{2}{3}$ pt.

4. The integers of the preceding denominations with the product in the lowest denomination form the answer: 2 pk. 5 qt. $\frac{2}{3}$ pt.

$\frac{2}{3}$ bu.

4

$\frac{8}{3} = 2\frac{2}{3}$ pk.

8

$\frac{16}{3} = 5\frac{1}{3}$ qt.

2

$\frac{2}{3}$ pt.

Ans., 2 pk. 5 qt. $\frac{2}{3}$ pt.

418. Rules for Reduction Descending.—I. To reduce a denominate fraction to a required lower denomination, multiply the given fraction by the number of units of the next lower denomination that equal one unit of the given denomination; and thus proceed until the required denomination is obtained, indicating the successive operations, and cancelling, if possible.

II. To reduce a denominate fraction to lower denominations, multiply the fraction by the number of units of the next lower denomination that equal one of the given denomination; and, in similar manner, reduce any fraction in the product, thus proceeding until the required denomination is obtained. The integers, together with the product in the lowest denomination form, the answer.

419. Reduce the following:

to pence.	to pwt.	to ℈.	to sq. ft.	to pints.	to inches.
1. $1\frac{7}{10}$ s.	*5.* $\frac{15}{16}$ oz.	*9.* $1\frac{1}{2}$ ℔.	*13.* $25\frac{1}{2}$ sq. yd.	*17.* $\frac{3}{4}$ gal.	*21.* $4\frac{2}{3}$ yd.
2. $\frac{4}{5}$ s.	*6.* $\frac{7}{8}$ oz.	*10.* $5\frac{5}{6}$ ʒ.	*14.* $7\frac{5}{8}$ sq. rd.	*18.* $\frac{2}{3}$ hhd.	*22.* $\frac{4}{5}$ rd.
3. £$2\frac{1}{2}$.	*7.* $4\frac{3}{4}$ oz.	*11.* $\frac{3}{8}$ ℥.	*15.* $\frac{2}{3}$ R.	*19.* $\frac{1}{4}$ bl.	*23.* $\frac{2}{25}$ yd.
4. £$\frac{1}{3}$.	*8.* $2\frac{1}{4}$ lb.	*12.* $\frac{1}{5}$ ℔.	*16.* $\frac{1}{4}$ A.	*20.* $5\frac{1}{2}$ gal.	*24.* $\frac{3}{8}$ fur.

420. Reduce to lower denominations:

1. $\frac{1}{3}$ pk.	*4.* $\frac{2}{3}$ cu. yd.	*7.* $\frac{7}{8}$ yd.	*10.* $\frac{3}{4}$ cwt.	*13.* $\frac{1}{4}$ ℈.	*16.* $2\frac{1}{2}$ pwt.
2. $2\frac{1}{2}$ pk.	*5.* $\frac{4}{5}$ C.	*8.* $2\frac{5}{6}$ rd.	*11.* $\frac{1}{5}$ T.	*14.* $\frac{2}{3}$ ʒ.	*17.* $\frac{3}{5}$ oz.
3. $\frac{4}{5}$ bu.	*6.* $\frac{1}{2}$ cu. ft.	*9.* $\frac{3}{8}$ mi.	*12.* $4\frac{1}{4}$ T.	*15.* $\frac{5}{6}$ ʒ.	*18.* $5\frac{3}{8}$ lb.

421. Written Problems. 1. What is the cost of $\frac{1}{2}$ pound of arsenic at 10 cents per scruple?

2. How much will five watch cases cost, each weighing $\frac{1}{3}$ of a pound, at 4 cents per pennyweight?

3. If coal sells at $3.28 per hundred weight, at the same rate, what will $3\frac{1}{2}$ tons cost?

4. At 5 cents a quart, what is the value of $25\frac{1}{2}$ gall oil?

5. A field is enclosed by a fence $5\frac{2}{3}$ miles long. How much did the fence cost at \$.48 per foot?

6. How many rotations will a wheel 75 inches in circumference make in travelling $\frac{7}{8}$ of a mile?

7. Each step being $2\frac{1}{2}$ feet long, how many steps will a man make in walking $3\frac{1}{4}$ miles?

8. If a train travels 360 miles in a day, how far does it travel each minute?

9. How much will a stationer receive for $\frac{1}{3}$ of a bundle of paper sold at \$.20 a quire?

10. If a bar of silver weighs $\frac{4}{5}$ of a pound, how many dozen spoons, each weighing 16 pennyweights, can be made from the bar?

11. How many cups, each holding $\frac{2}{3}$ of a quart, can be filled from 5 gallons of milk?

12. A field containing $\frac{1}{2}$ A. can be divided into how many lots each containing $\frac{1}{8}$ sq. rd.?

13. At $\frac{1}{2}$ a shilling each, how many dozen collars can be bought for £12?

14. Into how many equal pieces $\frac{1}{4}$ of a foot in length, can a piece of brass wire, which is 3 yards in length, be divided?

422. To reduce a denominate number to a fraction of a higher denomination.

1. Reduce $\frac{1}{8}$ of a square yard to acres.

Explanation.—1. Divide by $30\frac{1}{4}$ to reduce the given fraction to the fraction of a square rod.

2. Divide the result by 40 to reduce it to roods.

3. Divide the result by 4 to reduce it to

$\frac{1}{8}$ sq. yd. =
$\frac{1}{8} \div 30\frac{1}{4}$ sq. r.d. =
$\frac{1}{8} \div 30\frac{1}{4} \div 40$ R. =
$\frac{1}{8} \div 30\frac{1}{4} \div 40 \div 4$ A. =
$\frac{1}{8} \times \frac{4}{121} \times \frac{1}{10} \times \frac{1}{4}$ A. =
Ans., $\frac{1}{9680}$ A.

ing the preceding divisions, the work may be abbreviated
llation. Answer, $\frac{1}{9680}$ A.

2. Reduce 4 bu. 3 pk. 0 qt. 1 pt. to bushels.

2	1 pt.
8	$0\frac{1}{2}$ qt.
4	$3\frac{1}{16}$ pk.
	$4\frac{3}{64}$ bu.

Explanation.—1. Write all the denominations in their order in a column, writing the lowest denomination first, and leaving space between each integer after the first and the name of the integer.

2. Reduce the units of the lowest denomination by dividing by 2 to $\frac{1}{2}$, a fraction of the next higher denomination, inserting the result after the integer of the higher denomination.

3. Reduce the $\frac{1}{2}$ qt. thus obtained, to pecks by dividing by 8, obtaining $\frac{1}{16}$ peck, which insert after the 3 units of the next denomination.

4. Reduce the $3\frac{1}{16}$ pecks thus obtained, first to 16ths, making $\frac{49}{16}$ of a peck, and then to bushels, by dividing by 4, making $\frac{49}{64}$, which insert after the integers in bushels place. Answer, $4\frac{49}{64}$ bu.

423. Rules for Reduction Ascending.—I. To reduce a denominate integer or fraction to a fraction of a required higher denomination, divide the given number by the number of units that equal one of the next higher denomination, and thus proceed until the required denomination is obtained, indicating the successive operations, and cancelling, if possible.

II. 1. To reduce any compound number to one of the higher denominations, write all the denominations of the number in a column, beginning with the lowest denomination.

2. Reduce by division the number of the lowest denomination to a fraction of the next higher denomination, and add the result to the number of units of the next higher denomination.

3. Reduce by division the result thus obtained, to the next higher denomination, and add the quotient to the number of units of that denomination; and thus proceed until the required denomination is obtained.

424. Reduce the following:

to pounds.	to acres.	to pounds.	to gallons.	to pounds.	to miles.
1. 15s.	*4.* $\frac{2}{3}$ sq. rd.	*7.* $\frac{1}{2}$ oz.	*10.* 3 qt.	*13.* 2 ʒ.	*16.* $\frac{1}{25}$ yd.
2. 10d.	*5.* $\frac{7}{8}$ sq. yd.	*8.* 19 gr.	*11.* $3\frac{1}{3}$ qt.	*14.* $\frac{1}{2}$ ʒ.	17. $\frac{1}{25}$ ft.
3. 3 far.	*6.* $\frac{1}{4}$ *sq. ft.*	*9.* $3\frac{1}{4}$ pwt.	*12.* $1\frac{3}{4}$ pt.	15. 12 gr.	18. 7

425. Reduce the following compound numbers:

to acres.	to cu. yd.
1. 1 R. 3 P. 5 sq. yd.	*4.* 3 cu. yd. 12 cu. ft. 100 cu. in
2. 21 P. 20 sq. yd. 7 sq. ft.	*5.* 5 cu. yd. 250 cu. in.
3. 3 R. 14 sq. ft.	*6.* 10 cu. yd. 10 cu. in.
to miles.	**to bushels.**
7. 100 rd. 4 yd. 2 ft.	*10.* 5 pk. 3 qt. 1 pt.
8. 75 rd. 5 ft. 10 in.	*11.* 2 bu. 2 qt. 1 pt.
9. 125 rd. 5 yd. 5 in.	*12.* 4 bu. 3 pk. 1 pt.
to gallons.	**to tons.**
13. 3 qt. 1 pt. 2 gi.	*16.* 5 cwt. 5 lb. 5 oz.
14. 4 gal. 1 qt. 1 pt.	*17.* 10 cwt. 10 lb. 10 oz.
15. 1 gal. 1 pt. 1 gi.	*18.* 3 T. 30 lb. 3 oz.

426. Written Problems. 1. William had 14 shillings; John, 8 shillings; and Edwin, 2 pounds 3 shillings. How many pounds had they altogether?

2. Six boys gathered equal parts of 187 quarts of blackberries. How many bushels did each boy gather?

3. Mr. Wood buys 4 casks of wine, containing in all 832$\frac{2}{3}$ quarts. If he pays $8.50 per gallon, what did the wine cost?

4. A farmer sells 15 bu. 3 pk. 2 qt. of currants at $4. per bushel, and receives in exchange sugar at $.11 per pound. How many hundred-weight of sugar did he receive?

5. A lot containing 1,315 square feet, sells at the rate of $750 an acre. What was the selling price of the lot?

6. A sheet of zinc measures 15 inches in length, and 10 inches in breadth. How many square yards are there in the sheet?

7. If a block of stone is 3 feet long, 1$\frac{1}{2}$ feet wide, 1 foot high, how many cubic yards does it contain?

8. How many yards of $\frac{3}{4}$ yard wide carpet will be required for a room 12 feet long and 15 feet wide?

many cords are there in a pile of wood 80 feet
t wide, and 8 feet high?

ADDITION AND SUBTRACTION.

427. Addition and subtraction of fractional compound numbers may be performed by using the forms employed with integral compound numbers; or, by first reducing the fractions to integers of lower denominations.

NOTE.—Fractions frequently occur in addition and subtraction of integers under the tables of Long Measure, and of Square Measure, and, sometimes, in Liquid Measure.

428. Find the sums:

1. What is the sum of 4 mi. 4 yd. 2 ft.; 3 mi. 25 rd. 1 ft.; and, 7 mi. 18 rd. 3 yd.?

mi.	rd.	yd.	ft.	in.
4	0	4	2	0
3	25	0	1	0
7	18	3	0	0
14	43	7	3	0
14	44	2(½)	0	0
		(½=)	1	6
14	44	2	1	6

Explanation.—1. Write the numbers so that units of the same denomination form separate columns, and draw two lines underneath them.

2. The sum is found to be 14 mi. 44 rd. 2½ yd.

3. Inclose the fraction in a parenthesis, repeat it in the line under the column, and reduce it to integers of lower denominations, adding them to the integers previously found in the sum. Answer, 14 mi. 44 rd. 2 yd. 1 ft. 6 in.

2. What is the sum of £½, ⅘s., and ¾d.

	s.	d.	far.
£½ =	10	0	0
⅘ s. =	0	9	2⅖
¾ d. =	0	0	3
	10	9	5⅖
Ans.	10	10	1⅖

Explanation.—1. Write the fractions in a column, and reduce each separately to integers of a lower denomination.

2. The sum is then found to be 10s. 10d. 1⅖ far.

3. What is the sum of ¾ bu., ⅝ pk., and ½ qt.?

4.

mi.	rd.	yd.	ft.
5	90	3	1
4	76	14	2
3	85	2	1

5.

mi.	rd.	yd.	ft.	in.
14	50	2	0	11
3	75	4	2	0
26	19	2	1	9

6.

mi.	rd.	yd.	ft.	in.
29	7	0	0	5
18	0	4	2	0
32	6	5	1	

7.				8.					9.				
A.	R.	sq. rd.	sq. yd.	mi.	rd.	yd.	ft.	in.	A.	R.	sq. rd.	sq. yd.	sq. ft.
40	3	37	19	29	14	2	2	9	91	2	30	22	5
35	0	23	22	10	33	4	0	7	18	1	29	19	7
27	2	16	17	21	80	2	2	10	33	3	17	30	8

429. What is the value of the following:

1. $\frac{2}{3}$ A. + $\frac{4}{5}$ R. + 20 P.?
2. $\frac{1}{2}$ mi. + $\frac{2}{5}$ rd. + $\frac{3}{8}$ yd.?
3. $\frac{3}{10}$ T. + $\frac{3}{4}$ cwt. + 75 lb.?
4. $\frac{4}{5}$ C. + $\frac{2}{3}$ cd. ft. + 15 cu. ft.?
5. $\frac{3}{4}$ lb. + $\frac{5}{6}$ oz. + $\frac{1}{2}$ pwt ?
6. $\frac{1}{3}$ bu. + $\frac{2}{5}$ pk. + $\frac{1}{4}$ qt.?
7. $\frac{2}{3}$ gal. + $\frac{4}{10}$ qt. + $\frac{1}{2}$ pt.?
8. $\frac{1}{2}$ sq. mi. + $\frac{2}{3}$ A.?
9. $\frac{3}{4}$ cu. yd. + $\frac{4}{5}$ cu. ft.?
10. $\frac{1}{5}$ bu. + $\frac{3}{8}$ pk. + 2$\frac{1}{2}$ qt.

430. Find the differences:

1. What is the difference between 5 A. 3 R. 37 sq. rd. 25 sq. yd. 7 sq. ft. and 3 A. 2 R. 39 sq. rd. 27 sq. yd. 8 sq. ft.?

Explanation.—1. Find the remainder as in subtraction of integral compound numbers.

2. Inclose any fraction in the answer in a parenthesis and repeat it in the line below, and then reducing the fraction to integers of lower denominations, add them to the integers in the other part of the remainder. Answer, 2 A. 0 R. 37 sq. rd. 28 sq. yd. 1 sq. ft. 36 sq. in.

A.	R.	sq. rd.	sq. yd.	sq. ft.	sq. in.
5	3	37	25	7	0
3	2	39	27	8	0
2	0	37	27($\frac{1}{4}$)	8	0
			($\frac{1}{4}$=)	2	36
2	0	37	28	1	36

2. What is the difference between $\frac{1}{2}$ A. and $\frac{2}{5}$ sq. rd.?

Explanation.—1. Reduce each number separately to lower denominations and then subtract, finding the answer as above.

	sq. rd.	sq. yd.	sq. ft.	sq. in.
$\frac{1}{2}$A. =	80	0	0	0
$\frac{2}{5}$ sq. rd. =	0	12($\frac{1}{10}$ =)	($\frac{2}{10}$ =)	129$\frac{3}{5}$
	79	17($\frac{1}{4}$)	8	14$\frac{3}{5}$
		($\frac{1}{4}$=)	2($\frac{1}{4}$=)	36
Ans.,	79	18	1	50$\frac{3}{5}$

3.				4.					5.			
mi.	rd.	yd.	ft.	mi.	rd.	yd.	ft.	in.	mi.	rd.	yd.	ft.
		5	1	15	25	2	0	9	23	100	0	0
		2	2	9	27	5	2	11	18	125	3	2

6.

A.	R.	P.	sq. yd.
57	1	30	18
50	4	33	22

7.

mi.	rd.	yd.	ft.	in.
50	207	1	0	5
35	270	4	2	10

8.

A.	R.	P.	sq. yd.
400	0	0	0
175	2	35	25

9.

A.	R.	sq. rd.	sq. yd.
109	0	15	25
78	1	27	28

10.

A.	R.	sq. rd.	sq. yd.	sq. ft.
83	3	0	20	0
17	0	29	25	8

11.

A.	R.	sq. rd.	sq. yd.
77	1	35	17
19	0	37	21

431. What is the value of the following:

1. $\frac{1}{2}$ A. — $\frac{3}{8}$ sq. rd.?
2. $\frac{2}{3}$ T. — $\frac{4}{5}$ cwt?
3. $\frac{3}{4}$ C. — $\frac{2}{7}$ cd. ft.?
4. $\frac{2}{5}$ A. — $24\frac{1}{2}$ sq. rd.?
5. $\frac{3}{8}$ lb. — $\frac{4}{5}$ ʒ.?
6. $\frac{3}{8}$ lb. — $\frac{4}{5}$ pwt.?
7. $\frac{5}{9}$ gal. — $\frac{1}{2}$ qt.?
8. $\frac{4}{5}$ bu. — $\frac{3}{4}$ qt.?
9. $\frac{4}{5}$ mi. — $\frac{5}{8}$ rd.?
10. $\frac{1}{2}$ cu. yd. — $\frac{1}{4}$ cu. ft.?
11. $\frac{2}{3}$ sq. rd. — $\frac{1}{3}$ sq. yard?
12. $\frac{3}{5}$ ℥. — $\frac{1}{6}$ ʒ.?
13. $4\frac{1}{2}$ rd. — $2\frac{1}{4}$ yd.
14. $5\frac{3}{8}$ lb. — $1\frac{1}{3}$ pwt.?
15. $7\frac{1}{6}$ bu. — $2\frac{1}{3}$ pk.?
16. $2\frac{1}{5}$ gal. — $\frac{1}{4}$ qt.?

432. Written Problems. 1. A man bought a farm of 158 A. 2 R. 6 P., and sold 94 A. 1 R. 9 P. How much was the remainder worth at $320 an acre?

2. Mrs. Williams had £15 9s. 5d. She spent £6 10s. 7d. How many dollars in value had she left?

3. A dealer bought 24 T. 4 cwt. 46 lb. of coal. He kept for his own use 9 T. 12 cwt. 96 lb. He sold the remainder at $.40 a cwt. How much did he receive from the sale?

4. A goldsmith had a bar of gold weighing 15 pounds. He used 2 lb. 6 oz. 6 pwt. 15 gr. in making rings, and 5 lb. 7 oz. 3 pwt. 9 gr. in making bracelets. How much gold had he left in the bar?

5. A man has 72 hhd. 56 gal. 2 qt. of oil, and he sells 34 hhd. 60 gal. 1 qt. 1 pt. What is the remainder worth at $.02\frac{1}{2}$ a pt.?

6. A man has to travel a distance of 201 mi. 160 rd. After going 110 mi. 180 rd., how many hours will it take him to travel the rest of the distance, at the rate of one mile in two minutes?

7. A farmer had 325 bu. 2 pk. of wheat, and sold 95 bu. 3 pk. to one man, and 106 bu. to another man. He sold the remainder at $1.25 a bu. How much money did he receive for the remainder?

8. A man bought 43 cu. yd. 9 cu. ft. 1,024 cu. in. of stone for a wall, and 55 cu. yd. 7 cu. ft. 965 cu. in. of stone for a cellar. How much more stone did he buy for the cellar than for the wall?

MULTIPLICATION AND DIVISION.

433. A denominate fraction may be multiplied or divided, as any other fraction is multiplied or divided.

NOTE.—Denominate fractions frequently occur in multiplication and division in Long Measure and in Square Measure, and may occur in any problem in which the multiplier or divisor contains a fraction.

434. Find the products:

1. What is the product of 15 mi. 150 rd. 5 yd. 2 ft. multiplied by 27?

Explanation.—1. Multiply as with denominate integral numbers.

2. Inclose any fraction in the product in parenthesis, and repeating it on the next line reduce it to integers of lower denominations.

	mi.	rd.	yd.	ft.	in.
	15	150	5	2	0
					27
	417	237	$4(\frac{1}{2})$	0	0
			$(\frac{1}{2} =)$	1	6
Ans.,	417	237	4	1	6

12 | 27 | 18 | 3)54
405 | 4050 | 135 | 18
417 | 320)4077 | 5½)153 yd.
12 + 237 | 2 2
11)306 half-yd.
27 + 9 = 4½ yd.

the integers thus obtained to the other integers in the other
the answer. Ans., 417 mi. 237 rd. 4 yd. 1 ft. 6 in.

2. What is the product of 16 bu. 3 pk. 5 qt. 1 pt. multiplied by $10\frac{2}{5}$?

bu.	pk.	qt.	pt.
16	3	5	1
			$10\frac{2}{5}$
$175(\frac{2}{5})$	$2(\frac{1}{5})$	1	$\frac{2}{5}$
	$(\frac{1}{5}=)$	$1(\frac{3}{5}=)$	$1\frac{1}{5}$
$(\frac{2}{5}=)$	$1(\frac{3}{5}=)$	$4(\frac{4}{5}=)$	$1\frac{3}{5}$
Ans., 175	3	7	$1\frac{1}{5}$

9	7	5	2)10⅖
166⅔	31⅓	52	5+⅖
175⅔	4)38⅓	8)57	
	9+2⅓	7+1	

Explanation.—1. Multiply as with integral denominate numbers.

2. Inclose any fraction in the product in parenthesis, and repeating it on a separate line, reduce it to integers of lower denominations.

3. Add the integers thus obtained to the integers in the other part of the answer. Ans., 175 bu. 3 pk. 7 qt. $1\frac{1}{5}$ pt.

NOTE.—Any compound number may be reduced to its lowest denomination, and then having been multiplied as in simple numbers, it may be reduced to higher denominations.

3.

mi.	rd.	yd.	ft.
5	92	3	2
			7

4.

mi.	rd.	yd.	ft.
12	60	1	$1\frac{1}{2}$
			12

5.

mi.	rd.	yd.	ft.
15	84	2	1
			23

6.

lb.	oz.	pwt.	gr.
5	10	16	18
			$8\frac{1}{2}$

7.

A.	R.	sq. rd.	sq. yd.	sq. ft.
6	2	35	24	5
				9

8.

A.	R.	sq. rd.	sq. yd.	sq. ft.	sq. in.
16	3	22	28	7	112
					15

9.

gal.	qt.	pt.	gi.
17	3	1	2
			$24\frac{2}{5}$

10.

bu.	pk.	qt.	pt.
14	2	6	1
			$18\frac{1}{2}$

11.

sq. mi.	A.	sq. ch.	P.	sq. l.
6	360	9	12	480
				$16\frac{1}{2}$

435. Find the value of the following:

1. $\frac{3}{8}$ of 1 mi. 100 rd. 5 yd. 2 ft.
2. $\frac{4}{5}$ of 50 A. 2 R. 30 P. 25 sq. yd.
3. $4\frac{1}{2} \times 3$ lb. 5 ℥. 3 ʒ. 2 ℈.
4. $2\frac{1}{3} \times 4$ T. 18 cwt. 28 lb.
5. $4\frac{3}{4} \times 2$ bu. 3 pk. 5 qt. 1 pt.
6. $\frac{4}{5}$ of 40 A. 1 R. 4 P. 20 sq. yd.
7. $15\frac{1}{2} \times 7$ gal. 3 qt. 1 pt. 1 gi.
8. $17\frac{3}{8} \times 5$ T. 15 cwt. 55 lb.

436. Find the quotients:

1. What is the quotient of 15 A. 1 R. 25 P. 20 sq. yd. divided by 25?

Explanation.—1. Beginning with the highest denomination, divide in succession the different orders of the dividend, reducing any remainder to the next lower denomination, and adding the result to the units of the lower denomination before dividing it.

	A.	R.	P.	sq. yd.	sq. ft.	sq. in.
25)	15	1	25	20	0	0
	0	61	21 465	$473\frac{3}{4}$	$213\frac{3}{4}$	1980
Ans.,		2	18	18	8	$79\frac{1}{5}$

2. What is the quotient of 10 gal. 3 qt. 1 pt. divided by $3\frac{1}{4}$?

Explanation.—1. Reduce the mixed number to an improper fraction.
2. Divide the compound number by the numerator of the fraction.
3. Multiply the quotient by the denominator of the fraction. Ans., 3 gal. 2 qt. $\frac{12}{13}$ pt.

	gal.	qt.	pt.
13)	10	3	1
	0	3	$0\frac{9}{13}$
			4
Ans.,	3	2	$\frac{12}{13}$

3 | 2 | 13)36 | $2+\frac{10}{13}$
12
4)14
3+2

NOTE.—Any compound number may be reduced to its lowest denomination, and then having been divided, the quotient may be reduced to higher denominations.

3.

mi.	rd.	yd.	ft.	in.
8) 12	150	3	2	8

4.

mi.	rd.	yd.	ft.	in.
16) 9	200	4	1	10

5.

£.	s.	d.	far.
$8\frac{2}{3}$) 25	18	9	3

6.

sq. mi.	A.	R.	sq. rd.	sq. yd.	sq. ft.	sq. in.
$4\frac{1}{4}$) 36	300	3	21	2	6	108

7.

bbl.	gal.	qt.	pt.	gi.
18) 15	25	3	1	3

8.

sq. mi.	A.	R	sq. rd.	sq. yd.	sq. ft.
9) 2	525	3	15	18	4

9.

mi.	rd.	yd.	ft.	in.
$6\frac{1}{2}$) 18	160	2	1	8

10.

A.	R.	sq. rd.	sq. yd.	sq. ft.	sq. in.
0	1	36	26	7	96

11.

lb.	ʒ	ʒ	ȝ	gr.
$7\frac{1}{5}$) 25	9	6	2	15

437. Find the values in lower denominations:

1.	$150\frac{2}{3}$ rd.	$\div\ 4\frac{3}{4}$.	*7.*	$16\frac{2}{3}$ lb. (Troy)	$\div\ 5\frac{1}{6}$.
2.	$65\frac{1}{8}$ A.	$\div\ 5\frac{2}{5}$.	*8.*	$10\frac{5}{8}$ 3.	$\div\ 4\frac{1}{2}$.
3.	$44\frac{1}{2}$ sq. ch.	$\div\ 9\frac{1}{3}$.	*9.*	$33\frac{1}{3}$ T.	$\div\ 7\frac{1}{7}$.
4.	$112\frac{7}{8}$ cu. ft.	$\div\ 14\frac{1}{2}$.	*10.*	£$47\frac{1}{8}$	$\div\ 8\frac{1}{4}$.
5.	$26\frac{3}{4}$ gal.	$\div\ 11\frac{1}{3}$.	*11.*	$18\frac{3}{8}$ mi.	$\div\ 6\frac{4}{5}$.
6.	$75\frac{2}{5}$ bu.	$\div\ 24\frac{1}{8}$.	*12.*	$105\frac{1}{2}$ sq. rd.	$\div\ 21\frac{1}{3}$.

438. Written Problems. 1. How much are $250\frac{1}{2}$ acres of land worth, if one acre is worth £25. 14s. 7d.?

2. A man walks 92 mi. 245 rd. 3 yd. 4 ft. in 23 hours. How far at the same rate does he walk in one hour?

3. If each tank holds 1 hhd. 1 bl. 26 gal. 3 qt. 1 pt., how much will 29 tanks hold?

4. If a dozen silver spoons weigh 1 lb. 1 oz. 14 pwt. 16 gr., how much will $16\frac{1}{2}$ dozen spoons weigh?

5. 8 T. 16 cwt. 56 lb. 12 oz. of coal are to be stored in $12\frac{1}{2}$ equal bins. How much will each bin hold?

6. What is the length of a fence which incloses a rectangular lot twice as long as it is wide, if the width is 27 rd. 4 yd. $1\frac{1}{2}$ ft.?

7. How much wheat will it take to fill $24\frac{1}{3}$ bins, if each bin holds 97 bu. 2 pk. 7 qt.?

8. A man has 65 lb. 7 oz. 15 pwt. 18 gr. in 21 equal bars of gold. How much is there in each bar?

9. 1,200 A. 120 sq. rd. 11 sq. yd. of land are to be divided equally among 3 brothers. How much land will each receive?

10. A man cuts 3 C. 3 cd. ft. 12 cu. ft. of wood in one day. How much wood will he cut, at the same rate, in $25\frac{2}{3}$ days?

11. A grocer sold 15 T. 15 cwt. of coal to 25 customers. What was the average weight of coal sold to each customer?

439. To find a number, a part being given.

1. If $\frac{3}{8}$ of a farm contains 4 A. 1 R. 30 sq. rd. 30 sq. yd., how much does the entire farm contain ?

NOTE.—Divide by 3, to find $\frac{1}{8}$; multiply by 8, to find $\frac{8}{8}$, or the entire farm.

2. Mr. Johnson sold $\frac{5}{8}$ of his mill for £20,000. 10s. 10d. What was the entire mill worth ?

3. If a cup holding 1 pt. 1 gi. holds $\frac{3}{11}$ as much as a pitcher, how much does the pitcher hold ?

4. The circumference of the front wheel of a wagon is 3 yd. 2 ft. 8 in. long, or $\frac{5}{8}$ of the circumference of the rear wheel. How long is the circumference of the rear wheel ?

5. A clock had 5 da. 5 hr. to run and this was $\frac{5}{7}$ of its entire period. What was its entire period ?

440. To find a number, a part being given, divide the given number by the numerator and multiply the result by the denominator of the fractional part.

441. Written Problems. 1. A tenant paid £22. 10s. 10d. for 3 months' rent. What rent per year did he pay ?

2. If Edward can plough 3 A. 1 R. 25 sq. rd. in $\frac{4}{5}$ of a day, how much can be plough in a day ?

3. Mr. Miller, having travelled 250 mi. 250 rd. 15 ft., found that he had $\frac{2}{5}$ of his journey still to travel. What was the length of the entire journey ?

4. A silver pitcher weighing 2 lb. 5 oz. weighed $\frac{5}{6}$ as much as a silver salver. How much did the salver weigh ?

5. A druggist sold $\frac{5}{8}$ of a quantity of quinine. He then had 1 ℔ 3 ℥ 4 ℨ 2 ℈ left. How much quinine did he have at first ?

6. [illegible]h silver spoon weighs 3 oz. 5 pwt., and 1 dozen of [illegible] $\frac{3}{8}$ as much as a silver tray, what is the weight of

7. By selling a horse for £95 10s., a farmer lost $\frac{3}{10}$ of the cost price. What did the horse cost him ?

8. On weighing $\frac{3}{8}$ of a load of hay, it was found to weigh 15 cwt. 75 lb. How much did the remainder of the load weigh ?

442. To find what part one number is of another.

1. What part of 2 tons are 80 pounds ?

2 tons equal 4,000 pounds ; 80 pounds are $\frac{80}{4000}$ or, $\frac{1}{50}$ of 2 tons.

2. 3 qt. 1 pt. are what part of 5 bu. 5 pk. ?

3 qt. 1 pt. = 7 pt. ; 5 bu. 5 pk. = 25 pk. = 25 × 8 × 2 = 400 pt. ; 7 pt. = $\frac{7}{400}$ of 400 pt. Ans.

3. What part of a day are 5 hr. 24 min. 15 sec. ?

4. What part of 1 cord are 24 cu. ft. 720 cu. in. ?

5. What part of 1 sq. mi. 30 sq. rd. 10 sq. ft. are 50 sq. rd. 150 sq. ft. 110 sq. in. ?

6. What part of 9 gal. 3 qt. 2 pt. 1 gi. are 3 gal. 2 qt. ?

7. What part of a cube, each of whose edges is 12 feet, is a block containing 12 cubic feet ?

443. To find what part one denominate number is of another, reduce both to the same denomination, and then write the part as the numerator and the whole as the denominator of a fraction, and reduce it to its lowest terms.

444. Written Problems. 1. A farm containing $\frac{3}{4}$ of a square mile had 10 sq. rd. of pasture land. What part of the farm was in pasture land ?

2. If Mary uses 1 quire 10 sheets of paper, what part of a ream does she use ?

3. If a hod holds $\frac{1}{3}$ bu. of coal, what part of load, or 25 bushels, does it hold ?

4. If Harry wastes 2 hr. 45 min. 35 sec. each day, what part of each week does he waste ?

MISCELLANEOUS PROBLEMS.

445. Oral Problems. 1. What will be the cost of 5 cords of wood, if 6 C. 2 cd. ft. cost $25 ?

2. If a ton of coal is worth $7.20, how much are 10 hundred-weight worth ?

3. A man undertakes a journey of 1 mi. 100 rd. If he travels 210 rd. in 20 minutes, how long will it take him to travel the entire distance ?

4. If a man sells 3 yd. 1 ft. of wire for $.10, for how much will he sell 1 yd. 1 ft. ?

5. What will be the cost of 3 qt. 1 pt. of wine if 1 gallon costs $6.40 ?

6. It takes 7 sq. yd. 1 sq. ft. of carpet to cover a floor. How many square feet of carpet will it take to cover a floor $\frac{3}{4}$ as large ?

7. A certain tank holds 15 gal. 3 qt. of water, and another holds 20 gal. 1 qt. What fractional part of the second tank are the contents of the first tank ?

8. If $\frac{2}{11}$ of a can holds 5 gal. 2 qt. of oil, how much oil will fill the can ?

9. What part of a pound is the entire weight of a watch, if the case, which weighs $\frac{5}{8}$ of the whole, weighs 5 ounces ?

10. Edward earned £5 per week and spent £1 10s. for board. What part of his earnings did he have left ?

11. What fraction of a dram is the weight of ten quinine pills, each weighing 2 gr. ? What part of an ounce ?

12. How many acres are there in the area of two tracts of land, one of which contains $\frac{1}{10}$ of a square mile, and the other, 320 square rods ?

13. Of $\frac{3}{4}$ of a bushel of pears, how many quarts remained after $\frac{3}{4}$ of a peck had been sold ?

14. [illegible] having 5½ pecks of potatoes, bought 4½ pecks [illegible]els did he then have ?

15. A bar contained $\frac{1}{12}$ of a cubic foot of silver. Into how many chains, each containing $\frac{1}{2}$ of a cubic inch, can the bar be made ?

16. A grocer had $1\frac{3}{4}$ tons of coal. He sold 15 hundredweight. How many bushels of coal are there in the remainder, if each bushel weighs one hundred pounds ?

446. Written Problems. 1. Reduce 15 sheets to the fraction of a ream.

2. Find the difference between $1\frac{1}{3}$ cubic yards and $26\frac{1}{4}$ cubic inches.

3. If 25 francs are equivalent to a pound sterling, what fraction of a shilling is a franc ?

4. Add $\frac{5}{6}$ of a day and $\frac{1}{12}$ of an hour. What is the sum ?

5. A glazier charges $.40 per square foot. What is his charge for a light of glass 30 inches wide by 50 inches long ?

6. How many yards of $\frac{3}{4}$ yard wide carpet will it take for a room 15 feet long by 12 feet wide ?

7. A wheel is 3 feet 4 inches round. How many times will it turn in passing over 1 mi. 50 rd. of road ?

8. If tomatoes cost $.12$\frac{1}{2}$ a quart, how much will $\frac{1}{8}$ of a bushel cost ?

9. Mr. Irving buys a city lot 42 by 150 feet, for which he pays at the rate of $25 per square foot. What price does he pay per acre ?

10. How many cubic feet can a man cut in 6 hours, if he cuts 2 cords 64 cubic feet in one day of 10 hours ?

11. The distance between two towns is $2\frac{1}{2}$ miles. How many steps will a man take in walking the distance, if each step is 2 feet 6 inches long ?

12. A man sells a piece of cloth containing $31\frac{1}{2}$ yards for 47 pounds 5 shillings. What is the cost per yard in *United States money ?*

13. Into how many village lots, each containing 1 A. 16 sq. rd., can a farm of 249 A. 2 R. 30 sq. rd. be divided ?

14. A man buys 4 bags of coffee weighing respectively 112 pounds 3 ounces ; 80 pounds 7 ounces ; 96 pounds 4 ounces ; 111 pounds 2 ounces. He pays $112. What is the cost per pound ?

15. If a pendulum vibrates 115 times in a minute, how many times will it vibrate in $\frac{1}{4}$ of a common year ?

16. What profit is made on each of 25 gold watches, that together cost £500, if each watch is sold for £25. 5s. ?

17. If lead pencils sell at $3.60 per gross, what will $3\frac{1}{3}$ dozen cost ?

18. From a tank containing 100 gal. 3 qt. of wine, $2\frac{1}{2}$ barrels were taken. What part of the wine was left in the tank ?

19. How many square rods are there in a farm $\frac{1}{2}$ mile square ?

20. How many times will a wheel revolve in going a distance of 52 miles, the circumference of the wheel being 16 ft. 6 in. ?

21. How long would a cannon-ball, moving at the rate of 1,000 feet in a second, be in passing from the earth to the moon, 240,000 miles ?

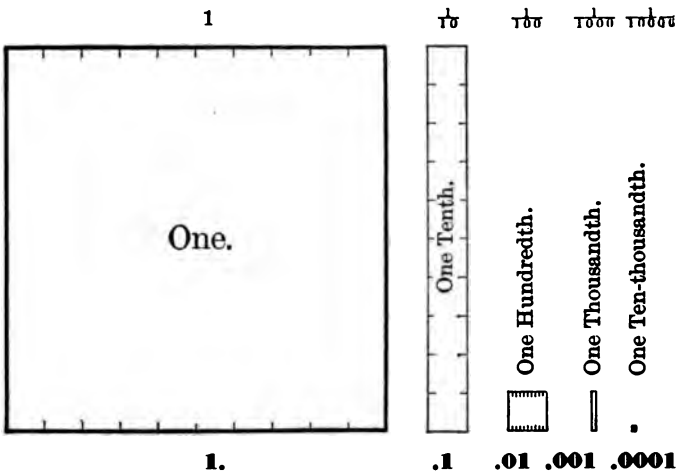
22. What is the difference between 8 rods square and 8 square rods ?

23. From a cube of iron, each of whose edges was 2 feet long, a piece containing 1,000 cubic inches was taken. What part of the cube was left?

24. How many seconds are there in $\frac{3}{15}$ of a degree of longitude ?

25. Mr. Banks earned £5. 5s. 5d. per week. Mr. Rollin earned $30 per week. What part of Mr. Rollin's earnings was [illegible] over Mr. Banks' earnings?

DECIMAL FRACTIONS.



447. Show by the diagram the difference between an integer and a fraction.

448. Decimal fractions are fractions in which the denominators are 10, 100, 1000, etc. (1 with ciphers annexed).

449. A **mixed decimal** is an integer and a fraction together expressed decimally, 5.5 ; 20.03.

450. A **complex decimal** is a decimal fraction followed by a common fraction ; as $.37\frac{1}{2}$, $.666\frac{2}{3}$.

451. Decimal comes from *decem,* the Latin word for ten, and it is applied to the system of whole numbers and fractions in which *ten* units of one order make one of the next *higher order.*

NOTATION AND NUMERATION.

452. The denominators of decimal fractions are not usually expressed, but are indicated by the number of places at the right of the decimal point, there being one place for every cipher in the denominator. Thus,

$\frac{1}{10}$	is written	.1	$\frac{1}{100}$	is written	.01	$\frac{1}{1000}$	is written	.001
$\frac{3}{10}$	"	.3	$\frac{3}{100}$	"	.03	$\frac{7}{1000}$	"	.007
$\frac{5}{10}$	"	.5	$\frac{5}{100}$	"	.05	$\frac{49}{1000}$	"	.049
$\frac{8}{10}$	"	.8	$\frac{29}{100}$	"	.29	$\frac{101}{1000}$	"	.101
$\frac{9}{10}$	"	.9	$\frac{78}{100}$	"	.78	$\frac{913}{1000}$	"	.913

453. Show by the diagram the meaning of the following:

	Decimal Notation.
One hundred-thousand	100,000.
Ten thousand	10,000.
One thousand	1,000.
One hundred	100.
Ten	10.
One	1.
One tenth . . . . . $\frac{1}{10}$ . .	.1
One hundredth . . . . . $\frac{1}{100}$ . .	.01
One thousandth . . . . $\frac{1}{1000}$ . .	.001
One ten-thousandth . . . . $\frac{1}{10000}$ . .	.0001
One hundred-thousandth . . $\frac{1}{100000}$ . .	.00001

454. Rule for Notation and Numeration.—1. The numerator is expressed as any whole number, and is preceded by a decimal point.

2. The denominator is indicated, being 1 with as many ciphers at its right, as there are decimal places at the right of the decimal point.

NOTE.—If the numerator does not occupy a sufficient number of places *to indicate* the denominator, ciphers are written between it and the *decimal point*.

455. Express in decimal notation the following fractions:

1.	$\frac{7}{10}$.	6.	$\frac{3}{100}$.	11.	$\frac{1056}{10000}$.	16.	$\frac{20564}{100000}$.	21.	$\frac{365043}{1000000}$.
2.	$\frac{9}{10}$.	7.	$\frac{157}{1000}$.	12.	$\frac{248}{10000}$.	17.	$\frac{1460}{100000}$.	22.	$\frac{70385}{1000000}$.
3.	$\frac{8}{100}$.	8.	$\frac{69}{1000}$.	13.	$\frac{75}{10000}$.	18.	$\frac{378}{100000}$.	23.	$\frac{6712}{1000000}$.
4.	$\frac{16}{100}$.	9.	$\frac{4}{1000}$.	14.	$\frac{9}{10000}$.	19.	$\frac{89}{100000}$.	24.	$\frac{819}{1000000}$.
5.	$\frac{43}{100}$.	10.	$\frac{27}{1000}$.	15.	$\frac{68}{10000}$.	20.	$\frac{7}{100000}$.	25.	$\frac{15}{1000000}$.

456. Read the following decimal fractions:

1.	.3	6.	.345	11.	.0209	16.	.30156	21.	.000113
2.	.9	7.	.062	12.	.2056	17.	.20037	22.	.4156032
3.	.14	8.	.047	13.	.0038	18.	.00021	23.	.0030304
4.	.07	9.	.008	14.	.0007	19.	.635425	24.	.01560415
5.	.22	10.	.101	15.	.5004	20.	.015043	25.	.000281709

457. Whole numbers and decimal fractions may be written and read together, as in the following:

Table.

ORDERS OR PLACES.	Millions.	,	Hundred thousands.	Ten thousands.	Thousands.	,	Hundreds.	Tens.	Units.	Decimal Point.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.
	9	,	5	0	6	,	4	3	0	.	3	0	7	5	6	1
			8	1	5	,	3	0	7	.	0	0	6	1	4	
NUMBERS TO BE READ.				3	0	,	0	5	0	.	0	5	0	7		
				2	6	,	1	0	8	.	5	0	2	1		
			7	4	5	,	0	0	7	.	0	0	9	6	2	
	4	,	3	0	8	,	0	1	9	.	1	0	4	5	0	9

458. Read above numbers, explaining the value of each place.

Thus: Nine million, five hundred six thousand, four hundred thirty *and* three hundred seven thousand, five hundred sixty-one *millionths*.

NOTE.—The *places* are integral and fractional; but the numbers before *and after the decimal point* are integral.

459. Each number is read as if it consisted of two whole numbers, separated by a decimal point read *and*. The part of the number at the right of the decimal point being only a numerator, is followed by the name of its lowest order, which is the name of the denominator.

460. Write from dictation the following numbers:

1. Seven *and* thirty-five *thousandths.*
2. Eighty *and* fourteen *ten-thousandths.*
3. Nine hundred one *and* one *thousandth.*
4. Three thousand six *and* two hundred eight *thousandths.*
5. Seven hundred *and* fifty-six *ten-thousandths.*
6. Twenty-three thousand nine *and* two hundred four *ten-thousandths.*
7. Sixty-seven *and* one thousand eighty-five *hundred-thousandths.*
8. Five hundred thousand *and* nine hundred six *ten-thousandths.*
9. Three hundred ten *and* ten thousand sixty *hundred-thousandths.*
10. Two million three hundred *and* forty-seven *hundred-thousandths.*
11. One thousand *and* thirty thousand four *ten-millionths.*
12. One million one thousand one *and* one thousand one *millionths.*
13. Fifty *and* three hundred million one thousand eighteen *billionths.*
14. One *and* one hundred ten *hundred-thousandths.*
15. Twenty-two thousand *and* twenty-two *ten-thousandths.*
16. Eighty thousand eight *and* eight *hundred-thousandths*
17. Ninety-nine thousand *and* one *hundred-thousandth.*

hundred thousand *and* one *ten-thousandth.*

housand ten *and* ten ten-thousandths.

461. Write from dictation the following numbers:

1. .027	5. .01016	9. .00871/2	13. 9.0363/3	17. 127.00281/6
2. .0503	6. .00037	10. .001251/3	14. 8.0011/8	18. 95.10061/9
3. .0005	7. .020019	11. .0230463/5	15. 17.09521/2	19. 300.25083/7
4. .6089	8. .0035063	12. .9500183/4	16. 20.10051/6	20. 34.010651/2

462. Illustrate the **PRINCIPLES OF NOTATION** by using the three following forms:

.05. .5 .005 Prin. IV.; thus, .5 = .50 ($\frac{50}{100} = \frac{5}{10}$)

I. *The name of the denominator of a decimal fraction is the name of the lowest order at the right.*

II. *The value expressed by a figure is made ten times as great by moving it one place to the left, or by moving the decimal point one place to the right.*

III. *The value expressed by a figure is made one tenth as great by moving it one place to the right, or by moving the decimal point one place to the left.*

IV. *Annexing ciphers at the right of a decimal fraction multiplies the numerator and the denominator of the fraction, but does not change the value of the fraction.* (*See Art.* 310, *Prin. III.*)

V. *Removing ciphers from the right of a decimal fraction divides the numerator and the denominator of the fraction, but does not change the value of the fraction.* (*See Art.* 310, *Prin. III.*)

VI. *Inserting a cipher between the decimal point and the numerator of a decimal fraction, divides the value of the fraction by ten.*

VII. *Removing a cipher from between a decimal point and the numerator of a decimal fraction multiplies the value of the fraction by ten.*

REDUCTION.

463. Reduction is changing a number, without changing its value.

464. I. **To reduce a common fraction to a decimal fraction.**

I. Reduce $\frac{1}{5}$ to a decimal fraction.

$\frac{1}{5}$ equals 1 divided by 5 ; 1 equals 1.0.

5)1.0
.2 Ans

2. Reduce $\frac{3}{4}$ to a decimal fraction.

$\frac{3}{4}$ equals 3 divided by 4 ; 3 equals 3.00.

4)3.00
.75 Ans.

3. Reduce $\frac{4}{5}$ to a decimal fraction.

NOTE.—At the right of the numerator, place a decimal point and as many ciphers as are necessary ; and then perform the operation of division indicated by the fraction.

4. Reduce $\frac{3}{8}$ to a decimal fraction. *See Principle IV.*
5. Reduce $\frac{5}{8}$ to a decimal fraction. " " "
6. Reduce $\frac{7}{16}$ to a decimal fraction. " " "

465. II. **To reduce a decimal fraction to a common fraction.**

1. Reduce .6 to a common fraction.

.6 also written $\frac{6}{10}$, equals in lowest terms $\frac{3}{5}$.

2. Reduce .75 to a common fraction.

.75 also written $\frac{75}{100}$ equals in lowest terms $\frac{3}{4}$.

3. Reduce .175 to a common fraction.

NOTE.—The decimal fraction is written in the form of a common fraction, and reduced to its lowest terms.

4. Reduce .048 to a common fraction.
5. Reduce .1028 to a common fraction.
6. Reduce .050025 to a common fraction.

466. III. **To reduce a decimal to a required equal decimal fraction.**

1. Reduce .05 to thousandths.

.05 equals $\frac{5}{100}$, or $\frac{50}{1000}$, written .050.

2. Reduce .030 to hundredths. *See Principle V.*

.030 equals $\frac{30}{1000}$, or $\frac{3}{100}$, written .03.

3. Reduce .025 to ten-thousandths.

NOTE.—Ciphers are annexed to or removed from the right of the decimal fraction.

4. Reduce .340 to hundredths. *See Principle V.*
5. Reduce .0550 to thousandths. " " "
6. Reduce .02500 to thousandths. " " "

467. IV. **To reduce an integer to a required decimal fraction.**

1. Reduce 5 units to hundredths. *See Principle IV.*

5 units equals $\frac{5}{1}$ or $\frac{500}{100}$, written 5.00.

2. Reduce 12 units to thousandths.

12 units equals $\frac{12}{1}$ or $\frac{12000}{1000}$, written 12.000.

3. Reduce 7 units to ten-thousandths.

NOTE.—Ciphers are annexed at the right of the decimal point.

4. Reduce 9 to hundred-thousandths.
5. Reduce 25 to millionths.
6. Reduce 125 to ten-millionths.

468. V. **To reduce a mixed number to a decimal.**

1. Reduce $4\frac{1}{2}$ to a decimal. $\frac{1}{2}$ equals .5 ; $4\frac{1}{2}$ equals 4.5.
2. Reduce $3\frac{3}{4}$ to a decimal. $\frac{3}{4}$ equals .75 ; $3\frac{3}{4}$ equals 3.75.
3. Reduce $5\frac{3}{8}$ units to a decimal.

NOTE.—The common fraction is reduced to a decimal fraction, and written at the right of the integer.

4. Reduce $10\frac{5}{8}$ to a decimal. *See Principle IV.*
5. Reduce $15\frac{4}{5}$ to a decimal. " " "
6. Reduce $24\frac{7}{8}$ to a decimal. " " "

7. Reduce to decimal fractions the following mixed numbers:

1. $14\frac{3}{5}$.	*3.* $36\frac{3}{16}$.	*5.* $205\frac{4}{32}$.	7. $570\frac{11}{64}$.
2. $25\frac{1}{2}$.	*4.* $72\frac{3}{10}$.	6. $369\frac{15}{16}$.	8. $428\frac{5}{25}$.

469. VI. **To reduce a mixed decimal to an equal mixed decimal.**

1. Reduce 5.6 to hundredths. *See Principle IV.*

5.6 equals $\frac{56}{10}$ or $\frac{560}{100}$, written 5.60.

2. Reduce 2.37 to thousandths. *See Principle IV.*

2.37 equals $\frac{237}{100}$ or $\frac{2370}{1000}$, written 2.370.

3. Reduce 3.015 to hundred-thousandths. *See Principle IV.*

NOTE.—Ciphers are annexed to or removed from the right of the decimal fraction as required, and then the entire number is read as so many units of the lowest decimal order.

4. Reduce 15.2046 to hundred-thousandths.

5. Reduce 11.00101 to millionths.

6. Reduce 6.027 to ten-millionths.

NOTE.—In reducing a common fraction to a decimal fraction, *exact division* is not always possible. The remainder is then written at the right of the decimal fraction making a **complex decimal.**

470. VII. **To reduce a complex decimal fraction to a decimal fraction.**

1. Reduce .02½ to a decimal fraction.

.02½ equals $\frac{2\frac{1}{2}}{100}=\frac{2\frac{1}{2}}{100}\times\frac{10}{10}=\frac{25}{1000}$, written .025.

2. Reduce .025¼ to a decimal fraction.

.025¼ equals $\frac{25\frac{1}{4}}{1000}=\frac{25\frac{1}{4}}{1000}\times\frac{100}{100}=\frac{2525}{100000}$, written .02525.

3. Reduce .175¾ to a decimal fraction.

NOTE.—Reduce the common fraction to a decimal fraction, and annex it at the right of the given decimal.

4. Reduce .0345⅝ to a decimal fraction.

5. Reduce .00150⅜ to a decimal fraction.

6. Reduce .6037½ to a decimal fraction.

7. Reduce to decimal fractions the following complex decimal fractions:

1. $5\frac{1}{8}$.
2. $100\frac{3}{4}$.
3. $10\frac{5}{8}$.
4. $.0090\frac{1}{2}$.
5. $.0901\frac{3}{16}$.
6. $.010\frac{9}{16}$.
7. $.00\frac{3}{32}$.

471. If the denominator of a common fraction is 2 or 5, or is a multiple of these factors only, it may be reduced to an exact or **terminate decimal.**

$\frac{1}{2} = .5$; $\frac{2}{5} = .4$; $\frac{3}{20} = .15$; $\frac{4}{25} = .16$; $\frac{5}{8} = .625$.

472. If the denominator of a common fraction contains any other factor than 2 or 5, it can only be reduced to an **interminate decimal.**

$\frac{1}{3} = .333+$; $\frac{1}{6} = .1666+$; $\frac{1}{12} = .0833+$.

473. Interminate decimals are called **circulating decimals,** as the same figure or set of figures is found to be indefinitely repeated in the decimal.

$\frac{2}{3} = .66666+$; $\frac{5}{6} = .83838383+$; $\frac{3}{7} = .428571428571+$.

474. The **repetend** is the figure or set of figures *to be repeated* in circulating decimals.

NOTE.—If the repetend is a single figure, it is indicated by a dot placed over it; if the repetend consists of several figures, a dot is placed over the first and the last figure; thus, $.\dot{3}$; $.5\dot{6}$; $.\dot{3}\dot{2}$; $.2\dot{5}70\dot{8}$.

475. VIII. **To reduce a common fraction to a circulating decimal.**

1. Reduce $\frac{2}{3}$ to a circulating decimal.

$$3)\overline{2.000}$$
$$\overline{.666+}$$ Ans., $.\dot{6}$.

2. Reduce $\frac{5}{6}$ to a circulating decimal.

$$6)\overline{5.0000}$$
$$\overline{.8333+}$$ Ans., $.8\dot{3}$.

3. Reduce eight-ninths to a circulating decimal.

NOTE.—Division is performed as indicated by the fraction, and the repetend is then indicated by points.

4. Reduce $\frac{5}{9}$ to a circulating decimal.
5. Reduce two-sevenths to a circulating decimal.
6. Reduce $\frac{4}{9}$ to a circulating decimal.
7. Reduce to circulating decimals the following common fractions:

1. $\frac{3}{9}$.	3. $\frac{7}{11}$.	5. $\frac{15}{31}$.	7. $2\frac{2}{36}$.
2. $\frac{6}{9}$.	4. $\frac{9}{11}$.	6. $\frac{19}{31}$.	8. $9\frac{1}{15}$.

476. IX. **To reduce a circulating decimal to a common fraction.**

All circulating decimals are equal to common fractions with denominators composed of 9's, thus:

1. $\frac{1}{9} = .1111+ = .\dot{1}$; hence, multiplying by 3, $\frac{3}{9} = .3333+ = .\dot{3}$; and $\frac{4}{9} = .4444+ = .\dot{4}$; etc.

2. $\frac{1}{99} = .010101+ = .\dot{0}\dot{1}$; hence, $\frac{5}{99} = .050505+ = .\dot{0}\dot{5}$; and $\frac{42}{99} = .424242+ = .\dot{4}\dot{2}$.

3. $\frac{1}{999} = .001001001+ = .\dot{0}0\dot{1}$; and $\frac{325}{999} = .325325+ = .\dot{3}2\dot{5}$.

RULE.—The repetend is written as the numerator of a common fraction, the denominator being as many 9's as there are places in the repetend.

4. Reduce $.\dot{7}\dot{2}$ to a common fraction.

5. Reduce $.\dot{9}8\dot{1}$ to a common fraction.

477. Rules for Reduction of Decimals. I. To reduce a common fraction to a decimal fraction, place a decimal point and ciphers after the numerator; and then divide, as indicated by the fraction.

II. To reduce a decimal fraction to a common fraction, write the given fraction in the form of a common fraction, and reduce it to its lowest terms.

III. To reduce a decimal fraction to an equal decimal fraction, annex or remove ciphers at the right of the decimal fraction, until the required denomination is obtained.

NOTE 1.—Rule I. is applied in the reduction of *mixed numbers*, of *complex fractions*, and of common fractions when required to be reduced to *circulating* decimals; Rule III. is applied in the reduction of *integers* and of *mixed decimals*.

NOTE 2.—In reducing a circulating decimal, the repetend is used as the numerator of a fraction, with as many 9's in the denominator as there are figures in the repetend.

NOTE 3.—The signs + and − are used after a quotient has been sufficiently extended, to indicate a slight increase or decrease in the answer.

478. Reduce to decimal fractions the following:

common fractions.			mixed numbers.		
1. $\frac{1}{3}$.	5. $\frac{5}{8}$.	9. $\frac{3}{250}$.	13. $2\frac{1}{2}$.	17. $7\frac{1}{8}$.	21. $9\frac{3}{20}$.
2. $\frac{3}{4}$.	6. $\frac{3}{200}$.	10. $\frac{6}{300}$.	14. $3\frac{1}{5}$.	18. $2\frac{1}{3}$.	22. $8\frac{4}{25}$.
3. $\frac{2}{5}$.	7. $\frac{3}{5}$.	11. $\frac{8}{400}$.	15. $1\frac{3}{4}$.	19. $3\frac{3}{8}$.	23 $2\frac{2}{500}$.
4. $\frac{3}{8}$.	8. $\frac{7}{8}$.	12. $\frac{45}{600}$.	16. $5\frac{2}{3}$.	20. $4\frac{2}{3}$.	24. $5\frac{15}{400}$.

complex decimal fractions.			to circulating decimals.		
25. $.25\frac{1}{2}$	28. $.2\frac{1}{8}$	31. $.3\frac{5}{8}$	34. $\frac{2}{3}$.	37. $\frac{7}{9}$.	40. $.12\frac{2}{3}$
26. $.05\frac{1}{4}$	29. $.9\frac{3}{4}$	32. $.75\frac{3}{5}$	35. $\frac{8}{9}$.	38. $\frac{4}{3}$.	41. $.35\frac{4}{9}$
27. $.10\frac{1}{5}$	30. $.8\frac{7}{8}$	33. $.21\frac{4}{5}$	36. $\frac{5}{6}$.	39. $\frac{6}{7}$.	42. $2.09\frac{4}{7}$

479. Reduce to common fractions the following:

decimal fractions.			circulating decimals.		
1. .2	4. .125	7. .1055	10. $.\dot{8}$	13. $.\dot{2}\dot{4}$	16. $.2\dot{6}$
2. .8	5. .625	8. .2305	11. $.\dot{7}$	14. $.\dot{3}\dot{6}$	17. $.3\dot{7}$
3. .25	6. .075	9. .0004	12. $.\dot{6}$	15. $.\dot{3}0\dot{3}$	18. $3.25\dot{4}$

480. Reduce the following:

to tenths.	to hundredths.	to thousandths.	to ten-thousandths.
1. 7.	4. .6	7. 4.	10. 8.7
2. 50.	5. .5	8. 53.	11. 9.50
3. 1.5	6. 2.3	9. 77.4	12. 6.712

ADDITION AND SUBTRACTION.

481. Decimal fractions are added and subtracted as decimal integers are added and subtracted.

NOTE.—The numbers are written so that the decimal points stand in a column.

482. Find the sums:

1.	2.	3.	4.	5.
$1,230.05	$138.287	$9,781.25	$1,237.015	$20.07
3,180.87	79.135	5,630.95	12.005	.49
2,345.03	181.432	391.803	2,000.	300.007
1,795.55	293.056	99.427	100.10	90.812

6.	7.	8.	9.	10.
311.01	84.7	.007	.0001	187.0107
175.1	.03	7.07	.11	47.085
84.097	200.9	.7	2.	159.7
101.003	85.	70.	.1	73.027

483. Find the value of the following:

1. $\frac{3}{4} + .25 + 1.4 \div .4\frac{1}{2}$.
2. $.6 + .33\frac{1}{3} + 2.5 + \frac{4}{5}$.
3. $.37\frac{1}{2} + \frac{7}{8} + .125 + 3.25$.
4. $5\frac{3}{4} + .5 + 1.75 + .24$.
5. $2.87\frac{1}{2} + .62\frac{1}{2} + \frac{1}{3} + .16\frac{2}{3}$.
6. $2.6 + .66\frac{2}{3} + \frac{1}{8} + 9$.
7. $4.6 + .45 + 6.6\frac{1}{2} + \frac{7}{20}$.
8. $\frac{4}{15} + 1\frac{8}{15} + .2 + .36$.
9. $6.125 + \frac{5}{8} + .1 + \frac{1}{36}$.
10. $\frac{4}{5} + .35 + 2.33 + 11\frac{2}{3}$.
11. $1.65 + \frac{1}{10} + .5 + 2.4$.
12. $27\frac{2}{8} + .37\frac{1}{2} + .75 + .48$.

484. Find the remainders:

1.	2.	3.	4.	5.
\$9169.009	\$653.01	\$157.105	\$113.80	\$150.307
875.104	79.847	148.918	67.001	149.308

6.	7.	8.	9.	10.
19.9009	1000.01	647.032	199.7	11.001
14.01	898.089	76.0008	57.0509	1.0101

11.	12.	13.	14.	15.
797.01	1900.00	875.707	821.03	9000.09
758.098	.09	98.08	.0085	.909

485. What is the value of the following:

1. $8 - .008 + .08$?
2. $27.5 - .18\frac{5}{10}$?
3. $50.05 - \frac{75}{1000} + \frac{1}{4}$?
4. $100.1 - .3$?
5. $4\frac{2}{3} + 5\frac{1}{3} - .01$?
6. $7 - 6\frac{4}{5} + .2$?
7. $15 + \frac{7}{8} - .125$?
8. $100 + 1 - \frac{1}{10}$?
9. $47 - 37.37 + 3.5\frac{3}{4}$?
10. $4 + 58.50\frac{7}{8} - 16\frac{5}{8}$?
11. $23 - .023 + .0\frac{4}{5}$?
12. $51 + .3 - .00\frac{1}{4}$?
13. $23 - 22\frac{17}{32}$?
14. $50\frac{2}{8} - 18\frac{4}{8} + 15.75$?

486. Oral Problems. 1. Henry earned \$6.35, and spent \$3.75 for board and \$2½ for clothes. How much did he save in a week?

2. Charles began work at 2:45 P. M., and stopped at 5:25 P. M. How long did he work?

3. Mr. Grey bought wheat at \$1.75 per bushel, and sold it at \$1⅞ per bushel. How much did he gain?

4. Jane spent \$1.05 for calico, \$.12½ for lining, and \$.37½ for trimming. What was the amount of her bill?

5. What change should Mrs. Davis receive out of \$2 after buying a broom for \$.20, a pail for \$.45, and a brush for \$.60?

6. A milliner bought velvet at \$2⅝ and sold it for \$3.50. How much did she gain on a yard?

7. A dealer sold apples which cost him \$⅜ a peck for \$.32. How much did he lose on each peck?

8. Henry has \$2.50. If he spends \$1.12½, and \$.62½, how much money has he left?

9. Martha has \$3.25 and Mary has \$2.75. Martha spends \$¾. Which of the two girls then has the more money, and how much the more?

10. A farmer who owns 24.16 acres has ploughed 8.04 acres. How many acres remain unploughed?

487. Written Problems. 1. Mr. Clark earned \$63 a month. He owes \$21.50 for rent, \$12.13 for meat, \$15.37 for groceries, \$6.03 for coal, \$3.60 for milk, and \$10.05 for clothing. How much will he be in debt after paying out the \$63?

2. A man paid \$300 for a horse, \$275.75 for a carriage, and \$51.25 for a wagon. How much money has he left out of \$900.50?

3. A man leaves \$1,057.40 worth of property to his wife, \$908.55 worth to his son, and \$600 to his daughter. What was the *amount of his* property?

4. A merchant paying for goods receives out of a $1,000 bill $157.37. He bought laces to the amount of $426.19 and expended the rest of the money in ribbon. How much money did he spend for ribbon?

5. A man who owed $1,080.25 paid $238.10 at one time, and $409 at another. How much did he still owe?

6. Bought a farm for $8,056$\frac{3}{4}$ and built a house for $3,015.25. Bought live stock for $6,024.18. How much more did the farm and house cost than the stock?

7. Mr. Sloane bought a ton of coal for 6\frac{7}{8}$ and a cord of wood for 5.37\frac{1}{2}$. $.25 was deducted for cash payment. What change did he receive out of $15?

MULTIPLICATION.

488. To multiply when the multiplier is a whole number:

1. Find the product of 27.034 multiplied by 27.

Explanation.—1. Multiply first as if there were no decimal point.

2. Multiplying 27034 thousandths by 27 gives a product of 729918 thousandths.

3. To indicate thousandths, the order of the answer, point off three decimal places in the product.

	27.034
	27
	189 238
	540 68
Ans.,	729.918

2.	3.	4.	5.	6.
$1,257.038	$1,281.47	$595.009	$370.134	$74.087
8	11	12	9	8

7.	8.	9.	10.	11.
11.0543	67.4031	2190.0102	830.707	37.0778
62	507	189	433	312

To multiply a decimal fraction by a whole number, multiply as with integers, and then point off in the product, as many decimal places as there are in the multiplicand.

490. To multiply any number by 10, 100, 1000, etc.:

1. Find the product of .009 multiplied by 10.

$.009 = \frac{9}{1000}$
$10 = 10$
$.090 = \frac{90}{1000} = \frac{9}{100}$

Explanation.—1. Multiplying 9 thousandths by ten gives 90 thousandths, written .090.

2. .090 reduced by removing the cipher at its right equals .09, Ans.

Or, merely move the decimal point one place to the right.

2. Find the product of .009 multiplied by 100.

$.009 = \frac{9}{1000}$
$100 = 100$
$.900 = \frac{900}{1000} = \frac{9}{10}$

Explanation.—1. 9 thousandths multiplied by 100 is 900 thousandths, or .900.

2. .900, reduced by removing the ciphers at the right, is 9 tenths, or .9, Ans.

Or, merely move the decimal point two places to the right.

491. To multiply by 10, 100, etc., move the decimal point as many places to the right as there are ciphers in the multiplier, annexing ciphers when necessary.

492. Find the products in the following:

1. **10** ×	2. **10** ×	3. **100** ×	4. **100** ×	5. **1000** ×
$.0099	.001	$.8015	10.01001	.05
$.099	.1001	$20.091	9.009109	5.0015
$.99	55.5	$477.0099	.7	181.78
$.9	183.04	$.00818	505.05	65.006
$9.9	67.7	$19.8918	.0008	31.0804

NOTE.—To multiply by 5, 50, 500, etc., move the decimal point one, two, three, etc. places to the right and divide by 2; to multiply by 25, 250, etc., move the decimal point two, three, etc. places to the right and divide by 4; to multiply by 125, move the decimal point three places to the right and divide by 8.

493. Multiply the numbers in the preceding article by 5, 25, 50, 125, 250.

Thus: $5 \times .0099 = 2).099$, or, .0495.

494. To multiply when the multiplier is a decimal.

1. Find the product of $.48 multiplied by .9.

Explanation.—1. Multiplying as with whole numbers, the product is 432. This product is the product of the two numerators.

$.48 = \frac{48}{100}$
$.9 = \frac{9}{10}$
$.432 = \frac{432}{1000}$

2. As the denominators of the multiplicand and multiplier are hundredths and tenths, the denominator of the answer is thousandths. As 2 decimal places indicate hundredths and 1 decimal place, tenths, 3 decimal places (the sum of 2 and 1) represent thousandths, the denominator of the answer.

2.	3.	4.	5.	6.
$1.01708	$7.031	$253.09	$510.006	$1000.40
.13	.27	.0011	.005	1.3

7.	8.	9.	10.	11.
19.7	13.008	670.2	253.04	.4789
.83	8.45	40.7	.832	.152

495. Rule for Multiplication of Decimals.—Multiply as in whole numbers and point off in the product as many decimal places as there are in the multiplicand and the multiplier together.

496. Find the products in the following:

1. **7.1** ×	*2.* **.59** ×	*3.* **.037** ×	*4.* **3.04** ×	*5.* **2.0707** ×
137.2	17.4	2.065	8.702	65.005
63.05	11.082	.017	18.9	11.1001
11.019	90.37	1.01	4.02	2.0003
515.7	14.057	.103	.019	17.504

497. Oral Problems. 1. If a train of cars travels 23 miles each hour, how far does it travel in 100 hours?

2. At $4.50 per ton, what will 1,000 tons of coal cost?

3. Mr. Thompson owned .4 of a ship, and sold .3 of his What part of the ship did he still own?

4. At $5.25 per pair, how much will 40 pairs of shoes cost ?

5. If a laborer earns $3.75 by working 10 hours, how much will he earn in 100 hours ?

6. What will 40 yards of broadcloth cost at $2.20 per yard ?

498. Written Problems. 1. What will be the entire cost of 45 cargoes of coal, containing an average of 375.4 tons at $3.37½ per ton ?

2. If each pound of cotton is worth $.11¾, what is the cost of 47⅜ bales having an average weight of 133.47½ pounds ?

3. If each of 12 equal fields yields 85.30½ bushels of wheat, how much is the entire growth worth at $1.12½ per bushel ?

4. A house is worth $33,500. What is the price of an adjoining lot worth .375 as much as the house ?

5. What will it cost to inclose a circular field with a fence at $.37½ per foot, if the diameter of the field is 469.25½ feet in length, the length of the circumference being 3.1416 times that of the diameter ?

DIVISION.

499. To divide when the divisor is a whole number :

1. Find the quotient of .5 divided by 8.

$$\begin{array}{r} {\scriptstyle 2\ 4} \\ 8\overline{).5000} \\ .0625 \end{array}$$

Explanation.—1. Write the numbers and divide as in whole numbers, annexing at the right of the dividend as many ciphers as are required to give a quotient without a remainder.

2. As 5 tenths does not contain the divisor, write a cipher under it in the quotient.

3. 5 tenths equals 50 hundredths, which divided by 8 gives 6 hundredths with 2 remainder. Write 6 under the hundredths.

4. The 2 hundredths remainder equal 20 thousandths, which divided by 8 gives 2 thousandths with 4 thousandths remainder. Write 2 under the thousandths.

5. The 4 thousandths remainder equal 40 ten-thousandths which divided by 8 gives 5 ten-thousandths, or, .0625.

2. Find the quotient of 25.308 divided by 12.

Explanation.—1. Dividing as in whole numbers, the quotient is 2109.

$$12\overline{)25.308}\quad 2.109$$

2. Point off as many places in the quotient as there are in the dividend, for the denomination of the quotient is the same as the denomination of the dividend. As the dividend is thousandths, the quotient is thousandths. Ans., 2.109.

3. Find the quotient of 2.142 divided by 126.

Explanation.—1. Write the numbers as in long division, drawing a straight line above the dividend over which to write the quotient.

2. As the divisor is not contained in 21 tenths, write a cipher over tenths in the quotient.

$$\begin{array}{r} .017 \\ 126\overline{)2.142} \\ 1\,26 \\ \hline 882 \\ 882 \\ \hline \end{array}$$

3. 214 (hundredths) contains the divisor 1 (hundredth) time with a remainder of 88 (hundredths.) Write 1 hundredth in the quotient.

4. The 88 hundredths equal 880 thousandths, which, with the 2 thousandths, make 882 thousandths. 882 thousandths contain the divisor 7 thousandths times. Write 7 in the quotient over the thousandths. Answer, 1 hundredth and 7 thousandths, or 17 thousandths.

4.	5.	6.	7.	8.
7).010101	8)31.048	13)17.9139	17)9.0304	9)11.007
9.	**10.**	**11.**	**12.**	**13.**
43).0192	38)121.619	72)439.22	18)1.8094	86)2.41

500. To divide when the divisor is a whole number, divide as with whole numbers and point off as many places in the quotient as there are in the dividend.

NOTE.—Ciphers may be annexed after the decimal fraction in the dividend, so that the dividend will contain the divisor, or so that it may give a quotient sufficiently extended.

501. Find the quotients in the following:

1. **25** in	2. **32** in	3. **56** in	4. **135** in	5. **306** in
18.03	3.0085	.8934	70.15	1.284
1.077	.035	24.56	8.265	7.3
524.3	150.15	37.239	37.195	23.216
7.12	45.75	1.432	2.0685	14.28
13.03	1.9	19.008	80.5	1.175

Extend the answers to four decimal places.

502. To divide when the divisor is 10, 100, 1000, etc.

1. Find the quotient of .009 divided by 10.

$$10\overline{).009}\quad .0009$$

Explanation.—Dividing by 10 gives .0009 for the quotient, in which the decimal point is one place further to the left.

2. Find the quotient of .009 divided by 100.

$$100\overline{).009}\quad .00009$$

Explanation.—Dividing by 100 gives .00009 for the quotient, in which the decimal point is two places further to the left.

503. To divide by 10, 100, 1000, etc., move the decimal point as many places to the left as there are ciphers in the divisor.

NOTE.—To divide by 5, 50, 500, etc., move the decimal point one, two, three, etc., places to the left, and multiply by 2; to divide by 25, 250, etc., move the decimal point two, three, etc., places to the left and multiply by 4; to divide by 125 move the decimal point three places to the left and multiply by 8.

504. Find the quotients in the following:

1. **10** in	2. **10** in	3. **100** in	4. **100** in	5. **1000** in
$3.00	$7.00	$45.00	.02	20.
.45	7.05	3.50	4.04	2.
1.25	.75	17.85	5.6	.2
.04	8.08	9.06	.07	2.2
2.70	.90	18.90	.008	.05

505. Divide the numbers in the preceding article by 5, 25, 50, 125, 250.

506. To divide when the divisor is a decimal.

1. Find the quotient of .04131 divided by .09.

Explanation.—1. Move the decimal point two places to the right, and then divide as if the divisor were a whole number. Removing the decimal point in the divisor and moving that in the dividend has the effect of multiplying both divisor and dividend by the same number, and does not change the result.

$$.09\overline{).04|131}\quad .459$$

2. 9 in 41 (tenths), **4** and 5 over; in 53 (hundredths), **5** and 8 over; in 81 (thousandths), **9**. Answer, .459.

2. Find the quotient of 2.3821 divided by .041.

Explanation.—1. Move the decimal point in the divisor and in the dividend three places to the right. This is the same as multiplying them both by 1000, and does not affect the result. Divide as if the divisor were a whole number.

$$\begin{array}{r} 58.1 \\ .041\overline{)2.382|1} \\ 2\,05 \\ \hline 332 \\ 328 \\ \hline 41 \end{array}$$

2. 41 in 238 (tens), **5** and 33 over; in 332 (units), **8** and 4 over; in 41 (tenths), **1**. Answer, 58.1.

3. Find the quotient of .0651 divided by .021.

4.	**5.**	**6.**	**7.**
.0051)2.000004	.00407(30.03	.088).70604	.927).004
8.	**9.**	**10.**	**11.**
3.1).000604	50.5).001	.101)80.006	4.17)400.

Extend the quotient not further than six decimal places.

507. Rule for Division of Decimals.—Remove the decimal point of the dividend as many places to the right as here are decimal places in the divisor (annexing ciphers to if necessary) and then divide as if the divisor r.

508. Find the quotients:

1. **2.008** in	2. **.083** in	3. **.0007** in	4. **5.7** in	5. **900.2** in
.4329	15.06	1.09	860.	.00705
.0598	7.009	.007	41.093	.0008
.864	20.632	4.08	7.7	.6
.3021	8.041	.006	18.08	.403

Extend the quotients not further than four decimal places.

509. Divide the numbers in the columns of the preceding article by respectively 250, 300, 4,500, 160, and 27,000.

NOTE.—Ciphers at the right of a divisor may be removed, if the decimal point of the dividend is moved a corresponding number of places to the left.

510. Oral Problems. 1. What is the cost of one melon, if 25 cost $6.25 ?

2. If 100 quires of paper cost $6.00, how much does each quire cost ?

3. What number multiplied by 1000 gives $8,450 ?

4. At $11.25 per hundred-weight, what is the cost of one pound of beef ?

5. What number multiplied by .7 equals .035 ?

6. If a merchant gains $3,500 per year, how long will it take him to gain $35,000 ?

511. Written Problems. 1. If the dividend is 22848.75 and the quotient .8124, what is the divisor ?

2. If .035 of a farm is worth $189.91, what is the entire farm worth ?

3. A city lot is worth $2,300. This is .147 of the value of the building on it. How much is the building worth ?

4. How many tons of coal at $4.37 per ton, can be bought for $8,285.52 ?

5. If the tax raised by a city with a population of [illegible] *is $3,296,592.64, how much* per capita is the tax ?

MISCELLANEOUS PROBLEMS.

512. Oral Problems. 1. A merchant bought cloth at $1\frac{1}{8}$ a yard and sold it at $1.50. How much did he gain on 4 yards?

2. At $.04 each, how many oranges can be bought for $3?

3. What is the cost of 56 pounds of currants at $.12\frac{1}{2}$ a pound?

4. What will 36 yards of gingham cost, if 3 yards cost $.50?

5. How many pounds of coffee at $.25 a pound can be bought for 4 pounds of tea at $.37\frac{1}{2}$ a pound?

6. A man bought 4 pairs of socks at $.87\frac{1}{2}$ a pair. How much change should he receive out of $5?

7. The product of two factors is .63, and one of the factors is 7. What is the other?

8. If one pound of crackers can be bought for $.16\frac{2}{3}$, how many pounds can be bought for $2.50?

9. The product of 3 factors is .045. One of them is .5, and another is .03. What is the third factor?

10. If a man earns $9 in 6 days, how much will he earn in 11 days?

11. At $.75 a bushel, how many bushels of apples can be bought for $5.25?

12. What is the cost of $.12\frac{1}{2}$ of a ton of coal at $6 a ton?

13. James bought a knife for $.45 and sold it at a gain of $.33\frac{1}{3}$ of the cost. For how much did he sell the knife?

14. John has $1.25, which is $.62\frac{1}{2}$ as much as Mary has. How much money has Mary?

15. Reduce .025 to a common fraction in its lowest terms.

16. Henry has $16 and William has $20. What decimal part of William's money is Henry's?

17. A stationer buys 2 gross of lead pencils for $2.88 and *sells them at* $.03 each. How much does he gain on

18. A man bought 90 books for $1.80 and sold them all for $5.40. How much did he gain on each book ?

19. Three boys earn $75 in 6 weeks. How much does each boy earn in one week ?

20. What number is that from which if .16⅔ of itself be subtracted, the result will be 9 more than 26 ?

21. If a man can carve a score of picture frames in a month, in what decimal part of a year can he carve 200 frames ?

22. 5 dozen eggs sell for $.75. For how much must they be sold apiece to gain $.45 on the whole ?

23. At $.60 a day, how many days will it take a boy to earn as much as his father earns in 15 days at $2⅔ per day ?

24. If a man buys pencils at $.24 a dozen, at what price must he sell them to gain $1.50 on 100 pencils ?

25. If Mr. Stone buys cloth at $1⅝ and sells it for $1.87½ per yard, how many yards must he sell to gain $28.

513. Written Problems. 1. A man engaged 45 men for 15 days at $1.66⅔ a day. If he had but $1,000, how much more did he require to pay them ?

2. What shall be the amount of a check to pay for 56 yards of carpet at $2.75 a yard, deducting $52.50 which the dealer owes the purchaser ?

3. A farmer exchanges 152 bushels of wheat at $1.12½ a bushel, and 25 pounds of butter at $.37½ a pound, for cloth at $1.87½ a yard. How many yards of cloth should he receive ?

4. Mr. Merrill bought peaches at $3.25 a basket and sold them so that he lost $90 on 150 baskets. What was the selling price per basket ?

5. Mr. Dawson bought a farm of 75.66⅔ acres for $10,669. How much does he lose on each acre by selling it for $122.16⅔ ?

6. If a man earns $45.50 a month and spends $32.75 a month, how many years will it take him to save $1,530 ?

7. A man bequeathed his property, worth $5,000, to his wife and three sons. To the oldest son he left $842.75 ; to the second, 2.06 times as much ; and to the third, .8 as much as to the second. The remainder was left to his wife. How much did the wife receive ?

8. If .1875 of a gallon of wine costs $1.218¾, how much will 5.12 gallons cost ?

9. If 162 is .72 of a certain number, what is 1.08⅓ times the number ?

10. Bought land at $55.875 an acre and sold it at $96.375, making $5,143.50. How many acres were bought ?

11. .00875 of a man's money is $11.375. He buys 116 acres of land at $67.62½ an acre. How much money must he borrow to pay for the land ?

12. Mr. Holmes borrows a sum of money, the interest, or sum to be paid for its use, being $51.275 or .0166⅔ of the sum he receives. How much does he borrow ?

13. An agent's commission for selling a house for $6,050 is $204.1875. What decimal part of the selling price does the owner receive ?

14. If an agent receives .05 of the amount he expends, how much will he receive for buying 2,360 pounds of cotton at $.1975 per pound ?

15. A man owns .95 of a factory worth $75,620, and sells .675 of his share. What is the value of the part he still owns ?

16. When coal sells for $.003 per pound, what is the cost of .6875¼ of a ton ?

17. A wagon loaded with hay weighs 3,200 pounds, the wagon weighing 625 pounds. What is the hay worth at *$8.75 per ton ?*

18. How many barrels of flour at $9.37½ a barrel can be exchanged for 150 tubs of butter, each containing 56 pounds, at $.37½ a pound?

19. A drover selling cattle that cost $75,035 for $80,235, made $8.66⅔ per head. How many cattle were there?

20. What is the cost of 47.256 M. shingles at $4.47½ per M.?

21. Bought 28,500 pounds of hay at $12½ a ton, and sold it at $.87½ per hundred-weight. What was the gain?

28,500 lbs. = 285 cwt. = 14.25 T.

22. What is the cost of 10.528 M. feet of boards at $13.25 per thousand feet?

23. When the quotient is 36, the divisor .3375, and the remainder .1967, what is the dividend?

24. Bought 1,000 sheep at $4.87½ a head, and sold them at $5.37½. What was the total gain?

25. How many cords of wood at $5¾ a cord can Mr. Baker buy out of $750.25 and have $238½ remaining?

26. How many pounds of coal has a man, if it can be put into his cellar by using boxes containing respectively 19, 20, or 22 pounds an exact number of times?

27. If three trains start at the same time and place, and make round trips to the same port in respectively 36, 48, and 60 hours, how long will it be before they all start again from the same place?

514. To find the cost, when the number and the price per hundred, per thousand, or per ton are given.

1. What cost 5,450 feet of timber at $4⅜ per hundred?

Divide 5,450 by 100, to find the number of hundred feet.

2. If 1,000 bricks cost $23, what will 35,460 bricks cost?

Divide 35,460 by 1,000, to find the number of thousand bricks.

3. At $3.75 per ton, what will 563,800 pounds of coal cost?

Divide 563,800 by 2,000, to find the number of tons.

515. To find the cost, when the number and the price per hundred, per thousand, or per ton are given, divide the number by one hundred, one thousand, or two thousand, and multiply the quotient by the price.

516. Find the cost, prices and quantities being given as follows:

Per hundred.

ft. $	lb. $	ft. $
1. 7,128 @ 2.25	*5.* 5,015 @ 21.75	*9.* 6,846 @ 19.55
2. 3,430 @ 1.60	*6.* 4,163 @ 11.25	*10.* 9,015 @ 21.40
3. 1,850 @ 1.75	*7.* 7,015 @ 11.40	*11.* 5,128 @ 14.70
4. 9,010 @ 2.35	*8.* 8,620 @ 9.50	*12.* 7,084 @ 23.00

Per thousand.

ft. $	lb. $	lb. $
13. 3,960 @ $16\frac{2}{3}$	*17.* 3,105 @ 27	*21.* 7,130 @ 31.25
14. 2,010 @ $13\frac{1}{2}$	*18.* 5,045 @ 35	*22.* 830 @ 19.63
15. 1,125 @ $9\frac{1}{2}$	*19.* 3,627 @ 21	*23.* 3,005 @ 28.50
16. 963 @ $8\frac{1}{3}$	*20.* 855 @ 19	*24.* 120 @ 16.00

Per ton.

lb. $	lb. $	lb. $
25. 8,000 @ 7	*29.* 1,080 @ $6\frac{1}{4}$	*33.* 34,125 @ 3.50
26. 6,300 @ 5	*30.* 875 @ $9\frac{1}{2}$	*34.* 23,080 @ 5.75
27. 4,120 @ 8	*31.* 1,235 @ $3\frac{3}{4}$	*35.* 250 @ 8.25
28. 782 @ 5	*32.* 25,020 @ $4\frac{1}{4}$	*36.* 13,175 @ 4.50

The answer may be expressed with the nearest number of cents.

517. Oral Problems. 1. A man bought 150 pounds of sugar at $8.50 per cwt. What did the sugar cost him?

2. What is the cost of 12,250 feet of timber at $12 per thousand?

3. If 100 dozen pairs of gloves can be bought for $300, what will be the cost of 6 pairs?

Henry bought $4\frac{1}{2}$ pounds of pepper at the rate of 10 pounds. What did he pay for the pepper?

5. How many pounds of butter can be bought for $700, if 100 pounds cost $20 ?

6. What will 2,500 pounds of coal cost at $4 a ton ?

518. Written Problems. 1. If the roof of a building is 75 feet long and 20 feet wide, and it requires 80 shingles to a square yard, what will the necessary shingles cost at $8.25 per thousand ?

2. What will a pile of wood 125 feet long, 8 feet high, and 4 feet wide cost, at $4.50 a cord ?

3. A bricklayer charges for paving, $4.50 per hundred square feet. What will be his charge for a pavement 60 feet long and 11 ft. 6 in. wide ?

4. What will 4,500 pounds of nails cost at the rate of $5.25 per keg, each keg containing 100 pounds of nails ?

5. Mr. Carson bought 6,272 pounds of flour, paying $6.25 a barrel (196 lbs.). How much did he pay for the flour ?

6. What will 3,540 ounces of camphor cost, at $.22 per pound ?

519. To find a number, a decimal part being given.

1. 12 is .3 of what number ?

$\frac{1}{10}$ is $\frac{1}{3}$ of 12 ; $\frac{10}{10}$ is 10 × $\frac{1}{3}$, or $\frac{10}{3}$ of 12. Divide 12 by 3 and multiply by 10; or, divide 12 by .3. Ans., 40.

2. 18 is .06 of what number ?

$\frac{1}{100}$ is $\frac{1}{6}$ of 18 ; $\frac{100}{100}$ is 100 × $\frac{1}{6}$, or $\frac{100}{6}$ of 18. Divide 18 by 6 and multiply by 100 ; or, divide 18 by .06. Ans., 300.

3. A grocer makes $.20 or .4 of the cost on each pound of tea that he sells. How much did he pay per pound ?

4. Frank lost $.36 or .12 of his money. How much money had he at first ?

5. A man spends $14 or .35 of his week's wages, how much does he *earn per week ?*

520. To find a number, a decimal part of it being given, divide the given number by the fractional part, expressed as a decimal.

521. To find what part one number is of another:

1. 6 is what decimal part of 8?

1 is $\frac{1}{8}$ of 8; 6 is $\frac{6}{8}$ or .75 of 8.

2. .04 is what part of .5?

.5).040
.08 Ans.

3. William had 32 marbles and lost 4; what part of his marbles did he lose?

4. James spent $.24 for a Speller and $.40 for a Reader; what part of the price of the Reader was that of the Speller?

522. To find what decimal part one number is of another, divide the part by the whole number, and express the result as a decimal fraction.

523. Find the number of which:

1. 15 is .05	5. 5.4 is .125	9. 89.5 is $\frac{1}{4}$
2. 24 " .12	6. 198 " .495	10. 16.25 " .375
3. 48 " .16	7. 184.5 " .1236	11. $4.66\frac{2}{3}$ " $\frac{7}{9}$
4. 35 " .75	8. $\frac{1}{688}$ " .012	12. 16.8 " $.6\frac{1}{4}$

524. Find what decimal part:

1. .2 is of .05	5. 5.7 is of .513	9. .89 is of 107.8
2. 40 " .8	6. 34.1 " 579.7	10. $16\frac{2}{3}$ " $1.87\frac{1}{2}$
3. .6 " .009	7. $.6\frac{1}{4}$ " 825	11. 2.1 " 16.8
4. 8.75 " .175	8. 1150 " 57.5	12. $11\frac{1}{2}$ " .115

MISCELLANEOUS PROBLEMS.

525. Oral Problems. 1. What is the cost of 75 slates, if each slate costs $.14?

2. How many yards of ribbon will it require to make 4 *sashes, if it requires* 3.75 yards for each sash?

3. If one pound of butter is worth $\$.37\frac{1}{2}$, how many pounds can be bought for $1.50 ?

4. What is the price of one pound of sugar, if 100 pounds can be bought for $6.25 ?

5. Edward spends $.65 a day ; how many days will it take him to spend $65 ?

6. What number divided by 9 will give .005 ?

7. What is the cost of 1,000 tons of coal at $5.25 a ton ?

8. Jane has $.12 and Anna has .5 as many cents as Jane. How many cents has Anna ?

9. A laborer builds .3 of a wall in one day. What part of the wall will he build in .2 of a day ?

10. What number multiplied by 7 will give 1.4 ? 21 ? .035 ?

11. Frank has $2.70, and William has $.09. How many times William's money is Frank's ?

12. If a bushel of wheat is worth $\$1.87\frac{1}{2}$, how much is $.33\frac{1}{3}$ of a bushel worth ?

13. If a farmer can plough .625 of a field in one day, how many fields of the same size can he plough in 16 days ?

14. If sugar is bought for 8 ¢ and sold for 9 ¢ per pound, what decimal part of the cost is the profit ?

15. Henry's money, $8.08, was .8 as much as his brother's money. How much money did his brother have ?

16. If wheat is worth $\$1\frac{1}{8}$ per bushel, how many bushels can be bought for $1,125 ?

17. A gross of buttons being worth $.75, how many of the buttons can be bought for $500 ?

18. At $2.50 per gross, how much will 14,400 eggs cost ?

19. If a butcher buys meat for $\$_{12}^{1}$ and sells it for $\$.12\frac{1}{2}$ per pound, how much does he gain by selling 24 hundred weight ?

20. A coal dealer gains $.50 on each ton of coal. If he *sells 200,000 pounds of coal,* how much money does he g

21. Fifteen is five thousandths of what number ?

22. Twenty-five thousandths is what decimal part of five hundredths ?

526. Written Problems. 1. What will 4.875 tons of coal cost, at $6.50 per ton ?

2. How much does a carpenter earn in 100 days, if he earns $101.25 in 45 days ?

3. The circumference of a circle is nearly 3.1416 times the diameter. How many yards does a wheel 8 feet in diameter pass over in making 75 rotations ?

4. A farmer exchanges 4 tons of hay at $11.25 per ton for flour, at $7.50 per barrel. How many barrels of flour does he receive ?

5. What will $\frac{3}{4}$ yd. wide carpet for a room 12.5 by 15.25 feet cost, at $.95 per yard ?

6. If coal at the mine costs 3.37\frac{1}{2}$ and the freight is $.87$\frac{1}{2}$, what must 10,080 pounds of coal be sold for by the long ton, to gain 24.62\frac{1}{2}$?

7. A grocer sold 15 dozen eggs at $.23 per dozen, 14 pounds of sugar at $.11$\frac{1}{2}$, 3 dozen oranges $.37$\frac{1}{2}$ per dozen. What was the amount of the sales ?

8. How many yards of fence will it require for a lot, 41.5 feet wide by 87.25 feet deep ?

9. If a barrel contains 2$\frac{1}{2}$ bushels of apples, how many barrels can be bought for $93.75, apples selling at $1.50 per bushel ?

10. At 10.87\frac{1}{2}$ per M., what will 14,746 feet of timber cost ?

11. Find the entire cost of 115$\frac{1}{2}$ bushels of wheat at 1.37\frac{1}{2}$ a bushel, and 112$\frac{1}{4}$ bushels of oats at $.87$\frac{1}{2}$ a bushel.

12. Mary bought a piece of silk for $63.50 ; she found, on *measuring* the silk, that she had only $61.25 in value. How a yard were given her ?

13. Bought 1,125 pounds of pork at $7.50 per hundred, and sold it at $8.75 per hundred. What was the gain?

14. If 75 loaves of bread made from a barrel of flour costing $6.75 were sold for $7.87½, what was the gain per loaf?

15. A man lost .45 of his property; and then he lost .22 of what remained. What part of his property was left?

16. If .66⅔ of the value of a certain property is $16,666⅔, and .75 of this part is sold, what is the value of the entire property and of that which is unsold?

17. A sold 2 lots, at $1,250 each, gaining .15 on the one and losing .15 on the other. Did he gain or lose, and how much?

18. What is the value of a pile of wood 19.5 feet long, 8.4 feet wide and 6.3 feet high, at $4.25 per cord?

19. If a bin is 10 yards long, 12.5 feet wide, and 3 yards deep, how many cu. ft. does it contain?

20. How many yards of yard-wide carpet are required to cover a floor 15 ft. long and 4 yd. wide?

21. A man having 140 A. of land, sold 50 A. 2 R. What was the remainder worth at $75 per acre?

22. What is the value of 15 barrels of flour, if each pound is worth 3½ cts.

23. What is the least quantity of tea that will fill an exact number of boxes containing either 40, 56, or 60 pounds?

24. What is the least number of words which each one of three stenographers can write in an exact number of minutes, if their speed is respectively 110, 150, and 180 words per minute?

25. What is the G. C. D. of $\frac{15}{16}$, $\frac{3}{4}$, and $\frac{1}{2}$?

26. What is the L. C. M. of $\frac{15}{16}$, $\frac{3}{4}$, and $\frac{1}{2}$?

NOTE.—1. To find the G. C. D. of fractions, find the G. C. D. of the numerators and the L. C. M. of the denominators.

NOTE.—2. To find the L. C. M. of fractions, find the L. C. M. of the numerators, and the G. C. D. of the denominators.

DENOMINATE DECIMALS.

527. A **denominate decimal fraction** is a decimal fraction, whose integral unit is a denominate number.

.4 rd., .25 sq. ft., .003 pwt. are *denominate decimal fractions.*

REDUCTION.

528. To reduce a denominate decimal fraction to lower denominations.

1. Reduce .027 of a mile to lower denominations.

Explanation.—1. Multiply by 1,760 the number of units of the next lower denomination equal to one unit of the given denomination. This gives 47.52 yd.

.027 mi.	.52 yd.
1760	3
1232	1.56 . . ft.
352	12
47.520 . . yd.	6.72 . . in.

Ans., 47 yd. 1 ft. 6.72 in.

2. Reduce in similar manner the .52 yd., the decimal part of the product thus obtained, and thus proceed until the required or the lowest denomination is obtained.

3. The entire product of the last denomination, together with the integers of the preceding denomination, form the answer. Answer, 47 yd. 1 ft. 6.72 in.

529. To reduce a denominate decimal fraction to lower denominations, reduce, as with integers, the given decimal fraction, and each succeeding decimal fraction to the next lower denomination until the required or the lowest denomination is obtained. The integers of the different denominations together with the last decimal fraction form the answer.

NOTE.—A decimal denominate fraction of one denomination may be any required denomination by successive reductions of imal.

530. To reduce a denominate decimal fraction to a higher denomination.

1. Reduce .2 ℈ to the decimal of a pound.

3).2 ℈
8).0666⅔ . . . ʒ.
12).0083⅓ . . . ℥.
.0006$\frac{17}{18}$ . . lb.
Ans., .0006$\frac{17}{18}$ . . lb.
or, .000694̇ . . lb.

Explanation. — 1. Divide .2 ℈ by 3 to reduce to ʒ, obtaining .0666⅔ ʒ.
2. Divide .0666⅔ ʒ by 8 to reduce to ℥, obtaining .0083⅓.
3. Divide .0083⅓ ℥ by 12 to reduce to lb., obtaining .0006$\frac{17}{18}$.
4. The answer, .0006$\frac{17}{18}$ lb., a complex decimal fraction, may be reduced to .000694̇ lb., a circulating decimal.

2. Reduce .3 pwt. to the decimal of a pound.
3. Reduce .07 sq. rd. to the decimal of an acre.

531. To reduce a denominate decimal fraction to a higher denomination, divide by the number of units of like denomination which make a unit of the next higher denomination; and proceed, in similar manner, until the highest, or the required denomination is obtained.

NOTE.—To reduce a mixed decimal to higher denominations, proceed as above, and then reduce the decimal fraction, if any, to lower denominations.

532. Reduce the following:

to lower denominations.

1. .3 T.	5. .7 A.	9. £2.9.
2. 1.4 lb. Troy.	6. .2 Cd.	10. 1.5 lb.
3. .9 bu.	7. 2.8 sq. ch.	11. 4.7 sq. rd.
4. .42 gal.	8. 3.3 mi.	12. 5.4 hhd.

to higher denominations.

13. 133.2 gr.	17. 40,000.3 oz.	21. 6,040.3⅓ ft.
14. 256.55 pt.	18. 55,000.21 sq. ft.	22. 5,094.4½ sq. yd.
15. 7,512.5 pwt.	19. 99.09¼ dr.	23. 980.5¾ far.
16. 302.08 ℈.	20. 409.15 d.	24. 60,505.09 cu.

533. Reduce the following:

to grains.	to pints.	to inches.
1. .4 ℔.	4. .85 gal.	7. .62⅜ yd
2. .36 lb. (Troy).	5. .023 bu.	8. .07 rd.
3. .09 lb. (Avoir.)	6. .3 hhd.	9. .12 mi.

to acres.	to pounds.	to tons.
10. 6,540.25 sq. yd.	13. 560.12½ ℈	16. 65.37½cwt.
11. 200,500.14 sq. ch.	14. 537.6⅔d.	17. 4,560.12¾ lb.
12. 75,124.205½ sq. in.	15. 1,040.19 far.	18. 53,010.27 oz.

ADDITION AND SUBTRACTION.

534. Addition and subtraction of denominate decimals are performed in the same manner as addition and subtraction of denominate integers; or, the denominate decimals are reduced to the same denomination and then added or subtracted as simple numbers.

535. Find the sums:

1. What is the sum of .08 of a barrel, .75 of a gallon, and .1 of a quart?

Explanation.—1. Reduce the .08 bl. to lower denominations, obtaining 2 gal. 2 qt. and .16 pt.

2. Reduce the .75 gal. to lower denominations, obtaining 3 qt.

	gal.	qt.	pt.
.08 bl. =	2	2	.16
.75 gal. =		3	0
.1 qt. =			.2
	2	5	.36

3. Reduce the .1 qt. to lower denominations, obtaining .2 pt.

4. Add the quantities thus reduced. Answer, 2 gal. 5 qt. 36 pt.

2. What is the sum of .7 bu., .9 pk., and .5 qt.?

3.	4.	5.	6.	7.
.62½ mi.	.12 rd.	.4 A.	.02 bu.	.01 sq. ch.
	.04 yd.	.08 sq. rd.	.18 pk.	.09 P.
	.32 ft.	.15 sq. yd.	.35 qt.	.22 sq. yd.

536. Find the value of:

1. $\frac{1}{2}$ pk. + .4 qt. + 3.$\dot{3}$ pt.
2. 3$\frac{3}{4}$ yds. + 5.25 rd. + 1.6$\frac{2}{3}$ ft.
3. 3$\frac{3}{5}$ qt. + .37$\frac{1}{2}$ pk. + 6.$\dot{4}$ pt.
4. .12$\frac{1}{2}$ lb. + 5$\frac{3}{4}$ oz. + .06$\frac{1}{4}$ lb.
5. 2.15 lb. + 1$\frac{3}{4}$ gr. + 2.5 pwt.
6. .2 A. + 3.5 sq. rd. + 1.2$\frac{3}{5}$ sq. ft.
7. .5 sq. ch. + 1$\frac{3}{4}$ P. + .4 A.
8. .75 C. + 2.2$\frac{1}{2}$ cd. ft. + 3$\frac{3}{5}$ cu. ft.
9. 4.2s. + £.6 + 30.87$\frac{1}{2}$d.
10. $\frac{1}{3}$ T. + 100.75 lb. + 14.37$\frac{1}{2}$ oz.

537. Find the remainders:

1. What is the difference between .09 of a bushel and .72 of a pint?

.09 bu. = 5.76 pt.
.72 pt. = .72 pt.
2)5.04 pt.
Ans., 2 qt. 1.04 pt.

Explanation.—1. Reduce the .09 bushels to pints, the denomination of the subtrahend, obtaining 5.76 pt.
2. Subtract .72 pt. from 5.76 pt., obtaining 5.04 pt.
3. Reduce 5.04 pt. to higher denomination by dividing by 2. Answer, 2 qt. 1.04 pt.

2. What is the difference between .09 of a gallon and .72 of a pint?

3. What is the difference between .15 of a mile and 17.25 yards?

4.	5.	6.	7.	8.
.03 T.	.08 C.	.2 lb.	.09 A.	.3 mi.
.4 lb.	.24 cd. ft.	.7 dr.	.25 sq. yd.	.35 rd.

538. What is the value of:

1. $\frac{3}{4}$ bu. − .5 qt.?
2. .62$\frac{1}{2}$ qt. − 3.$\dot{2}$ gi.?
3. 1$\frac{2}{3}$ yd. − .8 ft.?
4. $\frac{3}{8}$ A. − .9 R.?
5. .4 lb. − 1.6 dr.?
6. .15 C. −.4 cd. ft.?
7. .87$\frac{1}{2}$ T. − 125.04 lb.?
8. $\frac{3}{8}$ A. − .25 sq. ch.?
9. 5.2$\frac{1}{4}$ gal. − 10.5 qt.?
10. .2$\dot{3}$ lb. − 8.25$\frac{1}{2}$ pwt.?
11. $\frac{3}{4}$ mi. − 113.66 rd.?
12. .2 sq. yd. − 3 sq. ft.

MISCELLANEOUS PROBLEMS.

539. Oral Problems. 1. A carpenter, having a plank measuring 5 ft. 5 in., cut from it a piece equal to .4 of the whole. How many feet of plank had he left in the piece?

2. A grocer had a box of tea containing 6 lb. 4 oz. After selling .2 of the tea, how many pounds had he left?

3. A goldsmith used 12 grains of gold in making chains and 1.5 times as much in making studs. What fraction of a pound was the gold that he used?

4. The length of a rectangular lot is 25 rods, and the width is .6 of the length. By what part of a mile does the length exceed the width?

5. A farmer has 2 acres of land sown in wheat, and .8 as much sown in oats. How many more square rods are sown in wheat than in oats?

6. A man travelled $\frac{3}{4}$ of a mile one hour, and $5.62\frac{1}{2}$ furlongs the next hour. How many more rods did he travel the first hour than the second?

7. A pail contains 4 gallons of water. After taking out of it .9 of its contents, how many pints does it contain?

540. Written Problems. 1. Mr. Stone engaged a man to make a walk around his house, the entire length of the walk being 16.5 rods. After $44.5\frac{1}{3}$ yards of the walk had been made, what decimal part of a mile remained to be done?

2. Mr. Fisher buys 8.3 bushels of peaches. He uses 9.4 pecks himself, and gives away 2 quarts. How many bushels has he left?

3. A grocer buys 3 barrels of liquor. He sells to one party 30 gal. 3 qt. 1 pt., and to another 60.5 gallons. How many quarts has he left?

4. buys at one time $3\frac{1}{2}$ tons of coal, and at another, rt. How many tons does he buy altogether?

5. A miner finds .3 of a pound of gold at one time, 13.4 pennyweight at another time, and 4.53 ounces at another. What fraction of a pound does he find altogether ?

6. A man divides his farm among his three sons. To the oldest, he gives 13.7 A.; to the second, 8 A. 2.6 R.; to the third, 4 A. 1.7 R. What was the size of his farm expressed in acres, roods, and rods ?

7. John cut .38 of a cord of wood, and Henry cut 5 C. 12 cu. ft. of wood. How much the more wood did Henry cut ?

MULTIPLICATION AND DIVISION.

541. Multiplication and division of denominate decimal fractions follow the rules of multiplication and division, and the rules of decimal fractions.

NOTE.—To divide one compound number by another, reduce both numbers to the same denomination and then divide.

542. Find the products:

1. Find the product of 5 bu. 3 pk. 2 qt. multiplied by 5.4.

bu.	pk.	qt.
5	3	2
		5.4
31	1(.2)	2.8
	(.2 =)	1.6
31	1	4.4 Ans.

4	1	2
27.	16.2	8)10.8
31	4)17.2	1 pk. + 2.8 qt.
	4 bu. + 1.2 pk.	

Explanation.—1. Multiplying 2 qt. by 5.4 gives 10.8 qt., which reduced gives 1 pk. with a remainder of 2.8 qt. Write the 2.8 qt. between the lines, and add the 1 pk. to the product of the pecks.

2. Multiplying 3 pk. by 5.4 gives 16.2 pk., making with the 1 peck 17.2 pk., which reduced equals 4 bu. with a remainder of 1.2 pk. Write the 1.2 pk. between the lines, inclosing the 2 in a parenthesis (.2), and add the 4 bu. to the product of the bushels.

3. Multiplying 5 bu. by 5.4 gives 27 bu., which with the 4 bu. make 31 bu. Write 31 bu. between the lines.

4. Reduce *the .2 pk. to qt.* and add. Answer, 31 bu. 1 pk. 4.4 qt.

2. 7 gal. 3 qt. 1 pt. 3 gi. = 6.8

3. 1 lb. 7 oz. 9 pwt. 20 gr. = .55

4. 3 pk. 5 qt. 1 pt. = 8.4

5. 12 cu. yd. 16 cu. ft. 18 cu. in. = 12.5

6. 2 mi. 35 rd. 11 ft. 15 in. = 6.5

7. 5 A. 17 sq. rd. $68\frac{1}{16}$ sq. ft. = .04

543. Find the value of:

1. (4 cwt. 6 qr. 5 lb. 10 oz.) × 3.2

2. (15 hr. 45 min. 30 sec.) × .8

3. (3 C. 17 cu. ft. 576 cu. in.) × .3

4. (24° 50′ 50″) × .76

5. (11 mo. 5 da. 6 hr.) × 1.5

6. (2 T. 5 cwt. 25 lb.) × .48

544. Find the quotients:

1. What is the quotient of 33 gal. 3 qt. 1 pt. divided by .4?

Explanation.—1. Dividing by .4 gives 82.5 gal. 7.5 qt. 2.5 pt.

2. Reducing the .5 of a gal. and the .5 of a qt. the fractions of the higher denominations, gives for the first 2 qt., and for the other 1 pt.

3. Add the integers thus obtained to the other integers in the result, finding the entire sum of the lowest denomination gives the answer. Answer, 84 gal. 2 qt. 1.5 pt.

	gal.	qt.	pt.
.4)33		3	1
	82(.5)	7(.5)	2(.5)
	(.5=)	2(.5=)	1
	84	2	1.5

NOTE.—A denominate number may be reduced to its lowest denomination before dividing and then reduced to higher denominations.

2. What is the quotient of .5 lb. divided by .2 pwt.?

.5 lb. = 120 pwt. .2)120.0

600 Ans.

545. Find the quotients:

1. 1412 C. 12 cu. ft.

2. 1.5)1 mi. 60 rd. 9 ft. 10 in.

3. 45 sq. rd. 84 sq. ft.

4. .314 hhd. 35 gal. 7 qt.

546. What is the value of:

1. 25 yr. ÷ 4 hr.?	4. .5 yd. ÷ .2 ft.?	7. 3.5 ℔. ÷ .5 ℥?
2. .6 cir. ÷ .75 ?	5. 40 lb. ÷ 5 oz.?	8. 24 C. ÷ 1.5 cd. ft.?
3. 8 reams ÷ .33⅓ qr.?	6. 15 lb. ÷ 8 pwt.?	9. 25 A. ÷ 2.5 sq. rd.?

MISCELLANEOUS PROBLEMS.

547. Oral Problems.—1. Ellen has 4s. 8d., and Jane has .87½ as much as Ellen. How much money has Jane?

2. John has a piece of gold; .6 of its weight is 240 grains. What part of an ounce does the whole piece weigh?

3. At $.02½ a quart, how many bushels of tomatoes can be bought for $1.60?

4. What will be the cost of paving a walk 20 yards long and .5 yard wide, at $.10 per square foot?

5. Mrs. Brown bought 2 qt. 1 pt. of milk. What decimal part of a gallon did she buy?

6. A man having 160 rods to travel, has travelled .25 of the distance. What fraction of a mile has he still to travel?

7. An apothecary bought .8 of a pound of quinine and sold ¼ of it. How many ounces had he left?

548. Written Problems. 1. What will it cost to fence a field 32.4 rods square at $.22 a yd.?

2. What is the value of 25 bushels of pears at $.18 a quart?

3. How many square feet of boards will it take to make a floor 16.6 feet by 8.5 yards?

4. Paid $20.64 for 5 T. 3 cwt. 20 lb. of coal. What was the price per ton?

5. A piece of timber contains 12.5 cu. yd. It is 7.5 ft. wide and 5 ft. thick. What is its length?

6. What will be the cost of 350 acres of land, if .6 of a rood costs $27?

7. A grocer bought 57 pounds of butter at $.23 a pound. He sold .66⅔ of it at $.35 a pound, and the remainder at $.28 a pound. How much did he gain ?

8. A man sold .87½ of his hay or 2 T. 3 cwt. for $3,010. At the same rate what is the value of all his hay ?

9. A farmer sold .66⅔ of his potatoes and had 9 bu. 2 pk. left. What was the value of all his potatoes at $.60 a bushel ?

10. A man owned a lot 25 feet wide and 80 feet long. He bought another lot .9 as wide and .8 as long. How many square yards did the second lot contain ?

549. To find a decimal fractional part of a denominate number, multiply the number by the decimal fraction.

550. Find the following decimal parts :

1. .3 of 4 oz. Troy.
2. .75 of 1 sq. mi.
3. .09 of 1 lb. 3 cwt. 2 oz. avoir.
4. .64 of 56 rd.
5. .33⅓ of 12 cords.
6. .16⅔ of $1.87½.

Reduce the answers of 1 and 2 to lower terms.

551. To find what part one denominate number is of another, expressed as a decimal fraction, reduce the numbers to the same denomination, divide the part by the given whole number, and then reduce the answer to a decimal fraction.

552. Find what decimal part :

1. 7 pwt. is of 1 lb.
2. 3 cwt. " " 1 T.
3. 1 quire 16 sheets is of 2 Rm.
4. 15 doz. is of 1 great gross.

553. To find a denominate number, a decimal fractional part of it being given, divide the given number by the decimal fraction.

554. Find the number of which :

1. 3 pk. 4 qt. is .87½.
2. 8 oz. 15 pwt. is .25.
3. *60 rd. 2 ft. is .16.*
4. 4 sq. mi. is .32.
5. 2,592 cu. ft. is .12.
6. 10 sq. ft. is .45.

MEASUREMENT.

555. Masonry is usually estimated by the cubic yard or by the cubic foot, $24\frac{3}{4}$ ($16\frac{1}{2} \times 1\frac{1}{2} \times 1$) cubic feet being called a *perch.*

In rough estimates 25 cubic feet make a perch. In walls 22 cu. ft. of stone make one perch, $2\frac{3}{4}$ cu. ft. being allowed for mortar.

556. Embankments and excavations are estimated by the cubic yard.

A cubic yard of earth makes a *load.*

557. Materials are estimated by cubic measure ; the work, by cubic measure, or by square measure.

In estimating *materials*, an allowance of $\frac{1}{10}$ is made for doors, windows, etc.; in estimating *work*, it is not usual to make allowance for doors, windows, or corners, and the outside (or girth) of the wall is measured, so that each corner is measured twice.

558. Brickwork is estimated by the cubic foot, and, for pavements, by square measure.

27 common bricks without mortar or $22\frac{1}{2}$ bricks with mortar are allowed for each cubic foot. A common brick contains $8 \times 4 \times 2$ cu. in. Bricks, however, vary in size, 20 bricks with mortar being frequently called a cubic foot.

559. Oral Problems. 1. What will 4,750 bricks cost at $20 per thousand ?

2. If 5,500 bricks cost $121 what is the rate per thousand ?

3. How many common bricks are required for a wall 18 feet long, 50 feet high and 2 feet thick ?

4. What will it cost to cement the floor of a cellar 15 feet by 12 feet at $.50 per square yard ?

5. How many common bricks are required to pave a yard 12 ft. 6 in. long by 10 ft. 8 in. wide ?

560. Written Problems. 1. A pavement is 4 feet wide and 75 feet long. How many bricks did it require?

2. What will it cost to dig an excavation 50 feet long, 20 feet wide, and 7 feet deep, at $.35 per load?

3. A cellar 65 feet long and 8 feet deep has 711⅓ cubic yards of earth removed from it. How wide is the cellar?

4. A stone wall 18 ft. 3 in. long, 5 ft. 2 in. high, and 2 ft. 4 in. wide, requires how many perches of stone?

5. How many common bricks are there in a cubical pile, each edge of the pile being 16 feet long?

BOARD MEASURE.

561. Board measure is used by carpenters and by dealers in lumber.

Lumber is timber sawed or split for use; as, beams, joists, boards, planks, etc.

562. A **board foot** is the unit in board measure; it is 1 foot long, 1 foot wide, and 1 inch thick.

12 board feet make 1 cubic foot.

563. Timber is generally bought and sold by cubical measure.

40 feet of round, or 50 feet of hewn timber make one ton.

564. To Reduce Board Feet to Cubic Feet.—Divide by 12.

565. To Reduce Cubic Feet to Board Feet.—Multiply by 12.

566. To find the Number of Board Feet.—Multiply together the length in feet, the breadth in feet, and the in inches.

t the product, and two of these factors are given, the third found by division.

567. Rules for Timber.—I. For round timber, find the square of one-fourth of the mean girth in feet and multiply it by the length in feet. The product is the number of cubic feet.

II. For hewn timber, find the area of the base in square feet or of the mean cross section, if the timber tapers, and multiply it by the length in feet. The product will be the number of cubic feet.

NOTE.—The area of the cross section of round or of hewn timber may be found in square inches and then multiplied by the length in feet. If the product is divided by 144 the result is cubic feet.

568. Oral Problems. 1. If there are 9 board feet in a piece of lumber, the length being 9 feet, and the thickness 3 inches, what is the width?

2. What is the cost of boarding 8 feet long, 5 feet wide, 2 inches thick, at $40 per thousand?

3. How many board feet are there in 5 planks, each 6 feet long, 3 inches wide, 2 inches thick?

4. If a plank is 12 ft. long, 1 in. thick, and has width tapering from 2 to 6 in., how many board feet does it contain?

569. Written Problems. 1. How many board feet are there in 5 boards, 20 feet long, 9 inches wide, and $\frac{3}{8}$ inches thick?

NOTE.—Thickness less than 1 inch is counted 1 inch.

2. How many board feet are there in a piece of hewn timber 18 feet long, 18 inches thick, with width tapering from 16 to 20 inches?

3. At $25 per thousand feet, how much is the cost of 24 planks, each $1\frac{1}{2}$ inches thick, $1\frac{1}{2}$ feet wide, and 15 feet 6 inches long?

4. A floor cost $6, planks 1 inch thick, costing $20 per thousand feet. If the floor is 5 yards wide, what is its length *in feet?*

TIME.

570. The measurement of time is essential in many business operations, as in calculating wages, interest, etc.

571. To find the difference in time between two dates.

1. What is the difference in time between June 9, 1886, and May 8, 1889?

Explanation.—1. Write the later date first and under it the other date, writing for the month the number representing its order in the calendar year.

	yr.	mo.	da.
	1889	5	8
	1886	6	9
Ans.,	2	10	29

2. Subtract as with any compound numbers. **Answer, 2 yr. 10 mo. 29 da.**

Or,

Explanation.—1. Find the number of complete years.

2. Find the number of complete months remaining.

3. Find the number of days that remain.

From	To	
June 9, 1886,	June 9, 1888	= 2 yr.
June 9, 1888,	April 9, 1889	= 10 mo.
April 9, 1889,	May 8, 1889	= 29 da.

Ans., 2 yr. 10 mo. 29 da.

Answer, 2 yr. 10 mo. 29 da.

572. Find the time between:

April 5, 1885 and	October 27, 1898 and	February 28, 1889 and
1. September 4, 1889.	4. August 31, 1891.	7. January 31, 1893.
2. December 1, 1890.	5. June 2, 1890.	8. February 29, 1884.
3. July 4, 1883.	6. November 1, 1885.	9. March 31, 1886.

EQUIVALENT UNITS.

573. In solving problems on bins, cisterns, tanks, or on minerals, grains, liquids, etc., it is frequently necessary to pass from units of one scale or table to those of another; as, from cubic inches to bushels; or from avoirdupois pounds to troy pounds.

* For equivalent metric units see the Appendix.

574. Copy, and commit to memory the following table of

Equivalent Units:

1. 231 cu. in. equal 1 gal.
2. 2,150.42 cu. in. equal 1 bu.
3. 1 cu. ft. water equals 62½ lb.
4. 1¼ cu. ft. equal 1 bu.
5. 5,760 gr. equal 1 lb. Troy, apoth.
6. 7,000 gr. equal 1 lb. avoir.
7. 60 geographical, or 69.16 (69⅛) statute miles, equal 1 degree.

* 1 bushel equals	1 barrel equals
8. 60 lb. wheat, clover seed.	*12.* 100 lb. nails.
9. 56 lb. corn, rye.	*13.* 196 lb. flour.
10. 48 lb. barley.	*14.* 200 lb. fish, beef, pork.
11. 32 lb. oats.	*15.* 280 lb. salt.

* Varies for different States. 100 lb. of dry fish equals 1 quintal.

NOTE.—35 to 40 cu. ft. equal 1 ton anthracite; 35 to 45 cu. ft. equal 1 ton bituminous coal.

575. Oral Problems. 1. What is the value of a bin of wheat at $1.12½ per bushel, if the bin is 5 feet long, 3 feet deep, 2 feet wide?

2. What is the weight of water in a trough 20 feet long, 2 feet wide, and 2 feet 6 inches deep?

3. What is the difference of weight in grains, between a pound of gold and a pound of lead?

576. Written Problems. 1. At 8 cents a quart, what is the value of the milk required to fill a tank 2 feet 6 inches deep, with a base 18 inches square?

2. What weight of water will fill a rectangular cistern 20 feet deep, and 4 feet by 6 feet at the surface?

3. How many gallons of water will fill a cellar 10 by 18 by 40 feet?

4. At $3.87½ per ton, what is the value of the coal filling a rectangular coal pocket 30 by 20 by 15 feet, counting 40 cubic feet to the ton?

5. A druggist buys copperas at 3 cents per pound (avoirdupois); how much does he gain on each apothecaries' ℔ retailed *at 15 cents?* (See Arts. 182, 183.)

MISCELLANEOUS PROBLEMS.

577. Oral Problems. 1. A merchant, in selling a book, gained 2 pence, which was .2 of the cost. How many pence did the book cost the merchant?

2. If John's money, $5.25, is .05 as much as Edward's money, how much money has Edward?

3. Thomas is 8 years 4 months old, and his father is 50 years old. Thomas' age is what decimal part of his father's age?

4. If a box contains 200 lb. of coal, how many boxes of the same size will be required to hold 2 T. 4 cwt.?

5. Bought 5 lb. 4 oz. of lard at 16 ¢ per lb.; paid for it with a dollar bill. What change is due?

6. If 3 pecks of nuts cost $.96, what do the nuts cost per bushel?

7. How many 3 ft. cubes can be cut from a block 9 feet long, 6 feet high and 3 feet wide?

8. Bought beans for 12 ¢ a half peck. How much will be lost per bushel, if 4 bushels are sold for $3.60?

9. If a metal is worth 8¼ ¢ per pound, what will 4 T. cost?

10. A man has 2 A. of land and sells 120 square rods. How many acres has he left?

11. A room is 8 yd. long and 15 ft. high. What decimal part of the length is the height?

12. What will it cost to carpet a passage-way .75 of a yard wide by 12 feet long with 27 inch-wide carpet, at $3 per yard?

13. Bought cloth at $5 per yard, and sold it so as to gain .05 of the cost. How many yards must be sold to gain $10?

14. If .625 of a pound of tea costs $.37½, what will a pound cost?

15. A piece of timber contains 108 cu. ft. Its width is 4 [illegible]ickness, 36 inches. What is the length in

16. What is the number of board feet in a plank 8 feet long, 6 inches wide and 3 inches thick ?

17. What is the width of each board, if 2 similar boards contain 96 board feet, the length of each being 6 feet and the thickness 2 inches ?

18. What is the cost of a yard of cloth, if .75 of a yard cost $2.70 ?

19. If a boy earns $.66⅔ per day, how much will he earn in 4 weeks of 6 working days each ?

20. What is the length of a rectangular field 40 rods wide, and containing 20 A.?

21. How many yards of matting 1 yard wide will cover a floor 12 feet long by 9 feet wide ?

22. What is the price of a plank 10 feet long, 6 feet wide, 1½ inches thick, if a thousand board feet cost $30 ?

23. At 6 ¢ per yard, how much will it cost to border with lace a table cover 5 feet long and 4 feet wide ?

24. What will 1,200 feet of boards cost at $25 per M ?

578. Written Problems.—1. If the driving wheel of a locomotive is 11 ft. 8 in. in circumference and makes 12,000 turns in travelling from one city to another, what is the distance in miles between the cities ?

2. A man having a farm of 200 acres, gives ⅖ to his son and .45 of the remainder to his daughter. He gives to a village a church lot 10 rods wide by 12 rods long. The rest he gave to his wife. How much does his wife receive ?

3. A room contains 4,200 cu. ft. The width is 5 yards and the height 14 feet. What is the length ?

4. How many steps, 2 feet 6 inches each, will be taken in walking around a farm 1 mile square ?

5. How many board feet are there in a piece of timber 12 feet long, 2½ inches thick, 14 inches wide at one end and *inches at the other ?*

6. The distance from New York to New Haven being 73 mi. 80 rd., at what rate does a train travel, to cover the distance in 2 hours 10 minutes?

7. A grocer has 209 bu. of pears. How much at 15 ¢ each must he pay for a sufficient number of barrels to contain the pears, each barrel holding 2 bu. 3 pk.?

8. What is the length of a pole standing .125 of its length in mud, 18 feet in water, and $\frac{1}{2}$ of its length in the air?

9. A man's garden, which is 8 rods long and 99 feet wide, is .08 of his land. How many acres of land has he?

10. If a man buys 16 gallons of wine for $104, and retails it at $.85 per pint, how much does he gain on 3 gallons?

11. What is the value of a piece of timber 25 feet long, 16 inches square, at $.12 per cubic foot?

12. How many bricks with mortar will it take to make a wall around a garden 6 rods long and 4 rods wide, the wall being 9 feet high and 3 feet 6 inches thick?

13. What is the cost of 15 planks, each 12 feet long, 14 inches wide, $3\frac{1}{2}$ inches thick at $45 per M. board feet?

579. Make receipted bills for the following statements:

1. Cincinnati, May 28, 1882. Edward Price bought of Jas. McFarland, 4 bbl. New Orleans sugar @ $6.25; 3 sacks coffee @ $31.50; 2 tubs butter, each containing 56 lb., @ $.19 per pound; 15 lb. cheese @ $.18; 80 lb. ham @ $.11 a pound.

2. A farmer sold 175 bushels of corn @ $.79 a bushel; 197 bushels wheat @ $1.19 a bushel; 50 bushels oats @ $.60 a bushel; 3 tons hay @ $14.75 a ton.

3. New York, July 5, 1885.—John Hopkins bought of William Warren & Co. 3 reams extra foolscap @ $2.15 per ream; 2,000 envelopes @ $1.60 per M.; 6 gross lead pencils @ [illegible] oss; 10 boxes steel pens @ $.45 per box, 500 slate [illegible] 75 per hundred.

4. Chicago, Dec. 20, 1875. Albert Longley bought of Robert Upton & Co. 56 yd. Hope Muslin @ \$.09½ a yard; 48 yards French Cambric @ \$.28 a yard; 2½ dozen Lisle Hose @ \$4.00 a dozen; 24 yards Linen Lawn @ \$.16⅔ a yard; 15 yards India Mull @ \$.63½ a yard.

5. Henry Farnham, carpenter, labored 42 days @ \$2.25 a day for Frank Elmore, and supplied the following materials: 45 pounds of nails at \$.05½ a pound; 1,500 bd. ft. lumber @ \$12.40 a thousand.

580. Copy, and complete the following:

1888.	Cash Account.	Dr.		Cr.	
July 1	Amount on hand,	\$500	00		
8	Received of H. Lambert to balance account,	11	60		
16	Expense—paid rent,			\$30	00
"	" paid freight on Mdse.,			4	50
"	Received of H. B. Thompson on acct.,	38	00		
18	Received of J. Rockwell on acct.,	26	00		
20	Deposited in First National Bank,			450	00
25	Paid Geo. Wilbur on account,			25	00
31	Expense—paid gas bills,			4	00

What is the amount on hand at the end of the month?

NOTE.—The account is debited (Dr.) to the extent of items received, and credited (Cr.) by items paid out.

PERCENTAGE.

581. 1. John had $1 and spent 1 ¢. What part of his money did he spend?

2. Mary bought a yard of elastic for 2 ¢, and paid for it with a dollar. What part of her dollar did she spend?

3. A man invested money and received a dividend of 3 ¢ on every dollar. What part of a dollar did each dollar earn?

NOTE.—The answers to the above should be expressed as common and as decimal fractions.

582. The unit of exchange in the United States is the dollar ($1.00), and earnings, profits, losses, interest, etc., are reckoned as so many cents on the dollar; that is, as so many cents on the hundred cents, or *by the hundred.*

583. Percentage is a process of reckoning *by the hundred.*

584. Percentage is, also, the result obtained by taking so many hundredths of a number.

18 is a *percentage* of 24, being .75 of 24.

585. ***Per cent.*** is an abbreviation of the Latin phrase *per centum*, meaning *by the hundred.*

586. The ***rate per cent.*** is the number of the hundredths taken.

12 is 5 *per cent.* of 240, as it is .05 of 240; 5 being the *rate per cent.*

587. The **sign of per cent., %,** is used with the rate per cent. expressed as a whole number; thus, .05 or 5 per cent. is written 5 %; .16⅔ or 16⅔ per cent. is written 16⅔ %.

588. The number of which a percentage is taken is called the

of 240 is to be found, 240 is called the *base.* The base *per cent.*.. as any number is 100 % of (1.00 ×) itself.

NOTATION AND NUMERATION.

589. Read and write the following, either of the four equivalent expressions being given:

$1 = 1.00 = 100$ *per cent.* $= 100\ \%$.

1. $\frac{1}{2}$	.50	50 %	50 *per cent.*	*5.* $\frac{3}{4}$	.75	75 %	75 *per cent.*
2. $\frac{1}{4}$	.25	25 %	25 *per cent.*	*6.* $\frac{2}{5}$	.40	40 %	40 *per cent.*
3. $\frac{1}{5}$	.20	20 %	20 *per cent.*	*7.* $\frac{3}{5}$	.60	60 %	60 *per cent.*
4. $\frac{1}{10}$	.10	10 %	10 *per cent.*	*8.* $\frac{4}{5}$	.80	80 %	80 *per cent.*

Express similarly, $\frac{2}{10}$, $\frac{3}{10}$, etc.

590. Copy and commit to memory the preceding and the following:

1. $\frac{1}{3}$	$.33\frac{1}{3}$	$33\frac{1}{3}\ \%$	$33\frac{1}{3}$ *per cent.*	*5.* $\frac{1}{8}$	$.12\frac{1}{2}$	$12\frac{1}{2}\ \%$	$12\frac{1}{2}$ *per cent.*
2. $\frac{2}{3}$	$.66\frac{2}{3}$	$66\frac{2}{3}\ \%$	$66\frac{2}{3}$ *per cent.*	*6.* $\frac{3}{8}$	$.37\frac{1}{2}$	$37\frac{1}{2}\ \%$	$37\frac{1}{2}$ *per cent.*
3. $\frac{1}{6}$	$.16\frac{2}{3}$	$16\frac{2}{3}\ \%$	$16\frac{2}{3}$ *per cent.*	*7.* $\frac{5}{8}$	$.62\frac{1}{2}$	$62\frac{1}{2}\ \%$	$62\frac{1}{2}$ *per cent.*
4. $\frac{5}{6}$	$.83\frac{1}{3}$	$83\frac{1}{3}\ \%$	$83\frac{1}{3}$ *per cent.*	*8.* $\frac{7}{8}$	$.87\frac{1}{2}$	$87\frac{1}{2}\ \%$	$87\frac{1}{2}$ *per cent.*

$\frac{1}{12} = 8\frac{1}{3}\ \%$; $\frac{1}{16} = 6\frac{1}{4}\ \%$; $\frac{1}{7} = 14\frac{2}{7}\ \%$.

NOTE.—The common fractional form in Arts. 589 and 590 are frequently preferred in the solution of problems in percentage and its application, even when other forms are given in the statement.

591. Read as per cent. the following:

1. .08	*4.* $.16\frac{2}{3}$	*7.* .25125	*10.* $.0066\frac{2}{3}$	*13.* $.0014\frac{2}{7}$	*16.* .0035
2. .097	*5.* .001	*8.* .42875	*11.* .0025	*14.* $.0087\frac{1}{2}$	*17.* .0645
3. $.12\frac{1}{2}$	*6.* .003	*9.* .15375	*12.* $.0006\frac{1}{4}$	*15.* $.0062\frac{1}{2}$	*18.* $.0033\frac{1}{3}$

NOTE.—Read the part of a decimal at the right of hundredths as a fractional part (common or decimal) of 1 per cent.

592. Write as decimals the following:

1. 25 %	*4.* $12\frac{1}{2}\ \%$	*7.* $\frac{1}{2}\ \%$	*10.* 7.5 %	*13.* .03 %
2. 12 %	*5.* $66\frac{2}{3}\ \%$	*8.* $\frac{1}{3}\ \%$	*11.* .6 %	*14.* $33\frac{1}{3}\ \%$
3. 125 %	*6.* $37\frac{1}{2}\ \%$	*9.* $\frac{5}{8}\ \%$	*12.* 9.25 %	15. 2.08 %.

593. Explain the difference between:

1. 8 % and .8 % ?	4. .125 and $\frac{1}{8}$ % ?	7. .00$37\frac{1}{2}$ and $37\frac{1}{2}$ % ?
2. .30 and .3 % ?	5. .$87\frac{1}{2}$ and .$87\frac{1}{2}$ % ?	8. $\frac{1}{4}$ % and 25 % ?
3. 75 % and .75 % ?	6. .25 and $\frac{1}{4}$ % ?	9. .80 and .8 % ?

594. Percentage being an application of decimal fractions to concrete numbers, and decimal fractions being a form of notation for common fractions, the problems in percentage and in each of its applications may be classified as the problems in common and in decimal fractions are classified.

595. Illustrate the PROBLEMS in common and in decimal fractions, by using the following forms:

$\frac{3}{4}$ or .75 45 60 Prob. III., thus, 45 ÷ $\frac{3}{4}$ = 60.

I. *To find a part of a number.*

II. *To find what part one number is of another.*

III. *To find a number, a part of it being given.*

IV. *To find a number, the number plus or minus a part of it being given.*

PROBLEM I.

596. To find a part of a number.

Percentage* = *Base* × *Rate.

NOTE.—The other problems in percentage are deduced from the preceding; thus, Prob. II. : *Rate* = *Percentage* ÷ *Base; etc.*

1. What is 6 % of $1,845 ?

Explanation.—1. 6 % is .06 of a number.
2. .06 of $1,845 is $110.70. Ans.

$1,845
.06
$110.70

597. Find the following:

5 % of	6 % of	7 % of	8⅓ % of
1. $12,045.	4. $1,030.	7. $12,543.	10. 9,654
2.	5. $23,415.	8. $6,502.	11. 7,467
3.	6. $26,500.	9. $1,947.	12. 1,509

12½ % of	**125 %** of	**106¼ %** of	**$\frac{1}{8}$ %** of
13. 1,608.	*17.* 6,632.	*21.* 25,604.	*25.* 2,568.
14. 2,568.	*18.* 7,052.	*22.* 33,716.	*26.* 1,056.
15. 3,505.	*19.* 13,566.	*23.* 21,053.	*27.* 17,189.
16. 4,218.	*20.* 11,275.	*24.* 18,071.	*28.* 6,055.
$\frac{8}{8}$ % of	**.87½ %** of	**.57 %** of	**1.3 %** of
29. 160,256.	*33.* 35,240.	*37.* 32,496.	*41.* 2,563.
30. 73,624.	*34.* 42,416.	*38.* 40,016.	*42.* 9,102.
31. 96,532.	*35.* 91,563.	*39.* 91,052.	*43.* 8,145.
32. 10,479.	*36.* 70,365.	*40.* 72,314.	*44.* 12,063.

598. Oral Problems. 1. A man lends $1,200 at 6 per cent. and $1,500 at 5 per cent. What was the difference in the amounts of interest due?

2. A man lost 25 per cent., and sold 50 per cent. of a flock of sheep numbering 240. How many sheep did he then have?

3. Mr. Thomson lends $500 at 6 per cent. and $300 at 7 per cent. How much interest does he receive in one year?

4. If a boy gathers 8 bushels of nuts and sells 75 per cent. of them at $1.50 a bushel, how much money does he receive?

5. A mechanic earns $2 a day. If he saves 62½ per cent. of his earnings, how much does he save in one week of six days?

6. A farmer had 150 bushels of wheat. He sold 20 per cent. of it to one man and 30 per cent. to another man. How many bushels did he sell?

7. A school has 500 pupils. If 40 per cent. are girls, how many girls are there in the school?

8. A bookseller buys 12 astronomies at $1.00, and in selling them he gains 9 per cent. How much money does he gain?

9. If 10 shares of railroad stock costing $80 a share are sold at a gain of 25 per cent., how much is the gain?

599. Written Problems. 1. Mr. Graham bought 25 yards of cloth at $1.75, and selling it gained 20 per cent. on each yard. *What did* he make by the sale?

2. What is 15 per cent. of 15 guineas 15 shillings?

3. A sold a farm valued at $7,500 at a loss of 18 per cent. How much money did he lose?

4. What is 18 per cent. of 1 pound sterling?

5. 83⅓ per cent. of an army of 45,000 men are infantry. How many men are there in the infantry?

6. If Mr. Barrett sells 16⅔ per cent. of 1,500 barrels of flour to A, and 50 per cent. to B, how many barrels in all does he sell?

7. Mr. Swan's income is $3,500 a year; and his expenses are 80 per cent. of his income. How much does he spend each year?

8. An orchard contains 1,600 trees. 12½ per cent. are cherry trees; 50 per cent. are apple trees. How many cherry and apple trees together are there?

9. Mr. Williams bought 1,500 pounds of wool. He used .50 in the manufacture of blankets, 37½ per cent. in flannel, and 12½ per cent. in yarn. How much wool in all did he use?

10. Mr. Frame received $11,500 salary. He spent 83⅓ per cent., and gave away 6 per cent. as much as he spent. How much did he give away and spend altogether?

600. In solving many problems, the *percentage* having been found, it is added to or subtracted from the base, giving the **amount** or the **difference.**

601. The **amount** is the *base* plus the *percentage.*

602. The **difference** is the *base* minus the *percentage.*

603. Oral Problems. 1. Henry bought a knife for $.75 and sold it at a gain of 12½ %. How much did he receive for the knife?

2. A man having 7 miles to walk to the post office, has walked ⅗ of the distance. How many miles distant is he from the post office?

3. Mr. Williams bought a city lot at $125 per foot. What was it worth per foot, when it had increased 10 % in value ?

4. Bought 7 pk. 4 qt. of peaches, and found 5 % of them worthless. How much was good ?

5. A grocer bought 5 gallons of milk, and sold 60 % of it. How many quarts had he left ?

6. From a log 40 feet long, a piece of timber, 37½ % of the whole length, is cut. How long is the part that is left in the log ?

7. Henry earns $4.50 a week, and spends 66⅔ % of his wages. How much money does he save in a week ?

604. Written Problems. 1. Mr. Hart bought a farm for $5,250. He paid 25 % cash, and 15 % in 2 years. How much of the purchase money was then due on the farm ?

2. A merchant bought 500 yards of cloth, and found 5 % of it defective. After selling 8 % of the remainder, how many yards had he left ?

3. Mr. Wright engaged a man to cultivate his wheat fields, and in pay gave him 35 % of the product. If the fields yielded 12,500 bushels of wheat, how many bushels of wheat did the owner receive ?

4. If an agent sells 75 books at $8½ each, how much will the firm receive, if he deducts 6 % for his services in selling each book ?

5. A grocer buys 50 pounds of butter at $.25 a pound. How much does he pay for the butter, if 3 % is deducted for cash payment ?

6. Mrs. Burns buys 40 yards of carpet ¾ of a yard wide. She uses 10 % of it for a rug, and the remainder to carpet a floor. How many square yards does she use for the floor ?

7. If a man spends 37½ % of his money which amounts to $5,376, and gives his wife 37½ % of the remainder, how much has he left ?

PROBLEM II.

605. To find what part one number is of another.

Rate = Percentage ÷ Base.

1. If the increase in a herd of 400 cattle is 25, what is the rate of increase?

Explanation.—1. Divide 25 by 400, to find what part the increase is of the herd.

400) 25.
Ans., .06¼

606. Find what per cent.:

1. 14 is of 280.	7. 65 is of 585.	13. 5.5 is of 511.5.
2. 21 " 350.	8. 73 " 1,168.	14. 12.3 " 565⅘.
3. 36 " 440.	9. 42 " 294.	15. 15⅗ " 265.2.
4. 45 " 580.	10. 14¾ " 118.	16. 14.1 " 3,595½.
5. 19 " 340.	11. 16 " 148.	17. 1.62 " 24 3/10.
6. 35 " 1,040.	12. 15 " 127½.	18. 2.45 " 51.4½.

607. Oral Problems. 1. Henry had $.40 and spent $.35. What per cent. of his money did he have left?

2. The length of a room is 8 yards, and the width is 15 feet. What per cent. of the length is the width?

3. A man having 3 acres of land, sows 80 square rods in corn. What per cent. of his land does he sow in corn?

4. From a plank 10 feet long, a carpenter cut a piece 10 inches long. What per cent. of the whole length is the length of the piece cut off?

5. If a merchant sells 15 yards of cloth from a piece containing 90 yards, what per cent. of the cloth does he sell?

6. What per cent. of 96 is ⅜ of 20?

7. Edward bought 2 pounds of caramels. He ate 4 ounces of the candy and gave away 8 ounces. What per cent. of the [illegible]d he left?

bushel of peaches, a grocer loses 2 pk. 4 qt. does he lose?

9. Frank has £2. 8s. and Joseph has 12 shillings. What per cent. of Frank's money is Joseph's?

10. A merchant buys 2 gross of buttons and sells 9 dozen. What per cent. of the whole does he sell? What per cent. has he left?

608. Written Problems. 1. Mr. Wilson's income is $1,800 and his expenses, $1,260. What per cent of his income is he able to save?

2. A farmer owning 120 acres of land, sows 57 A. 96 sq. rd. with wheat, and the remainder with oats. What per cent. of his farm does he sow with oats?

3. Mr. Woods buying a house for $8,250, made a cash payment of $4,950. What per cent. of the cost did he owe? What per cent. did he pay?

4. If a grocer has 8 bushels of potatoes, and sells 4 bu. 3 pk. 6 qt., what per cent. of the whole has he left? What per cent. did he sell?

5. After walking 1 mi. 4 fur. 20 rd., what per cent. of a distance of 3 miles has a man walked?

6. A room is 10 yards long and 24 ft. 6 in. wide. The width is what per cent. of the length of the room?

7. Mr. James owns $\frac{5}{8}$ of a ship worth $16,400. He sells $\frac{4}{5}$ of his share. What per cent. of the value of the ship does he sell?

8. The cost of a house was $6,080 and the cost of the lot $3,800. What per cent. of the cost of the house was the difference in their cost?

9. If a grocer bought 2 bl. 25 gal. of vinegar and sold 1 bl. 20 gal. 2 qt., what per cent. of the whole quantity did he sell?

10. A merchant's capital is $150,000. He sells an average value of $5,300 a month. What per cent. on his c[illegible] *does he sell per year?*

PROBLEM III.

609. To find a number, a part being given.

Base = Percentage ÷ Rate.

NOTE.—The formula may be deduced from Problem I.: *Percentage = Base × Rate.*

1. 154 cubic inches of a gallon of water were used. This being $66\frac{2}{3}$ % of an entire gallon, how many cubic inches of water are there in a gallon ?

Explanation.—1. The part is divided by the decimal representing it.

ANALYSIS: If 154 is $66\frac{2}{3}$ %, 1 % is 1–$66\frac{2}{3}$ part of 154, and 100 %, or the number, is 100 the quotient. Divide by $66\frac{2}{3}$ and multiply by 100; or, simply divide by $.66\frac{2}{3}$.

$.66\frac{2}{3})154$

$\frac{3}{2.00}$ $\frac{3}{462.}$

231

610. Find the number of which :

1. 8 is 5 %.	*7.* 9 is $37\frac{1}{2}$ %.	*13.* 15 is $33\frac{1}{3}$ %.
2. 12 " 9 %.	*8.* 24 " $66\frac{2}{3}$ %.	*14.* .168 is 1.2 %.
3. 15 " 8 %.	*9.* 15 " $62\frac{1}{2}$ %.	*15.* .45 " .3 %.
4. 20 " $16\frac{2}{3}$ %.	*10.* 130 " 40 %.	*16.* 1.3 " $1.6\frac{1}{4}$ %.
5. 25 " 20 %.	*11.* 255 " 60 %.	*17.* 1.732 is $5.4\frac{1}{8}$ %.
6. 4 " $6\frac{1}{4}$ %.	*12.* 360 " 45 %.	*18.* .20 is $.66\frac{2}{3}$ %.

611. Oral Problems. 1. 16 shillings is 80 per cent. of what sum ?

2. $7.50 is 75 per cent. of how much money ?

3. The tax on a certain property is $40 or $2\frac{1}{2}$ per cent. What is the property worth ?

4. 40 per cent. or 72 of the children in a school are boys. How many children are there in the school ?

5. A man in selling a barrel of flour, gains $.70, or $16\frac{2}{3}$ %. What did the flour cost him ?

6. Andrews receives $240 as a year's rent on his house. What is the value of his house, if the rent was 6 % of it ?

7. A man sold a city lot at a loss of $200, which was $4\frac{1}{8}$ per cent of the cost. What did the lot cost him ?

8. George is 18 years old, and his age is 75 per cent. of William's age. How old is William ?

9. Charles has 18 marbles, or $12\frac{1}{2}$ per cent. as many as Henry. How many marbles has Henry ?

10. Mary gathers 12 quarts of berries, or $33\frac{1}{3}$ per cent. as much as Jane. How many bushels and quarts does Jane gather ?

612. Written Problems. 1. $6\frac{1}{4}$ per cent. of $8\frac{1}{3}$ per cent. of 9,600 is 4 per cent. of what number ?

2. Frank spends $500, or $83\frac{1}{3}$ per cent. of his income. What is his income ?

3. The increase of a flock of sheep was 228, and this was 12 per cent. on the original number. How many sheep did the flock contain at first ?

4. The population of a town is 3,000 more this year than last year, having gained $16\frac{2}{3}$ per cent. What was the population last year ?

5. A man pays $6,750, which is 75 per cent. of his debts. What is the amount of his debts ?

6. If a farmer sold 750 sheep, or 40 % of his flock, how many did he have at first ?

7. Mr. Wilson pays $131.25 insurance on $\frac{3}{4}$ of the value of a building at a premium of $1\frac{1}{4}$ per cent. What was the value of the building ?

8. If the gain on a lot was $243 or $16\frac{2}{3}$ per cent., what was the value of the lot ?

9. If $55, or 20 per cent., was gained on 75 tons of coal sold at retail, how much per ton was paid for the coal at wholesale ?

10. A man sold 50 cords of wood, losing thereb
per cent. What did the wood cost him per cord

PROBLEM IV.

613. To find a number, the number plus or minus a part being given.

$$\textit{Base} = \begin{cases} \textit{Amount} \div \overline{1 + \textit{the rate.}} \\ \textit{Difference} \div \overline{1 - \textit{the rate.}} \end{cases}$$

1. What was the amount of a man's capital, if at the end of a year, having earned 20 %, it amounts to $36,480 ?

Explanation.— 1. If the capital has increased 20 % in value, it amounts to 120 %.

1.20)$36,480.0
Ans., $3,040.0

2. To find the original capital, divide the given number, $36,480, by the decimal per cent. (1 + rate) that represents it.

ANALYSIS.—The capital was 100 %; the gain being 20 %, the amount is 120 %. If 120 % is $36,480, 1 % is 1-120th of $36,480, and 100 % is 100 times as much. Divide by 120 and multiply by 100; or, divide by 1.20.

2. What was a man's capital if, after a loss of 25 %, it is $22,500 ?

Explanation.—1. The capital having decreased 25 % is now 75 % of the first sum.

2. To find the first capital, divide the given number by (1 − rate) the decimal representing it.

.75)$22,500.00
$30,000

614. Find the numbers which equal the following:

when increased

12½ %	8⅓ %	16⅔ %	37½ %	33⅓ %	62½ %
1. 27.	4. 39.	7. 42.	10. 33.	13. 56.	16. 39.
2. 36.	5. 26.	8. 63.	11. 44.	14. 48.	17. 52.
3. 45.	6. 52.	9. 56.	12. 55.	15. 64.	18. 78.

when decreased

12½ %	66⅔ %	1.5 %	.3 %	37½ %	2.2 %
12.	25. 5.91.	28. 4.985.	31. 6376.	34. 19.56.	
16.	26. 7.88.	29. 11.964.	32. 3985.	35. 43.9.	
9.	27. 19.7.	30. 49.85.	33. 399.	36. 63.57.	

615. Oral Problems. 1. 75 acres, or 25 % less than an entire farm, is planted in corn. How many square rods are there in the farm ?

2. A cloak was sold for $375, a loss of 25 per cent. What was the cost price ?

3. John cut ½ cord of wood, which was 33⅓ per cent. more than Edward cut How much wood did Edward cut ?

4. 12,500 bricks were found to be 25% more than were required for a building. How many bricks were required ?

5. What is the base, if the rate is 5 per cent., and the difference 190 ?

6. If the sum of 2 numbers is 24, and the greater is 40 per cent. more than the less, what are the numbers ?

7. A pound of tea sells for $.63, the gain being 16⅔ per cent. of the cost price. What is the cost price ?

8. Oranges sold at the rate of $.35 per dozen were sold at a loss of 30 per cent. How much per dozen was the cost price ?

9. The time past noon is 66⅔ per cent. of the time to midnight. What is the time ?

10. George has 45 marbles, which is 25 per cent. more than his brother has. How many marbles has his brother ?

616. Written Problems. 1. A carriage was sold for $640, which was 28 per cent. less than the first cost. What was the first cost ?

2. John spent 80 per cent. of his money and had $400 left. How much had he at first ?

3. A number increased by 64 per cent. of itself is 2,460. What is the number ?

4. A number diminished by 11 per cent. of itself is 2,225. What is the number ?

5. A sold 60 bushels of wheat at $1.50 a bushel, l
20 *per cent.* *How much* did the wheat cost him per b

6. What was the first cost of a horse that sold for $175 at a loss of 12½ per cent.?

7. 10 yards of cloth at $1.75 a yard were sold at a loss of 14⅔ per cent. What was the cost per yard?

8. A farmer sold 2,400 bushels of oats, which was 6¼ per cent. more than what he had left. How many bushels had he left?

9. If 120 bushels of peaches are sold for $330, at a gain of 10 per cent., what did they cost per bushel?

10. A jeweller sold 6 watches at $125 each, thereby gaining 42⅔ per cent. What did the watches altogether cost him?

MISCELLANEOUS PROBLEMS.

617. Oral Problems. 1. James has $36 and increases it by 16⅔ %. How much money has he then?

2. A man sold his carriage for 37½ % less than cost, thereby losing $45. What was the cost of the carriage?

3. Mary had $.72. She spent 8⅓ % and lost 12½ %. How much money had she left?

4. A farmer sold 40 % of ⅝ of his farm and had 75 acres left. How many acres had he at first?

5. A man is 75 years old, and ⅗ of his age is 62½ % of his wife's age. How old is his wife?

6. A man who earns $56 a month, spends 87½ % of his wages. How many dollars does he save in one month?

7. Mr. Graham sold his horse at a loss of $25, which was 12½ % of its cost. What was the selling price?

8. An agent charged $120 for selling goods at 4 % commission. For what sum did he sell the goods?

9. A grocer bought flour at $8 a barrel and sold it at $8.50 What per cent. of profit did he make?

an sold his piano for $450, losing 16⅔ %. What of the piano?

11. Bought a house for \$6,400 and paid 87½ % on it. How much is still due on the house?

12. If a grocer buys tea at \$.48 and sells it for \$.60, what per cent. of profit does he make?

13. At 3 % commission, how many dollars' worth of goods must an agent sell to make \$210?

14. Mr. Hall rents a house for \$350, which is 8⅓ % of the value of the house. What is the value of the house?

15. Bought a lot for \$8,000, paying at different times 12½ %, 25 %, and 37½ %. How much is still due on the lot?

16. A broker charged ¼ % commission for buying 300 shares of stock at \$100 each. What was his commission?

17. William earns \$7.50 a week. He pays 66⅔ % of his wages for board, and 20 % of the remainder for car-fare. How much money has he left?

18. It takes 2 % of a man's income of \$2,000 to pay the premium on his house insured for \$8,000. What rate of insurance does he pay?

19. A farmer had 250 sheep. He sold 20 % of them, and lost 12½ % of the remainder. How many sheep had he left?

20. A man sold 25 % of 80 % of 400 bushels of wheat at \$1¼ a bushel. How much money should he receive?

21. A man gave to his wife 66⅔ % of his month's salary. His wife, with 75 % of the money she received, bought a cloak for \$150. What was the man's salary per month?

22. If 16⅔ % is gained by selling a wagon for \$63, how many dollars would be lost by selling it at 16⅔ % below cost?

23. A furniture dealer sold two chairs for \$45 each. On one he gained 25 %, and on the other he lost 25 %. Did he gain or lose on both, and how much?

24. How much more per cent. is gained on an article bought at \$6 and sold at \$9 than on an article bought at \$5 and sold at \$7?

25. Bought a number of barrels of flour for $130. Paid $11 for storage and sold at a gain of 8⅓ % on the whole cost. What was the selling price?

618. Written Problems. 1. A man owns 46 % of a mill worth $9,540, and sells 25 % of his share. What is the value of the remainder of his share in the mill?

2. A's property is 6¼ % of B's, which is worth $3,556. C's property is 106 % of A's. What is the value of the three properties combined?

3. A man sold his lot for $5,816.25, losing 17½ %. For how much did he buy it per front foot, if he had 75 feet front?

4. A merchant, after spending 66 % of his money, and then 35 % of the remainder, has $54.74 left. How much money had he at first?

5. A horse dealer sold two horses at $825 each. He gained 33⅓ % on one and lost 33⅓ % on the other. How much was his whole gain or loss?

6. A man bought 37 cows and sold them for $2,898.45, thus gaining 13 %. What was the average cost of one cow?

7. A merchant in the course of a year sells goods to the amount of $[illegible], making an average gain of 16⅔ %. What [illegible] following a rent of $1,000 and other [illegible] receiving 8⅓ % of the rent?

8. [illegible] A, B and C, in partnership [illegible] as much as A and C, the [illegible] a loss of $[illegible] what was [illegible] share of the loss?

9. [illegible] $[illegible] worth $4,750. [illegible]

10. [illegible] inches at $1.75. What [illegible] inches at $1.75?

11. [illegible] $22.40 at $65 [illegible]

12. Property valued at $9,000 is given to an agent to sell, his commission to be $315. What rate per cent. is the commission?

13. An army loses in battle 1,750 men, or 2½ per cent. of the entire number. How large was the army?

14. At $41⅔ a month, the yearly rent of a house is 8 % of the value of the house. What is the value of the house?

15. Railroad stock purchased @ 87½ % is sold @ 95½ %. What is the gain on 125 shares?

100 % equals $100.

16. A gives a broker $11,306.25 to invest in bank stock, and to pay his commission of ¼ %. How many shares at 112½ % will A receive?

17. When gold was quoted at 103½, A exchanged a carriage worth $200 in gold for a horse worth $212 in greenbacks. Did he gain or lose, and how much?

18. What is 25 per cent. of 8,920 feet expressed in miles?

19. A and B starting at the same time travel toward each other, A at the rate of 6 miles per hour, B at the rate of 4 miles per hour. The distance is 120 miles. What per cent. of the distance will each travel?

20. An agent buys 25 hundred weight of sugar at 8½ cents per pound. His commission is 1¾ per cent. What amount must he receive to cover the entire bill?

619. The **Applications of Percentage** do or do not involve the element of *time*, and they are accordingly separated into two classes:

I. ***Without Time:*** 1. Profit and Loss; 2. Commission; 3. Stocks and Bonds; 4. Insurance; 5. Trade Discount; 6. Taxes; 7. Duties.

II. ***With Time:*** 1. Interest; 2. Bank Discount; 3. True Discount; 4. Partial Payments; 5. Equation of Payments; 6. *Exchange; 7. Averaging* Accounts.

PROFIT AND LOSS.

620. Profit and **loss** are the gain and the loss in business.

621. In applying the principles of PERCENTAGE to **Profit and Loss,** the ***cost*** is the *base;* the ***profit*** or the ***loss*** is the *percentage;* the ***selling price*** is the *amount* or the *difference;* and the ***rate per cent.*** of profit or of loss is the ***rate per cent.,*** as in percentage.

PROBLEM I.

622. To find a part of a number.

$$\left.\begin{matrix}\textit{Profit,}\text{ or}\\ \textit{Loss}\end{matrix}\right\} = \textit{Cost} \times \textit{Rate.}$$

NOTE.—The selling price equals the cost, plus the profit, or minus the loss.

1. What is the gain on cloth bought for $1.50 per yard, and sold at a profit of 10 %?

The profit equals 10 % or, .10 of $1.50, which is $.15. Ans.

2. If a carriage is bought for $175, and sold at a loss of 20 %, how much money is lost by the sale?

3. What is the selling price, if a house costing $4,500 is sold at a gain of 33⅓ %?

4. A bushel of peaches was bought for $1.20, and sold at a loss of 40 %. What was the selling price?

623. Find the profit or the loss on the following cost prices:

Groceries.			Fruits.		
12½ % on	16⅔ % on	20 % on	25 % on	33⅓ % on	66⅔ % on
1. $ .80	6. $ .60	11. $ .40	16. $ .20	21. $ .75	26. $ .30
2. $ .72	7. $ .90	12. $ .75	17. $ 32.	22. $ .18	27. $ .60
3. $ .64	8. $ .18	13. $1.00	18. $ .16	23. $ .30	28. $ .75
	9. $2.40	14. $1.25	19. $ .12	24. $2.40	29. $3.00
	$3.00	15. $5.00	20. $2.00	25. $3.30	30. $4.50

Dry Goods.				Carriages, etc.			
50 % on	15 % on	24 % on		27½ % on	18¾ % on	9.5 % on	
31. $ .10	*36.* $ .40	*41.* $ 2.50		*46.* $200.	*51.* $480.	*56.* $700.	
32. $ .18	*37.* $ .20	*42.* $ 3.50		*47.* $425.	*52.* $340.	*57.* $650.	
33. $2.50	*38.* $ .50	*43.* $14.00		*48.* $375.	*53.* $700.	*58.* $425.	
34. $3.00	*39.* $2.00	*44.* $25.50		*49.* $550.	*54.* $660.	*59.* $300.	
35. $3.50	*40.* $2.50	*45.* $37.00		*50.* $600.	*55.* $800.	*60.* $ 25.	

The first 35 should be worked without the use of slate.

624. Find the selling prices in the preceding, with the rate stated as profit or as loss.

Thus, Ex. 17: What is the selling price per quart of apples bought at the rate of $.32 per peck and sold at a profit of 25 %?

625. Make problems for the numbers in Arts. 623 and 624.

626. Written Problems. 1. If a house and lot are bought for $7,500 and sold at a gain of 45 %, what is the selling price?

2. A drover bought 125 sheep at $3.50 a head. What was his entire gain on selling the sheep at an advance of 35 %?

3. Mr. Collins bought property for $50 per foot, and sold 95 feet at a profit of 62 per cent. How much money should he receive?

4. If 500 hats are bought at $2.50 and sold at a loss of 8 %, what is the selling price?

PROBLEM II.

627. To find what part one number is of another.

$$\textit{Rate} = \left.\begin{matrix}\textit{Profit}\\ \textit{Loss.}\end{matrix}\right\} \text{ or } \div \textit{Cost.}$$

The amount or the difference may be given together with the cost, the profit or loss being readily found by adding or subtracting.

1. John bought papers at 3 ¢ and sold them at 2 ¢. What per cent. was his loss?

The loss was 1 ¢; 1 ¢ is ⅓ or .33⅓ or 33⅓ % of 3 ¢, the cost. Ans.

2. If a pair of shoes is bought for $2.50 and sold for $4.00, what is the rate per cent. of gain?

The gain is $1.50; $1.50 is what part of $2.50? Ans., $1.50 ÷ $2.50 = ⅗, or .60 or 60 %.

3. A man bought corn at $.50 a bushel and sold it for $.62½ a bushel. What was the rate of gain?

4. If a lot is bought at $36 a foot and sold at $65 a foot, what is the rate of gain?

5. A merchant bought cloth at $1.37½ a yard and sold it at $1.25 a yard. What was the rate of loss?

628. Find the rate per cent. of profit or of loss on the following cost prices:

Groceries.			Shoes, etc.		
12 % on	15 % on	20 % on	25 % on	40 % on	50 % on
1. $.24.	*5.* $.30.	*9.* $ .40.	*13.* $2.50.	*17.* $4.00.	*21.* $5.00.
2. $.36.	*6.* $.60.	*10.* $ .80.	*14.* $1.25.	*18.* $2.00.	*22.* $2.50.
3. $.48.	*7.* $.90.	*11.* $1.00.	*15.* $2.00.	*19.* $2.40.	*23.* $2.00.
4. $.60.	*8.* $.45.	*12.* $1.20.	*16.* $3.00.	*20.* $3.20.	*24.* $3.00.

Hardware.			Real Estate.		
75 % on	80 % on	95 % on	$45.50 on	$125.25 on	$430 on
25. $7.50.	*29.* $4.00.	*33.* $ 5.70.	*37.* $182.	*41.* $1,503.	*45.* $2,175.
26. $3.00.	*30.* $2.40.	*34.* $ 7.60.	*38.* $227.	*42.* $1,002.	*46.* $3,480.
27. $6.00.	*31.* $3.20.	*35.* $ 2.85.	*39.* $273.	*43.* $876.75.	*47.* $1,160.
28. $2.25.	*32.* $2.00.	*36.* $11.40.	*40.* $364.	*44.* $751.50.	*48.* $1,087.

The first 32 should be worked without the use of slates.

629. Find the rate per cent. in the preceding, with the cost price and the selling price given.

Thus: Ex. 13. What is the rate of gain or of loss on a pair of shoes costing $2.25 and sold for $2.50?

630. Make problems for the numbers in Art. 628, finding the rate per cent. of profit or of loss, with the gain or the loss and price given, or with the cost and the selling price

631. Written Problems. 1. If a house and lot are bought for $12,500, and sold for $13,000, what is the rate per cent. of gain ?

2. Mr. Nathan bought bedroom sets for $42.50 and sold them for $60. What rate per cent. of profit did he make ? What rate of profit would he make by selling them for $85 ?

3. A horse was bought for $175 ; his keeping cost $35. What was the % of net profit, the horse being sold for $210 ?

4. If 500 bushels of wheat are bought @ $1.12½ per bushel, and sold for $575, what is the rate per cent. of profit ?

5. A coal merchant bought a cargo, containing 375 tons of coal for $1,000. He retailed it at $4.50 per ton. What rate of profit did he make ?

6. If a farm costing $1,800 is divided into 20 equal village lots, and sold at $115 each, what is the rate per cent. of gain ?

7. A silver mine was bought for $175,000, and sold for $37,500. What was the rate per cent. of loss, and how much did a man lose who owned 45 shares costing $25 each ?

PROBLEM III.

632. To find a number, a part being given.

$$\textit{Cost} = \left.\begin{matrix}\textit{Profit}\\ \textit{Loss}\end{matrix}\right\} \text{ or } \div \textit{Rate}.$$

NOTE.—Selling price equals the cost price plus the profit, or minus the loss.

1. If by selling a house and lot at a loss of 15 %, a man loses $9,000, what did the house and lot cost him ?

If 9,000 is 15 %, 1 % is $600, and 100 % is $60,000.

NOTE.—Divide $9,000 by 15 and multiply by 100 ; or divide by .15.

2. If railroad stock is sold at a profit of $42,500, or 12½ %, what *was the cost of* the stock ?

3. A dry goods merchant bought winter cloaks, and sold them in the spring at a loss of $17.50, or 37.5 %. What did the cloaks cost him?

4. What was the selling price of a farm sold at a profit of $1,449, or 18⅜ % ?

633. Find the cost when in the following:

Notions.			Hats, etc.		
5 % is	4 % is	12½ % is	8½ % is	14⅜ % is	16⅔ % is
1. $.02.	6. $.04.	11. $.05.	16. $.50.	21. $ .20.	26. $2.00.
2. $.03.	7. $.05.	12. $.09.	17. $.25.	22. $ .50.	27. $1.50.
3. $.05.	8. $.06.	13. $.15.	18. $.40.	23. $ .15.	28. $3.50.
4. $.10.	9. $.08.	14. $.20.	19. $.80.	24. $ .40.	29. $1.25.
5. $.15.	10. $.10.	15. $.25.	20. $.75.	25. $1.50.	30. $ .75.

Saddlery.		Ships, etc.	
15 % is	17½ % is	37½ % is	63 % is
31. $ 4.50.	36. $15.75.	41. $ 7,500.	46. $ 15,750.
32. $ 5.40.	37. $ 5.25.	42. $11,250.	47. $ 52,920.
33. $ 6.30.	38. $11.37½.	43. $16,875.	48. $ 85,050.
34. $ 8.40.	39. $14.35.	44. $22,500.	49. $100,800.
35. $11.25.	40. $16.62½.	45. $46,875.	50. $ 85,680.

The first 30 should be worked without the use of slates.

634. Find the selling prices in the preceding.

635. Make problems for the numbers in Art. 633, requiring the cost price or the selling price, as in Arts. 632 and 636.

636. Written Problems. 1. If a ship's cargo was sold at a profit of $32,600, or 33⅓ % of the cost, for what sum was the cargo sold?

2. A consignment of melons was sold at an advance of $926. What was the selling price, if the advance was 1.4 % on the cost price?

3. If a grocer sells 20 lb. of butter, thereby losing altogether cent. of the cost, what was the cost?

4. A was defrauded of \$5,250, or 58⅓ per cent. of his share of an estate. What did A receive?

5. If carpet is sold at a gain of \$36, or 24 per cent., what is the selling price?

6. A tailor sells a suit of clothes, losing \$10, or 14⅔ per cent.; for what sum did he sell the clothes?

PROBLEM IV.

637. To find a number, the number plus or minus a part being given:

$$\textit{Cost} = \textit{Selling Price} \div \begin{cases} 1 + \textit{rate of gain}, \text{ or,} \\ 1 - \textit{rate of loss}. \end{cases}$$

1. A wagon was sold for \$60, and a whip for \$.90, the entire loss being 16 %. What was the entire cost?

If the loss was 16 %, the selling price was 84 %. If \$60.90 is 84 %, 1 % is found by dividing by 84, and 100 % by multiplying the result by 100. Dividing by .84 is the same as dividing by 84 and multiplying by 100.

2. What was the cost of maple syrup sold for \$1.60 per gallon, at a gain of 28 per cent.?

3. A bushel of cranberries was sold for \$1.80, at a loss of 10 per cent. What was the cost?

4. Wheat was sold for \$1.50 a bushel, at 7 2/3 per cent. less than first cost. What was the first cost?

638. Find the cost prices, the selling price with the rate of gain or loss being as follows:

Stationery.			**Books.**		
Gain, 10 %	20 %	30 %	Gain 12½ %	16⅔ %	14⅔ %
1. \$1.10.	6. \$.60.	11. \$.39.	16. \$.45.	21. \$ .35.	26. \$.40.
2. \$.66.	7. \$.54.	12. \$.65.	17. \$.36.	22. \$ .49.	27. \$.56.
3. \$.55.	8. \$.42.	13. \$.78.	18. \$.63.	23. \$ .63.	28. \$.72.
4. \$1.32.	9. \$.75.	14. \$.91.	19. \$.81.	24. \$ .84.	29. \$.96.
5. \$2.20.	10. \$.96.	15. \$.52.	20. \$.54.	25. \$1.05.	30. \$.88.

Grains.		Clothing.	
Loss, 14 %	37½ %	Loss, 5¾ %	19.5 %
31. $ .86.	*36.* $16.00.	*41.* $150.80.	*46.* $ 88.55
32. $1.07½.	*37.* $21.20.	*42.* $ 60.32.	*47.* $ 61.18.
33. $1.50½.	*38.* $28.60.	*43.* $ 75.40.	*48.* $ 32.20.
34. $1.72.	*39.* $41.25.	*44.* $120.64.	*49.* $ 96.60.
35. $1.29.	*40.* $45.00.	*45.* $ 64.09.	*50.* $144.90.

The first 30 should be worked without the use of slates.

639. Make Problems for the numbers in Art. 638, as in Arts. 637 and 640.

640. Written Problems. 1. A jeweller sells a stem winding watch for $200, or 33⅓ per cent. above cost. What was the cost ?

2. If a farmer sells a flock of sheep for $520, gaining 30 per cent. of the cost, what did the sheep cost him ?

3. A lumber merchant sells lumber for $2,650, which was 1⅔ per cent. less than cost. What did the lumber cost ?

4. What will a box of raisins cost, if 40 boxes sell at a gain of 40 per cent. for $224 ?

5. A bought 3 cords of wood at $5.50 a cord, but it proved to be 12½ per cent. less than full measure. How much did the wood actually cost him per cord ?

6. If a house is sold for $6,800 at a loss of 2¾ per cent., what was the cost of the house ?

7. A cheese weighing 16⅔ pounds was sold for $3.00. What was the cost price per pound, if 100 per cent. was made by selling the cheese ?

8. A sheet of music sells for $.30. If a profit of 1,400 per cent. is made, what is the actual cost to the publisher ?

9. If a stove is sold for $15.70 at a loss of 22.5 %, what was the cost price ?

10. A selling price being $36.63 at a loss of 19.6 %, what

?

MISCELLANEOUS PROBLEMS.

641. Oral Problems. 1. What is the gain on goods that cost $1.60 a yard and that are sold at a profit of 12½ per cent. ?

2. How much was lost on land that cost $96 per front foot and that was sold at a loss of 37½ % ?

3. On a pound of coffee sold for $.32 a grocer makes $.08. What per cent. of the cost does he gain ?

4. If a bookseller buys Readers at $.42 each and sells them for $.56 each, what is his rate per cent. of gain ?

5. Bought calico at $.08 a yard and sold it at a gain of 25 %. What was the selling price ?

6. A man bought paper at $.20 a quire and sold it at a loss of 20 %. What was the selling price ?

7. Mr. Mason bought flour at $7.50 a barrel and sold it for $5 a barrel. What was his rate per cent. of loss ?

8. By selling his knife, James makes $.08 or 16⅔ %. What was the cost of the knife ?

9. Received $200 for a horse that cost $150. What was the per cent. of gain ?

10. A piece of sheet music is sold at a gain of 66⅔ % or $.40. What was its cost ?

11. At what price must a book that cost $.40 be marked to deduct 20 % from the marking price and still gain 20 % ?

Find the selling price and then the marking price.

12. If $.48 or 10 % is lost on the sale of 4 baskets, what is the cost of 1 basket ?

13. By selling a wagon for $150, 16⅔ % was lost. What was the cost ?

14. What is the cost of an article on which 25 % is gained, after deducting 40 % from the marking price, which is $.50 ?

15. Mrs. French sold lace at a loss of 20 %, losing $.18 on 6 *yards. What did the* lace cost per yard ?

16. A man sold 6 dozen eggs for $1.08 and gained 50 %. What was the cost per dozen?

17. How much is gained on a vase marked $10, if 20 % is deducted from the marking price and 33⅓ % on the cost is gained?

18. 25 % was gained on a piano that was sold for $625. How much did the piano cost?

19. If a grocer sells milk at $.32 a gallon and makes 33⅓ %, what is the cost per quart?

20. What per cent. is deducted from the marking price of a slate marked $.16, and costing $.12, the gain being 16⅔ %?

642. Written Problems. 1. Bought 120 pounds of meat at $.12½ a pound and sold it at 20 % gain. What was the entire gain?

2. A picture dealer sells 2 pictures which cost $850 each. On the one he gained 12½ %, and on the other he lost 37½ %. What was his entire loss?

3. By selling coal for $8 a ton, a dealer makes $5.40 on 6 tons. What is his rate per cent. of gain?

4. Sold a piece of muslin containing 32 yards for $2.80. What was the rate per cent. of gain, the cost being $.06¼ per yard?

5. If a man owns ¾ of a factory valued at $16,000, and sells ⅔ of his share at a gain of 37½ %, how much does he receive?

6. What was the selling price of 125 yards of velvet worth $3.50 a yard, sold at a loss of 8 %?

7. A man sold his farm of 220 acres for $33,000. What per cent. did he gain, if the farm cost him $130 per acre?

8. If $374.40 is lost on 240 barrels of flour, what is the cost per barrel, the rate of loss being 24 %?

9. [illegible]ns sold his carriage for $224, which was ⅞ of [illegible]t per cent. would he have gained, if he had [illegible]?

10. By selling 1,500 yards of gingham for $270, a merchant made 12½ %. How much did it cost him per yard ?

11. A coal dealer gained $3.75, or 5 % of the cost, on 12 tons of coal. What did the coal cost per ton ?

12. A jeweller sold a clock for $68.75 and gained 37½ %. What would have been the selling price if he had lost 14 % ?

13. A grocer bought 7 gallons of wine, and lost 2 gallons by leakage. He sold the remainder for $8.05 a gallon, making 25 % on the whole. What was the cost per gallon ?

14. Sold two lots at $2,400 each, gaining 25 % on the one, and losing 25 % on the other. What was the entire gain or loss ?

COMMISSION.

643. A **commission merchant** or **broker** is one who buys or sells property for another, called the *principal.*

Agent is a more general term, an agent being one who acts for another in any business transaction ; *factor* is a more restricted term, the factor being a broker with possession of the property bought or sold.

644. The **commission** is the sum paid to the broker, or retained by him for his services.

645. A **consignment** is the goods sent to be sold ; the **consignor** is the party that sends the goods ; and the **consignee** is the party to whom the goods are sent.

646. In applying PERCENTAGE to **Commission** or **Brokerage,** the ***selling*** or the ***cost price*** is the *base ;* the ***commission*** or ***brokerage*** is the *percentage ;* the ***sum sent*** or the ***sum received*** by the principal is the *amount* or *difference ;* and the ***rate per cent. of commission*** is the *rate per cent.,* as in percentage.

NOTE.—The base is either the *selling price* or the *cost price,* as the broker's charge is on the selling price when he sells, and on the cost price when he buys.

PROBLEM I.

647. To find a part of a number.

$$\textit{Commission} = \left.\begin{matrix}\textit{Selling Price,}\\ \textit{Cost Price.}\end{matrix}\right\} \text{ or } \times \textit{Rate.}$$

NOTE.—The sum of money sent by the principal equals the cost plus the commission ; the sum received by the principal equals the selling price minus the commission. The other problems in Commission may be deduced from Prob. I.

1. What commission at $2\frac{1}{2}$ % does a broker receive for selling a consignment of wheat for $15,750 ?

The broker receives .025 of the selling price.

2. If a real estate broker buys a farm for $9,725, and charges $4\frac{1}{2}$ % commission, how much does he receive for his services ?

The broker receives .045 of the cost price.

3. An agent collects a bill of $2,550 and charges $3\frac{1}{2}$ % for collecting. What is his commission ?

4. How much does an agent make, if he buys a lot for $7,250 and charges $1\frac{1}{2}$ % for buying? What is the cost to the principal ?

5. If a commission merchant sells 105 barrels of flour for $8.25 a barrel, at a commission of 4 %, what does he make by the transaction ? How much does the principal receive ?

648. Make problems for the numbers in Art. 649 as in Arts. 647 and 651.

649. Find the amount of commissions at the given rates on the following cost or selling prices :

Fruits.			Vegetables.		
5 % on	6 % on	8 % on	3 % on	4 % on	5 % on
	4. $3.00.	7. $4.00.	10. $2.50.	13. $ .50.	16. $2.50.
	$1.50.	8. $1.50.	11. $4.25.	14. $ .75.	17. $6.40.
	$ .50.	9. $1.25.	12. $5.75.	15. $1.25.	18. $8.80.

Grains.			Real Estate.		
1¼ %	2½ %	2.5 %	1½ %	4½ %	2.75 %
19. $236.	*22.* $ 275.	*25.* $610.	*28.* $2,500.	*31.* $5,060.	*34.* $9,600.
20. $450.	*23.* $ 550.	*26.* $840.	*29.* $3,250.	*32.* $6,800.	*35.* $7,100.
21. $590.	*24.* $1,020.	*27.* $525.	*30.* $4,980.	*33.* $1,750.	*36.* $9,420.

The first 18 should be worked without the use of slates. In the written problems find the answer to the nearest number of cents.

650. Find the amount sent or received by the principal in the preceding.

651. Written Problems. 1. How much does a commission merchant receive for selling 1,500 bushels of wheat at $1.37½, if his charge is 2.5 % ?

2. A real estate broker bought 143 feet of land, at $75 per foot. What did he charge for his services at 1¾ % commission? What was the entire cost to the principal ?

3. A factor sold 1,200 clocks at $2.35 each, and charged 5 % for his services. How much did he receive ? How much did his principal receive ?

4. What does a house receive from the sale of 5,000 yards of cloth at $.62½ per yard, allowing 2½ per cent. to an agent for his services, and 5 per cent. for his guarantee ?

PROBLEM II.

652. To find what part one number is of another.

$$\textit{Rate} = \textit{Commission} \div \begin{cases} \textit{Selling Price,} \text{ or} \\ \textit{Cost Price.} \end{cases}$$

NOTE.—The sum sent or the sum received by the principal may be given, in which case the cost or the selling price is found by adding or subtracting.

1. If a broker receives $250 for selling a house and lot for $12,500, what is his rate per cent. of commission ?

$250 is $\frac{250}{12,500}$ of the cost price ; $\frac{250}{12,500} = \frac{1}{50} = .02$; that is, divide the *commission by the selling price.*

2. A Cincinnati firm sent $7,790 to their agent in New Orleans to invest in sugar. He deducted his commission of $190, and invested the balance in sugar. What rate per cent. did the broker charge ?

3. If a broker charges $25 for investing $10,000 in certain bonds, what is his rate of commission ?

4. Black & Co. sent 500 bu. of wheat to their agent in Chicago, who sold it for $750 ; the freightage was $15, and the drayage, etc., $5 ; the agent received $18.75. What was the rate of his commission ?

5. A lawyer charges $750 for collecting money for a client to whom he returns $6,750. What was the rate per cent. charged for his services ?

653. Find the rate per cent., the selling price, or the cost price, and the commission being as follows:

Notions.

$.25 on	$.50 on	$.33⅓ on
1. $ 5.00.	5. $50.00.	9. $ 6⅔.
2. $ 6.25.	6. $10.00.	10. $ 8⅓.
3. $25.00.	7. $25.00.	11. $16⅔.
4. $10.00.	8. $12.50.	12. $11⅑.

Cutlery.

$.75 on	$.40 on	$62½ on
13. $15.	17. $ 8.	21. $12½.
14. $25.	18. $20.	22. $31¼.
15. $18¾.	19. $10.	23. $15⅝.
16. $37½.	20. $13⅓.	24. $20⅝.

Plated Ware.

$2.25 on	$3.75 on	$4.37½ on	$5.25 on
25. $37.50.	29. $ 62.50.	33. $ 43.75.	37. $25.00.
26. $75.00.	30. $ 75.00.	34. $ 62.50.	38. $22.50.
27. $25.00.	31. $ 93.75.	35. $ 87.50.	39. $44.75.
28. $45.00.	32. $125.00.	36. $218.75.	40. $45.00.

Dry Goods.

$10.20 on	$35.65 on	$105.87½ on	$125 on
41. $204.	45. $ 713.00.	49. $1,512.50.	53. $4,200.
42. $340.	46. $ 891.25.	50. $2,117.50.	54. $3,075.
43. $255.	47. $1,782.50.	51. $5,293.75.	55. $2,980.
44. $170.	48. $1,426.00.	52. $4,235.00.	56. $5,600.

The first 12 should be worked without the use of slates.

654. Work the preceding with the amount received or sent by the principal and the commission given.

655. Make Problems for the numbers in Arts. 653 and 654.

656. Written Problems. 1. A real estate agent receives $200 for selling a house for $9,000. At what rate per cent. was his commission ?

2. If an agent sells a ship for $60,000, receiving $18,000, what rate did he receive ?

3. What rate per cent. was given for the sale of books for $5,760 when the agent received $172.80 as his commission ?

4. What rate does a commission merchant charge, who receives $6,179.25 to invest in merchandise, his commission being $117.70, and his guarantee being $176.55 ?

PROBLEM III.

657. To find a number, a part being given.

$$\left.\begin{matrix}\textit{Selling Price,}\text{ or}\\ \textit{Cost Price.}\end{matrix}\right\} = \textit{Commission} \div \textit{Rate.}$$

NOTE.—The sum received by the principal equals the selling price minus the commission; the sum sent by the principal equals the cost price plus the commission.

1. If a real estate broker charges $250, or 2 % for selling property, what is the amount of his sales ? How much does the principal receive ?

Divide by 2 and multiply by 100, or, divide by .02.

2. A commission merchant receives $225 for buying oats at 2 % commission. How much did he pay for the oats ? What was the entire cost to the principal ?

3. How much must a merchant send to his agent to buy grain and pay $540 as a 1½ % commission on the grain ?

4. What will Mr. Baker receive for his house and lot sold by an agent, who charged 2.5 %, or $150 for his services ?

658. Find the cost or the selling price, the rate and the commission being as follows:

Grains.

5 % is	4 % is	3 % is
1. $.06.	4. $.05.	7. $.06.
2. $.07.	5. $.06.	8. $.045.
3. $.09.	6. $.07.	9. $.036.

Coal.

2 % is	1 % is	½ % is
10. $.10.	13. $.085.	16. $.04.
11. $.15.	14. $.07.	17. $.035.
12. $.08.	15. $.065.	18. $.025.

Real Estate.

1½ % is	1¾ % is	1⅝ % is	2¼ % is
19. $37.50.	22. $105.70.	25. $ 85.50.	28. $340.00.
20. $19.20.	23. $176.40.	26. $ 56.70.	29. $520.71.
21. $82.50.	24. $168.91.	27. $152.85.	30. $776.90.

Ships.

2.75 % is	3½ % is	3.33⅓ % is	3.5 % is
31. $1,663.75.	34. $2,200.	37. $2,800.	40. $1,500.
32. $3,216.50.	35. $2,450.	38. $2,200.	41. $2,900.
33. $2,062.50.	36. $2,500.	39. $1,868.	42. $3,000.

The first 18 should be worked without the use of slates.

659. Find the sum sent by the principal in buying or received by him in selling in the examples in Art. 658.

660. Make problems for the numbers in Arts. 658 and 659.

661. Written Problems. 1. How many bushels of peaches were bought at $3.25 per bushel, when a commission of 1½ per cent. or $1.95 was paid to the agent?

2. An agent sells a flour-mill for $750, or 5 per cent. commission. What did the flour-mill sell for?

3. An auctioneer sold a consignment of goods, receiving $33.25 or 1¾ per cent. for his services. What was the amoun of his sales?

4. How many barrels of flour were sold, if each barrel commission of 3 per cent. or $21.75 was char

PROBLEM IV.

662. To find a number, the number plus or minus a part being given.

Cost Price $= ***Sum Sent by Principal*** $\div$ $\overline{\mathbf{1} + \textit{Rate}}$.

Selling Price $= ***Sum Received by Principal*** $\div$ $\overline{\mathbf{1} - \textit{Rate}}$.

1. What did a house and lot cost, if the purchaser gave $15,300 to cover the cost and the broker's commission of 2 %?

$15,300 is 102 %. Divide by 102 and multiply by 100; or, divide by 1.02.

2. If a man receives $14,202 as the proceeds of the sale of his grain after paying a commission of 2 %, for what sum was the grain sold?

$14,202 is 98 %. Divide by 98 and multiply by 100: or, divide by .98.

3. A shoe dealer remits to an agent $3,825.80 to invest in shoes, after retaining a commission of $1\frac{3}{4}$ per cent. How much was invested?

4. A has $12,625 returned, as the proceeds of a sale after paying a commission of $2\frac{1}{2}$ per cent. What was the full amount of the sales?

663. Make problems for the numbers in Art. 664, as in Arts. 663 and 665.

664. Find the cost price or the selling price, the rate per cent. and the sum sent in buying or the sum received by the principal in selling being as follows:

Buying Real Estate.		Selling Real Estate.	
5 % com.	1.5 % com.	5 % com.	2 % com.
1. $ 262.50.	6. $15,225.	11. $ 303.	16. $2,499.
2. $ 263.34.	7. $22,736.	12. $2,641.	17. $3,185.
3. $ 512.40.	8. $26,207.	13. $3,724.	18. $3,969.
4. $5,334.00.	9. $33,698.	14. $4,408.	19. $2,695.
5. $6,804.00.	10. $49,126.		20. $1,812

Buying Grain.		Selling Stocks, etc.	
5 % com.	2½ % com.	¼ % com.	⅛ % com.
21. $525.00.	*26.* $2,460.	*31.* $24,968.75.	*36.* $79,950.
22. $420.00.	*27.* $3,690.	*32.* $39,950.00.	*37.* $63,960.
23. $105.00.	*28.* $1,097.	*33.* $17,977.50.	*38.* $47,970.
24. $ 52.50.	*29.* $8,692.	*34.* $38,951.25.	*39.* $71,955.
25. $ 2.10.	*30.* $3,526.	*35.* $50,936.25.	*40.* $31,980.

The first 5 should be worked without the use of slates.

665. Written Problems. 1. Mr. Jones bought 24 yards of black silk, and charged 5 per cent. for his services. What did the silk cost per yard, if the whole amount was $56.70?

2. Mr. Taylor received $472.50 to buy 75 sheep, and to pay his commission of 5 per cent. How much did he pay for each sheep?

3. A builder is paid $260, with which to buy 25,000 feet of lumber, and retain his commission of 4 per cent. How much did the lumber cost per thousand feet?

4. A brick mason buys 14,400 bricks through an agent, paying in all $457.92, the agent receiving 6 per cent. How much per thousand did the bricks cost?

5. Mr. Dix paid $20,307.17 for ⅞ of a ship including the commission agent's charge of ¾ %. What was the value of the ship?

6. A stock-dealer bought 6 cows through an agent. He sent $525.30 to pay for the cows and a commission of 3 %. How much did each cow cost?

MISCELLANEOUS PROBLEMS.

666. Oral Problems. 1. How much will an agent receive for selling $600 worth of goods at 2 % commission?

2. If an agent buys 6 shares of R. R. stock at $100 per share and charges 3 % commission, how much does he receive?

3. For collecting a bill an agent charged 4 % or $60. How much did he collect?

4. Mr. Phillip retains \$8 or 2 % for collecting a bill. What was the amount of the bill ?

5. For buying \$500 worth of bonds an agent charges \$25. What was the rate of commission ?

6. Mr. Adams sends \$210 to an agent to buy goods and to cover a commission of 5 %. How much money did the agent expend for goods ?

7. An agent bought \$150 worth of cloth. If he charged a commission of 4 %, what amount of money was sent to him?

8. Mr. Johnson sells \$100 worth of fruit and sends to his principal \$97.50. What rate of commission was charged?

9. Mr. Martin sends \$636 to an agent for the purchase of a horse. If the agent deducts his commission of 6 % from this sum, what is the cost of the horse ?

10. If an agent sells 20 barrels of flour at \$6 per barrel and charges 5 %, how much money is due the principal ?

11. If an agent charges \$16 for collecting $\frac{2}{3}$ of a \$1,200 note, what rate of commission does he charge ?

12. If a broker charged \$10 or $\frac{1}{2}$ % for selling stock, what amount did he expend?

13. A real estate agent, with \$4,080, bought a lot, deducting his commission of 2 %. What did he pay for the lot ?

14. A broker charges \$2 for buying \$1,600 worth of stock. What is the rate of brokerage ?

15. If an agent charges \$12 or 3 % for buying goods, how many dollars' worth of goods does he buy ?

16. An agent, selling eggs, sent to his principal \$19, after deducting his commission of 5 %. What value did he sell ?

667. Written Problems. 1. An agent sold 15 barrels of flour at \$7.25, charging $2\frac{2}{3}$ per cent. commission. What was his commission ?

2. A horse trader sells a horse for \$175, receiving \$15 *commission. What* was the rate per cent. of commission ?

3. A broker bought 150 shares of railroad stock at $96 per share, the face value being $100. What was the amount he received, if he charged ¼ per cent. brokerage?

4. A Boston firm bought through an agent in Mobile 20,000 pounds of raw cotton @ $.06¼ per pound. If the firm paid $21.87½ commission, what rate did they pay?

5. A broker sold a consignment of cotton receiving $280 commission, at the rate of 1¾ per cent. What was the amount of the sale?

6. If a stock broker invests in a certain number of shares of railroad stock, receiving ⅛ per cent. commission or $45.00, what is the face value of the stock?

7. A broker was sent $8,010 to invest in N. Y. Central. His charges were ⅛ %. How many shares of stock did he buy?

8. A lawyer charged 10 % commission for collections. He collected $3,750 for a client. What was his commission?

9. A commission merchant sells 10 quintals of fish at $.11 per pound, charging 5 %. How much does he receive?

10. An agent bought for a Cincinnati firm 2,500 lbs. of coffee @ $.16⅔ a lb. What will he receive for his services, if he charges 3 per cent.?

11. Smith & Co. instructed their English agent to invest a certain sum of money in goods, paying him 4 per cent., which was $389.32. How many pounds sterling did he invest?

12. Mr. Williams sent an agent $812 to invest in chemicals for dyes. Allowing a commission of 1½ per cent. on the amount of the bill, how much was invested?

13. Mr. Davis sold a piece of property worth $21,000. He paid to the real estate agent 2½ per cent. for the sale. What was paid to the agent?

14. A farmer sells through a commission merchant 150 bushels of corn at $1.37½ a bushel and pays 2 per cent. commission. What does he pay the agent?

15. If gold is at $6\frac{1}{4}$ % premium, how much gold will \$6,375 in greenbacks buy ?

16. A factor sells a consignment of goods for \$1,800. He receives \$22.50 for his services. At what rate is he paid ?

17. What rate per cent. does a broker receive for selling 800 shares Union Pacific Railroad stock, quoted at \$75 a share, if he is paid \$100 brokerage ?

18. A French agent bought 500 dozen pairs of gloves in Paris for an American house. He was paid $1\frac{1}{2}$ per cent. or \$115.80 commission. How many Louis d'or did he invest in gloves ? How many francs ?

19. A lady bought silk at \$1.75 a yard. She charged $2\frac{1}{2}$ per cent. for her services. How many yards of silk did she buy with \$43.05, retaining her commission ?

20. An English agent sells goods for a N. Y. firm amounting to £5,000, charging them 4 per cent. commission. What is the amount of his commission in U. S. money ?

21. Mr. Wilson sent an agent in New Orleans \$3,500 to invest in sugar. The agent retained 4 % for services and \$10 for freight. How much was spent in sugar ?

22. A sent a consignment of goods to an agent to sell. The agent retained $1\frac{3}{4}$ per cent., or \$43.75, for his services. What was the amount of his sales ?

23. A commission agent invested a certain amount in groceries, receiving $2\frac{3}{4}$ per cent., or \$206.25 commission and \$16.75 guarantee. What was the amount invested ?

24. A broker bought 50 shares of railroad stock at \$85 per share, for which he received \$6.25 brokerage. He sold the shares for \$90 a share, and received \$12.50 brokerage. What rate did the broker charge for buying and selling ?

25. The proceeds of a sale of goods, after deducting $\frac{1}{8}$ per cent. and \$60 for various expenses, was \$4,000. What was the *entire amount sold ?*

STOCKS AND BONDS.

668. A **corporation** is a body created and authorized by law to transact a given business.

669. Stock is the capital invested in the business of a corporation.

670. A **share** is one of the equal parts into which the capital is divided.

Capital is generally divided into $100 shares; sometimes into $500, $50, or $25 shares. Unless otherwise specified, $100 shares are intended.

671. A **certificate of stock** is a corporation's paper acknowledging its indebtedness for a number of shares.

672. The **par value** of stock is the face value of one of its shares.

673. The **market value** of stock is the value at which a share is bought or sold in open market.

A stock is said to be *at par* when its market value *equals* its face value, *par* being the Latin word for equal. The expression, *at a premium*, means above par, and *at a discount*, below par.

674. A **dividend** is a percentage divided among the stockholders, as earnings on the capital invested.

Gross earnings are gross receipts ; net earnings are gross receipts less expenses.

675. An **assessment** is a percentage required by the corporation from the stockholders to cover losses, etc.

NOTE.—Brokerage or commission, dividends, assessments, premiums, etc., are computed on the face or par value of stocks or bonds ; and not on the buying or selling value.

676. A **bond** is a certificate of indebtedness issued by a corporation for money borrowed.

Bonds are of the denomination of $1,000, unless otherwise specified.

677. Bonds are generally named by naming the rate of interest together with the name of the nation, state, or corporation; thus, *U. S.* $4\frac{1}{2}$*'s; N. J.* 6*'s; N. Y. and N. H.* 5*'s.*

678. In applying PERCENTAGE to **Stocks and Bonds,** the ***par value*** is the *base;* the ***premium,*** the ***discount,*** or the ***dividend,*** etc., is the *percentage;* the ***market price*** is the *amount* or the *difference;* and the ***rate per cent.*** of premium or discount is the *rate per cent.*, as in percentage.

679. To find the cost of stocks or of bonds:

Multiply the market value including brokerage by the number of shares or of bonds.

NOTE.—Brokerage in stocks or bonds is computed on the face value. *In selling,* the principal receives the market value *minus* the brokerage; *in buying,* the principal pays the market price *plus* the brokerage.

1. How much did Mr. Brown pay for 24 shares of R. R. stock costing \$144 each, brokerage $\frac{1}{8}$ %?

$\frac{1}{8}$ % on face value, \$100, is \$$\frac{1}{8}$ ∴ 24 × \$144$\frac{1}{8}$ = ?

2. What is the cost of 560 shares of stock, at 28$\frac{3}{4}$ % premium, brokerage $\frac{1}{16}$ %?

3. Bought 212 shares of Telegraph stock at 8 % discount, $\frac{1}{4}$ % brokerage. What did the stock cost?

4. Mr. Rice engaged his broker to buy 56 shares of stock. If each share cost \$75$\frac{1}{2}$, and the brokerage was $\frac{1}{8}$ %, what was the entire cost to Mr. Rice?

5. If B sells 80 shares Ohio and Mississippi Railroad stock at 12$\frac{1}{2}$ per cent. discount, brokerage $\frac{1}{8}$ per cent., what does he receive for the stock?

6. On selling 50 U. S. 4$\frac{1}{2}$ % bonds at 109$\frac{7}{8}$, brokerage $\frac{1}{8}$ per cent., how much should the principal receive?

7. Bank stock bought at 15 per cent. discount, was sold at 101. *What was the gain,* brokerage $\frac{1}{8}$ % each way?

8. A man bought 150 shares of Missouri Pacific at 115, and sold 50 shares at 128, and the remainder at 136. How much was his gain, allowing $\frac{1}{16}$ for brokerage?

9. What profit is made by buying 95 shares of Rock Island R. R. stock at $112\frac{1}{2}$, and selling at $136\frac{1}{4}$, brokerage $\frac{1}{8}$ %?

10. If a speculator bought 500 shares of Union Pacific R. R. stock at $75\frac{5}{8}$, and sold at $71\frac{3}{4}$, how much did he lose, paying $\frac{1}{8}$ % brokerage?

11. How much is lost by selling 1,500 shares of stock at $87\frac{3}{8}$, if it cost $89\frac{1}{8}$, brokerage being $\frac{1}{16}$ %?

680. To find the number of shares of stock or of bonds.

Divide the entire cost by the cost of one share including the brokerage.

1. If insurance stock is sold at 4 % premium, how many shares can be bought for $4,170, brokerage being $\frac{1}{4}$ %?

2. At $3\frac{1}{2}$ % discount, how many shares of R. R. stock can be purchased for $19,325, brokerage $\frac{1}{8}$ %?

3. How many shares at $137\frac{1}{2}$ can be bought for $4,125?

4. If one share of bank stock costs $102\frac{3}{4}$ and the brokerage is $\frac{1}{4}$ %, how many shares can be bought for $6,157.50?

5. How many shares of stock at a discount of 15 %, with $\frac{1}{8}$ % brokerage, can be bought for $3,830$\frac{5}{8}$?

6. A broker sent his principal $56,437.50 on selling stocks at 13 % premium, and charging $\frac{1}{8}$ % commission. How many shares did he sell?

681. To find the amount of an investment having a given income.

Divide the entire income by the income from 1 share or bond, to find the number of shares, and multiply the result by the market value of one share.

income of $2,400 is derived from 4 % bonds, wha
nt invested, each bond being worth 104$\frac{1}{2}$?

2. How much must be invested in U. S. 3's at 107½ to afford a quarterly dividend of $750 ?

3. If stock, worth 109, yields 5 % semi-annually, how much must be invested to yield an annual income of $6,000, brokerage being ¼ % ?

4. How much must be invested in stock sold at 4 % discount and yielding 3 % to afford an annual income of $1,200 ?

PROBLEM I.

682. To find a part of a number.

Dividend = Par Value × Rate.

NOTE.—The market value equals the par value plus the premium or minus the discount.

1. What is the income from 250 shares of stock paying 5 % dividends ?

The face value of 250 shares is $25,000; .05 of $25,000 is $1,250.

2. If Mr. Matthews owns 145 shares of a 4½ % stock, what is his income from the stock ?

3. A man had $16,000 invested in manufacturing stock paying 6 %. He sold his manufacturing stock for $16,000 and bought 12 % bank stock at 159⅜, paying ⅛ % brokerage. How much did he increase his annual income ?

4. How much less than $1,000 per year is the dividend on two hundred and fifty 3½ % bonds ?

5. What is the income from 1,000 U. S. 3½'s and 2,500 shares of bank stock paying 7 % ?

PROBLEM II.

683. To find what part one number is of another.

Rate of Interest = Dividend ÷ Market Value.

1. What rate of interest does Mr. Bell's money earn if is invested in 8 % bank stock at 125 ?

If $125 earns $8, $1 earns 1–125th of $8, or .06⅖. Ans.,

2. At what rate is money invested, if it buys a 6 % stock at 96 ?

3. A man bought R. R. stock at 75⅞, brokerage ⅛ %. What rate of interest does he receive, if the stock declares a 5 % dividend ?

4. If stock is bought at 116½, brokerage ⅛, what is the rate of income, a dividend of 7 % being declared ?

5. R. R. stock bought at 12 % discount and $\frac{1}{16}$ % brokerage, declares 5 % dividends. What is the rate of interest paid on the investment ?

PROBLEM III.

684. To find a number, a part being given.

Market Value = Dividend ÷ Rate of Investment.

NOTE.—The market value equals the par value plus the premium, or minus the discount.

1. If a 5 % stock is bought so as to pay 6 % on the investment, what is the market value of the stock ?

If $5, the dividend on 1 share, is 6 % of the market value, 1 % is ⅙ of $5, and 100 % is 100 times as much. Divide by 6 and multiply by 100; or, divide by .06.

2. What is the market value of an 8 % stock earning 12 % on the investment ?

3. How much above par is a 5 % stock that earns 4 % on the investment ?

First find the market value.

4. How much below par is a 3 % bond that earns 3½ % on the investment ?

5. If a stock declares 8 % dividends, and pays 6¾ % income, is it above or below par, and how much ?

6. If a stock declares 7 % dividends, and pays 8 % income, is it above or below par, and how much ?

7. What is the market value of a 10 % stock that earns 12 % on the investment? That earns 8 % ?

PROBLEM IV.

685. To find a number, the number plus or minus a part being given.

$$\textit{Cost Value} = \textit{Market Value} \div \begin{cases} 1 + \textit{Rate of Gain}, \text{ or,} \\ 1 - \textit{Rate of Loss}. \end{cases}$$

1. By selling stock at 84 a man makes 5 % on his investment. How much did he pay for the stock?

He sold for \$84 or 105 %. \$84 ÷ 1.05 = \$80 or cost.

2. By selling stock at 105, 25 % is gained on the investment. How much was paid for the stock?

3. Mr. Williams sold his stock at 96 a share, thereby losing 20 % on his money. How much did the stock cost him?

4. If there is a loss of 10 % on stock sold for 99, what was the cost value?

5. A man gains 30 % on 20 shares of stock sold for \$2,340. What did the stock cost him per share?

6. If 20 % is lost on stock by selling it for 92, what was the cost of 35 shares?

MISCELLANEOUS PROBLEMS.

686. Oral Problems. 1. What is the annual dividend on 25 bonds yielding 4 %?

2. Mr. Clark bought 6 shares of R. R. stock at par, and sold them at 15 % premium. What did he gain?

3. If a man receives \$24 dividend on 8 shares of R. R. stock, what is the rate of dividend?

4. What rate of dividend is paid on \$100 stocks yielding \$4½ each?

5. What rate of interest does a man's money pay, if he *invests it in 6 % bonds* at 120?

6. If money is invested in 2 % stock at 150, what rate of interest does the money pay ?

7. What is the market value of 4 % stock, which pays 8 % on the money invested ?

8. The market value of stocks on which 50 % is gained is 120. What was the cost ?

9. A man sells stock at 90, gaining 20 %. What did it cost ?

10. How much is paid for stock, if the semi-annual dividend of 3 % on the investment is $4 per year ?

687. Written Problems. 1. What is the income on 825 shares of stock yielding 6 % dividend ?

2. What is the discount on 45 shares of R. R. stock at 84 % ?

3. Bought bonds at 20 % discount. If they yield 4 % dividend, what rate of interest do they bear ?

4. If 6 % bonds bear 4 % interest, what is their value ?

5. What is the market value of $4\frac{1}{2}$ % stock, which pays 6 % on the investment ?

6. A invests $4,650 in stock at 93 %. What will be his loss if the stock falls $\frac{1}{2}$ per cent. ?

7. A man sold $5,000 of 6 per cent. stocks at 72, and bought 8 % stock at 90. What is his annual income from the 8 % stocks ?

8. If 110 shares of 4 % stock bought at 92 are changed to 5 per cent. stock at 110, what is the increase in income ?

9. What is the gain on 1,200 shares of stock bought at 84 and sold at 88, brokerage being $\frac{1}{8}$ % each way ?

The cost is $84 + \frac{1}{8}$, or $84\frac{1}{8}$, and the selling price is $88 - \frac{1}{8}$, or $87\frac{7}{8}$. The gain on one share is the difference between $87\frac{7}{8}$ and $84\frac{1}{8}$, or $3\frac{3}{4}$. If $3\frac{3}{4}$ are gained on one share, $1,200 \times \$3\frac{3}{4}$, or $4,500, will be gained on 1,200 shares.

10. On how many shares of stock will $6,075 be gained, if e stock is bought at 45 and sold at 52, brokerage being $\frac{1}{8}$ %

INSURANCE.

688. Insurance is assurance against loss.

Fire insurance is *insurance* against loss by fire; *life insurance*, against loss by death; *marine insurance* against loss at sea; *accident insurance*, against loss by accident, etc.

689. The **policy** is the written contract given by the insurance company to the party insured.

690. The **premium** is the sum of money paid to the party insuring for the insurance.

691. In applying PERCENTAGE to **Fire Insurance** and to **Marine Insurance,** the ***amount of insurance*** is the *base;* the ***premium*** is the *percentage;* the ***amount of insurance plus or minus the premium*** is the *amount*, or the *difference;* and the ***rate per cent. of insurance*** is the *rate per cent.*, as in percentage.

The insured sometimes insures the value of his property plus the premium (*i. e. the amount*); the loss of the insurance company is the amount of the insurance minus the premium (*i. e. the difference*).

692. Life Insurance and Accident Insurance are not applications of percentage, but problems in multiplication and division.

The premium in Life Insurance is so many dollars on the $1,000, at a given age; and in accident insurance, so many dollars for $1,000, $3,000, or $5,000, for a given time.

PROBLEM I.

693. To find a part of a number.

Premium = Amount of Insurance × Rate.

1. What premium is paid for $10,000 of insurance at ¾ %?

1 % is $100; ¾ % is ¾ of $100 or $37½.

PROBLEM II.

694. To find what part one number is of another.

Rate = Premium ÷ Amount of Insurance.

NOTE.—Problems II., III. and IV. may be deduced from Prob. I.

1. If \$45 is paid for insuring property for \$3,000, what is the rate?

\$45 is $\frac{45}{3000}$ or .01½ of \$3,000.

2. What is the rate when \$12 is paid for \$1,500 insurance?

PROBLEM III.

695. To find a number, a part being given.

Amount of Insurance = Premium ÷ Rate.

NOTE.—The amount of the insurance is frequently a fractional part of the value.

1. For what sum is a ship insured, if the rate is 2½ %, and the premium \$500?

Divide by 2½ to find 1 %; multiply by 100 to find 100 %; or, divide by .02½.

PROBLEM IV.

696. To find a number, the number plus or minus a part being given.

$$\textit{Amount of Insurance} = \begin{cases} \textit{Amount} \div \overline{1 + \textit{Rate}}, \text{ or,} \\ \textit{Difference} \div \overline{1 - \textit{Rate}}. \end{cases}$$

1. For what sum was property insured, if the rate being 2½ %, the net sum received by the insured, the property having been destroyed by fire, was \$48,750?

The net receipts were 100 % less 2½ %, the rate of premium.

2. A merchant insured his ship, the policy covering the premium. The ship having been lost at sea, he received *\$180,687.50.* For what sum did he insure, the rate being 3¼ %?

3. The net loss to a fire insurance company by a fire was $63,979.25. The rate of insurance being $1\frac{1}{8}$ %, what was the amount of the insurance?

4. A dry goods merchant having lost his property by fire, received $148,650, having paid premium for 2 years. The rate per cent. for insurance being $.4\frac{1}{2}$, what was the amount of his insurance?

MISCELLANEOUS PROBLEMS.

697. Oral Problems. 1. How much will it cost to insure a factory worth $40,000 at 2 %?

2. What is the premium for insuring a man's life for $10,000, at $20 per thousand?

3. If $1 is paid for insuring a piano worth $500, what is the rate of insurance?

4. $600 premium is paid on a vessel insured at 3 %. What is the amount of insurance on the vessel?

5. If $5 premium is paid for insuring property worth $4,000, what is the rate of insurance?

6. A man whose property was destroyed by fire received from the insurance company, after deducting the premium, $990. If the rate of insurance was 1 %, what was the amount of insurance?

7. If a man pays $150 for insuring merchandise worth $10,000, what is the rate of insurance?

8. What is the cost of 5 years' insurance on property worth $5,000 at $\frac{2}{5}$ %, including a policy for $1?

9. At $\frac{3}{8}$ %, the cost of insuring a house was $24. For how much was the house insured?

10. If the amount insured, increased by the premium, was $408, and the rate 2 %, what was the amount insured?

11. The premium for insuring a stable and its contents at $1\frac{1}{4}$ % was $70. What was the amount of the insurance

12. If $816 covers the sum insured and the premium at 2 %, what is the amount insured ?

698. Written Problems. 1. What is the premium for insuring a house worth $12,540 for $\frac{3}{4}$ of its value, at $\frac{3}{8}$ % ?

2. Mrs. Walker's furniture, worth $720, is insured at $\frac{3}{8}$ %. In case of a fire, what is her loss ?

3. If a man pays $75 premium on his house insured for $\frac{1}{6}$ of its value at $\frac{3}{4}$ %, what is the entire value of the house ?

4. A grocer insured his stock, worth $35,600, for 10 years at $\frac{3}{4}$ %. What was the premium ?

5. Paid $16.81, including $1 for policy, for insuring 124 barrels of flour at 8\frac{1}{2}$ per barrel. What was the rate of insurance ?

6. What is the value of a house insured for $\frac{3}{4}$ of its value for 5 years at $\frac{2}{5}$ %, if the premium for that time is $127.50 ?

7. A ship-owner insures his vessel, worth $160,000, for $\frac{7}{8}$ of its value. What rate of insurance does he pay, if the premium is $6,000 ?

8. For what amount should a house worth $3,940 be insured at $1\frac{1}{2}$ %, to cover its full value and the premium ?

9. $105.40 is paid for insuring 620 tons of coal, worth $8.50 per ton, for $\frac{4}{5}$ of their value. What is the rate of insurance ?

10. The loss to an insurance company on a house insured for $\frac{3}{4}$ of its value for one year at $\frac{3}{8}$ % was $4,622.10. What was the value of the house ?

11. Mr. Perkins paid $99 premium for life insurance for 3 years. If this was $1\frac{3}{8}$ % of the amount of his insurance, for[illegible] was his life insured ?

[illegible]aying insurance on a house for 4 years at $\frac{7}{8}$ %, [illegible] by fire. If the house was insured for $\frac{4}{5}$ of its [illegible] loss to the insurance company was $7,720, [illegible] value of the house?

TRADE DISCOUNT.

699. The *discount* or *percentage* deducted by merchants from the marking price of goods is called **mercantile** or **trade discount.**

The *trade discount* is so many per cent. from the marking price, as, 5 % for cash payment, or so many per cent. off for goods bought by the trade; that is, by retailers, wholesalers, jobbers and manufacturers; as, 20 % off; 20 and 10 % off; 10, 10, and 10 %; 50, 33⅓, and 25 %.

700. The trade gives different discounts to different purchasers; as to one, 20 % off; to another, 20 and 10; that is, 20 % off from the marking price and 10 % from the remainder; and to another, 10, 15, and 20; that is, 10 off from the marking price, 15 off from the remainder, and 20 off from the second remainder, etc.

701. In applying PERCENTAGE to **Trade Discount,** the ***selling price*** or the ***cost price*** is the *base;* the ***discount*** is the *percentage;* and the ***cost price*** or the ***selling price*** is the *difference* or the *amount,* the ***rate per cent.*** being generally the result of combining a succession of rates.

702. The principal operations in Trade Discount involve a succession of discounts in finding the cost price, the selling price, the marking price, or the entire rate of discount, according to the following

703. Rules.—I. The selling price equals the continued product of the marking price and the remainders found by subtracting the discounts, each from 100 %.

Thus: What is the selling price of goods marked at $.75, and sold at a *discount of* 10, 20, and 15 %? Ans., $75 × .90 × .80 × .85.

704. II. The marking price equals the selling price divided by the continued product of the remainders found by subtracting the discounts, each from 100 %.

NOTE.—The selling price is given or found by multiplying the cost price by 1 plus the rate of gain.

705. Written Problems. 1. A bill of dry goods amounting to $625 on 30 days credit had 12 per cent. discounted for cash. How much was the discount?

2. A builder bought lumber to the amount of $500 on 60 days time, 10 per cent. off for cash. What was the net value of the lumber?

3. The marked price of a piano was $525. What was the net price, if 30 per cent. was deducted for cash?

4. A bought goods billed at $1,650, on 90 days credit, a discount of 10 per cent. and 4 per cent. off for cash. What is the net value of the goods?

5. A bill of $850 was discounted 8 and 5 per cent. What was the discount and the amount paid?

6. What is the difference between 20 per cent. discount and 10 per cent. and 10 per cent. on $2,000?

7. What will be the difference, if $6,000 is discounted at 15 % and 10 %, or 10 % and 15 % off?

8. If a book cost $.06 less when discounted 50 per cent. than when discounted 40 % and 10 %, what is the marking price?

9. If the selling price is $.56 after 30 per cent. and 20 % are deducted, what is the marking price?

10. Paid $513 for a bill of goods after a deduction of 10 % and 5 % had been made. What was the amount of bill?

11. A bill of $1,000 was discounted 10, 5, and 3 %. What was the amount paid?

,250 is paid for an account after a deduction of
. 10 % has been made, what was the original

TAXES.

706. A **tax** is a sum levied on the property or on the persons within a district or territory for the support of the government.

In the U. S., the tax may be a U. S. tax., or a State, city, county, or school district tax. The U. S. tax is not a *direct* tax on person or property, but on imports, exports, manufactures, etc.

707. A **property tax** is a tax levied on property, whether personal property or real estate.

Personal property is that which is movable ; real estate, that which is fixed.

708. A **personal tax** is a tax levied on the person.

It is so much, generally $1 or $2 per head, and it is, therefore, called a *poll* tax, polle being the German for head. The tax is also said to be *per capita* (Latin), meaning *by the head;* it is assessed on voters.

709. In applying PERCENTAGE to **Taxes,** the ***valuation*** of the property is the *base;* the ***tax*** is the *percentage;* and the ***tax rate*** is the *rate,* as in percentage.

The property tax is generally so many mills on the dollar, a mill being .1 %.

710. Written Problems. 1. Mr. Clayton has property valued at $16,000 in a town whose tax is $6,700 on property valued at $800,000, and including 1,500 polls at $2 each. What tax does he pay ; and what is the rate per cent. of tax ?

2. Mr. Bell's tax is $3,800. The rate being 4 %, what is the assessed value of his taxable property ?

3. The school tax in a country district is $875 on property valued at $218,750. A farmer in the district has property worth $5,000. What tax does he pay ?

4. The tax assessment in a town of 15,000 people is $51,000. Property is valued at $2,406,391 ; with 3,000 polls at $1 each. A collector is paid $\frac{1}{4}$ per cent. What is the rate per cent. of the tax ?

5. A district containing 500 polls, each assessed at $1, has a tax levy of $882, and pays a collector 2 per cent. The property assessed is valued at $25,000. What is A's tax, and the rate per cent., if his property is worth $1,250 ?

6. A school-house being built at a cost of $14,998.50, the district was assessed 8 mills on the dollar, the collector being paid 1 %. What was the value of the taxable property ?

DUTIES AND CUSTOMS.

711. A **duty** is any sum of money required by the government to be paid on the importation, exportation, manufacture, or consumption of goods.

Customs are duties levied on the importation and exportation of goods.

712. Duties are either *ad valorem* or *specific*.

713. Ad valorem duty is a tax levied on the cost of goods, estimating the cost in the country from which they are exported.

714. Specific duty is a tax levied on the weight or measure of goods exported or imported, without regard to cost.

NOTE.—Some goods are subjected to both a specific and an ad valorem duty.

715. In applying PERCENTAGE to **Duties**, the ***value***, the ***weight***, or the ***price***, is the *base* ; the ***duty*** is the *percentage* ; and the ***rate*** of duty is the *rate*, as in percentage.

fore estimating duties, the following allowances are made: the weight of boxes, casks, etc., **leakage**, for liquid **reakage**, for liquids in bottles.

716. Written Problems. 1. Imported an invoice of embroidery valued at £500; an allowance of 15 per cent. was made for damage received by shipment, the ad valorem duty being 25 per cent. What was the duty paid?

2. If the specific duty on 100 boxes of tin of 125 pounds each, tare $1\frac{1}{2}$ per cent., is 6 ¢ per lb., how much is the duty?

3. What is the specific duty on 350 yards of silk invoiced at 10 francs per meter, each meter being 3 feet $3\frac{3}{8}$ inches long, at $.75 a meter?

4. Invoiced 50 pieces of linen, each containing 24 yards, at 2 shillings per yard. If the ad valorem duty is 20 %, what will be the duty in pounds sterling? What in U. S. money?

5. Imported 100 casks of raisins, each containing 45 lb., duty $.0193 per lb., tare 12 %. What is the specific duty?

6. Imported 100 gross bottles of porter. If duty is $.50 per dozen, breakage 5 per cent., what is the amount of duty?

7. What is the ad valorem duty at 25 per cent. on an invoice of 60 chests of tea, containing 50 pounds each, at $.40 per pound, tare being 4 pounds per chest?

8. An importer buys 75 watches at Geneva: 60 are bought for 30 francs each, the balance at 50 francs each. What is the ad valorem duty at 40 per cent. in francs? In dollars?

9. What is the specific duty at 4 ¢ per lb. on 100 bags of Rio coffee, containing 56 pounds each, tare being 2 per cent.?

10. If a duty of $453.60 is paid on an invoice of alcohol at $.80 per gallon, a leakage of 10 per cent. being allowed, how many hogsheads were bought?

11. If clocks cost 15 francs each in France, what is the duty at 50 % in dollars?

12. What is the ad valorem duty at 30 % on an invoice of 150 sacks of coffee, containing 100 pounds each, at 20 cents *per pound, tare being* 2 pounds per sack?

INTEREST.

717. Interest is money paid for the use of money.

718. The **principal** is the sum of money on which interest is paid.

719. The **rate of interest** is the rate per cent. charged for one year's use of the principal.

720. The **amount** is the sum of the principal and the interest.

721. Interest is either *simple, compound,* or *annual.*

Simple interest is intended, unless otherwise specified.

722. Simple interest is the interest for the given time on the principal only.

723. Compound interest is interest on principal and interest, each year's interest increasing the principal for the next year.

724. Annual interest is interest on the principal, and each year's interest for the remainder of the term, or time.

The interest on interest is computed but once, and then from the end of the year in which it is earned for the remainder of the term.

725. Exact interest is interest computed for the exact number of days in the year and in the term for which the interest is paid.

When time is counted, 30 days to the month and 12 months to the year, the interest will not be exact, exact interest being for 365 or for 366 days in the year.

NOTE.—Although for short periods it is customary to use the exact [n]umber of days, exact interest (365 days to the year) is computed [u]nless otherwise specified.

GENERAL METHOD.

726. 1. A merchant borrowed \$1,250 at 5 % for 3 years 7 months 6 days. What was the interest ?

\$1,250	= Principal.	
.05		
12) \$62.50	= Int. for 1 yr.	
30) \$5.20⅚	= " " 1 mo.	
\$.173 11/18	= " " 1 da.	
\$187.50	= Int. for 3 yr.	
36.46	= " " 7 mo.	
1.04	= " " 6 da.	
\$225.00	= " " 3 yr. 7 mo. 6 da.	

Explanation.—1. The rate being 5 %, the interest for 1 year is .05 of the principal, or \$62.50.

2. The interest for one month is 1/12 of the interest for 1 year, or \$5.20⅚.

3. The interest for 1 day is 1/30 of the interest for one month, or \$.173 11/18.

4. The entire interest is 3 × \$62.50 or \$187.50, the interest for 3 years ; plus 7 × \$5.20⅚ or \$36.46, the interest for 7 months; plus 6 × \$.173 11/18 or \$1.04, the interest for 6 days. Ans., \$225.00.

Or,

\$1,250	= Principal.	
.05		
\$62.50	= Int. for 1 yr.	
3		
\$187.50	= Int. for 3 yr.	
31.25	= " " 6 mo.	
5.20⅚	= " " 1 mo.	
1.04⅙	= " " 6 da.	
\$225.00	Ans.	

Explanation.—1. The interest for 1 year is found by taking .05 of the principal.

2. The interest for 3 years is 3 times \$62.50, the interest for one year.

3. The interest for 6 months or half a year is one half of \$62.50, the interest for 1 year.

4. The interest for 1 month is one-sixth of \$31.25, the interest for 6 months.

5. The interest for 6 days is one-fifth of \$5.20⅚, the interest for 1 month. Ans. \$225.

NOTE.—Separate the given number of months or of days into parts, each of which is an easy fractional part of the years, months, or days previously found.

2. What was the interest on \$2,500 at 5 %, for 5 years 5 months 5 days ?

3. How much will the use of $1,440 cost for 4 years 4 months 15 days, at 6½ %?

4. How much interest will accrue on $9,000 in 6 years 3 months 6 days, at 8 %?

5. If a man lends $3,600 for 5 years 9 months 24 days at 7 %, how much interest does he receive?

6. A capitalist invests $4,500 in such a way that it bears 9 %. How much does he gain in 2 years 8 months 18 days?

7. If the principal is $5,040, the rate 5 %, and the time 10 years 11 months and 26 days, what is the interest?

PROBLEM I.

727. To find a part of a number.

Interest = Principal × Rate × Time in Years.

NOTE.—If the time is expressed in months, divide the result by 12; if in days, divide by 360 (or by 365 in exact interest), to reduce the time to years.

1. What is the interest on $2,225 for 4 yr. 6 mo., at 8 %?

Explanation.

$2,225 .08 $178.00 4.5 89000 71200 $801.000	1. Find .08 of the principal for 1 year's interest. 2. Multiply by 4.5 to find the interest for the entire time.	**Or,** Find the continued product of the principal, the rate, and the time in years.	$2,225 .08 4.5 $2,225 .36 13350 6675 Ans., $801.00

NOTE 1.—Express the time in years when it can be readily written as years and the decimal part of a year.

NOTE 2.—If there is a fraction in either the principal, the rate, or the time, it may be reduced to an improper fraction, in which case the denominator may be written on the left of the line as a divisor.

2. What is the interest on $3,200 for 3 yr. 3 mo., at 7 %?

3. If a man lends $5,800 for 5 yr. 4 mo. at 5 %, how much should he receive?

4. How much interest is due on $1,500 at 8 % for 9 months 6 days?

Explanation.

1. Find .08 of the principal for 1 year's interest.
2. Divide by 12 to find the interest for 1 month.
3. Divide the number of days taken by 3 to reduce the days to tenths of a month.
4. Multiply the interest for 1 month by the number of months.

Or, Find the continued product of the principal, the rate, and time in months and the decimal of a month, and divide by 12. Ans., $92.

$1,500
.08
12)$120.00
$10.00
9.2
2000
9000
$92.000

12 | 125
$1500
.08
9.2
$125
.736
750
375
875
$92.000

NOTE.—Express time in months when it can be readily written as months and a decimal part of a month. 3 days equal .1 of a month.

5. Find the interest on $2,800 at 7 % for 7 mo. 12 da.

6. What is the interest on $3,375 at 9 % for 10 months 24 days?

7. How much interest will $1,000 gain in 19 days at 5 %?

Explanation.

1. Find .05 of the principal for 1 year's interest.
2. Divide by 12 to find the interest for 1 month.
3. Divide by 2 to find the interest for 15 days.
4. Divide by 5 to find the interest for 3 days.
5. Divide by 3 to find the interest for 1 day.
6. Add the last three quotients for the answer.

Or, Find the continued product of the principal, the rate, and the time in days, and then divide by 360. Ans., $2.64.

$1000
.05
12)$50.00
2)$4.16⅔
5)$2.08⅓
3)$ .41⅔
$ .14
$2.64

9 | 25
$1000
.05
360 | 19
1.25
19
1125
125
9)23.75
$2.64

8. *What is the interest* on $4,200 at 8 %, for 22 day

9. If a man loans $2,850 at 6½ % for 14 days, how much interest should he receive?

728. Read at sight the interest on the following sums with time and rate, as given:

2½ yr. @ 4 %	4½ yr. @ 2 %	3½ yr. @ 3 %	3 yr. @ 4½ %	6 yr. @ 6½ %
1. $ 4.	4. $ 40.	7. $300.	10. $ 3.	13. $ 8.
2. $ 6.	5. $200.	8. $150.	11. $ 2.	14. $20.
3. $12.	6. $120.	9. $225.	12. $50.	15. $50.

729. Find the interest on the following sums at the given rate and time in months:

4 %, 10 mo. 3 da.	6 %, 9 mo. 9 da.	8 %, 8 mo. 18 da.	9 %, 3 mo. 27 da.	6½ %, 5 mo. 21 da.
1. $ 650.	4. $1,200.	7. $480.	10. $1,400.	13. $840.
2. $1,025.	5. $2,850.	8. $950.	11. $5,060.	14. $876.
3. $2,500.	6. $3,115.	9. $720.	12. $6,055.	15. $912.

730. Find the interest for the given rate and time in days on the following:

6 %, 45 days.	8 %, 30 days.	7 %, 60 days.	9 %, 2 mo. 14 da.	4½ %, 1 mo. 10 da.
1. $500.	3. $1,000.	5. $4,200.	7. $8,000.	9. $2,700.
2. $360.	4. $1,250.	6. $5,460.	8. $4,520.	10. $8,136.

1 or 2 months may be treated as 30 or 60 days.

6 % METHOD.

731. The interest on $1 at 6 %

For 1 year = 6 cents or .06 of the principal.
2 months = 1 cent " .01 " "
1 month = 5 mills " .005 " "
6 days = 1 mill " .001 " "

732. Find the interest on $1 at 6 % for

1. 4 mo. 12 da. = $.02 + $.002 = $.022.
2. 9 mo. 20 da. = $.045 + $.00$3\frac{1}{3}$ = $.048$\frac{1}{3}$.
3. 1 yr. 10 mo. 25 da. = $.11 + $.00$4\frac{1}{6}$ = $.114$\frac{1}{6}$.
4. 11 mo. 27 da. = ?
5. 1 yr. 7 mo. 14 da. = ?
6. 2 yr. 6 mo. 15 da. = ?
7. 3 yr. 11 mo. 21 da. = ?

733. Find the interest:

1. Find the interest on $785 for 5 yr. 7 mo. 11 da. at 6 %.

Explanation.—1. By table Art. 731 the interest on $1 for 5 yr. = $.30 or .30 of principal.
7 mo. = .035 " .035 "
11 da. = .001⅚ " .001⅚ "
$.336⅔ " .336⅔ "

$785
.336⅔
6)3925
654⅙
4710
2355
2355

2. The interest on $785 = .336⅔ × $785 = $264.414⅙

2. Find the interest on $630 for 3 yr. 9 mo. 16 da. at 6 %.

For 3 yr. = .18
9 mo. = .045
16 da. = .002⅔
.227⅔

$630
.227⅔
210
210
4410
1260
1260
$143.430

3. Find the interest on $1,225 for 4 yr. 5 mo. 9 da. at 6 %.

For 4 yr. = .24
5 mo. = .025
9 da. = .001¼
.266¼

.266¼ × $1,225 = ?

4. Find the interest on $2,550 for 5 years 3 months 7 days.

734. Find the interest by the 6 % method:

	Principal.	Time.		Principal.	Time.
1.	$ 480,	1 yr. 3 mo. 7 da.	7.	$ 5,250,	6 yr. 10 mo. 20 da.
2.	$ 525,	1 yr. 9 mo. 11 da.	8.	$ 7,140,	5 yr. 6 mo. 14 da.
3.	$1,050,	3 yr. 5 mo. 3 da.	9.	$ 8,025,	7 yr. 2 mo. 19 da.
4.	$1,360,	2 yr. 7 mo. 13 da.	10.	$10,570,	8 yr. 1 mo. 21 da.
5.	$2,500,	4 yr. 11 mo. 9 da.	11.	$12,090,	9 yr. 3 mo. 25 da.
6.	$3,100,	5 yr. 9 mo. 17 da.	12.	$11,225,	10 yr. 4 mo. 24 da.

Find the amounts in the preceding.

735. To find the interest for months by the 6 % method.

The interest on $1 *is half the number of months considered as cents, or hundredths of the principal.*

NOTE.—*The interest on* $1 for 1 month is one-half of a cent.

736. Find the interest:

1. What is the interest on \$1,600 for 1 yr. 4 mo., at 6 %?

\$1,600
.08
\$128.00

Explanation.—The number of months is 16. The interest on \$1 is \$.08; on \$1,600 it is 1,600 × \$.08.
Or, the interest is ($\frac{1}{100}$ of 16 =) .08 of the principal = .08 × \$1,600 = \$128.

Principal.	Time.	Principal.	Time.
2. \$800,	1 yr. 8 mo.	5. \$1,200,	1 yr. 3 mo.
3. \$528,	1 yr. 9 mo.	6. \$1,854,	1 yr. 4 mo.
4. \$640,	2 yr. 6 mo.	7. \$2,058,	1 yr. 8 mo.

737. Find the amounts in the preceding.

738. To find the interest for days by the 6 % method:

The interest on \$1 *is one-sixth the number of days considered as mills, or thousandths of the principal.*

NOTE.—The interest on \$1 for one day is $\frac{1}{6}$ of a mill.

1. What is the interest on \$3,250 for 2 months 12 days?

\$3,250
.012
\$39.000

Explanation.—The number of days is 72. The interest on \$1 is \$.012; on \$3,250 it is 3,250 × \$.012.
Or, the interest is ($\frac{1}{6}$ of 72 =) .012 of the principal = .012 × \$3,250 = \$39.

Principal.	Time.	Principal.	Time.	Principal.	Time.
2. \$240,	24 days.	5. \$635,	1 mo. 3 da.	8. \$1,050,	1 mo. 18 da.
3. \$175,	27 days.	6. \$849,	1 mo. 5 da.	9. \$2,250,	2 mo. 9 da.
4. \$346,	36 days.	7. \$715,	1 mo. 9 da.	10. \$3,126,	1 mo. 15 da.

Find the amounts in the preceding.

739. To find the interest at any per cent. by the six per cent. method, first find the interest at 6 %, then divide by 6, and multiply by the required number; or, find that part of the interest which the required per cent. is of 6 %.

$\frac{1}{2}$ of 6 %; 2 % = $\frac{1}{3}$ of 6 %; 4 % = 6 % − $\frac{1}{3}$ of 6 %; 5 % = 6; 7 % = 6 % + $\frac{1}{6}$ of 6 %; 8 % = 6 % + $\frac{1}{3}$ of 6 %; 9 % = 6 %.

740. Find, by the 6 % method, the interest on the following:

1. $784.11 for 9 mo. 21 da. at 9 %.
2. $8,000 for 15 months at 4½ %.
3. $1,154.87 for 75 days at 4 %.
4. $900 for 11 months at 4½ %.
5. $480 for 5 mo. 24 d. at 8 %.
6. $575 for 93 days at 5 %.
7. $800 for 115 days at 7 %.
8. $9,000 for 63 days at 10 %.
9. $240 for 2 years 10 months 16 days at 6 %.
10. $330 for 1 year 7 months 5 days at 7 %.
11. $1,200 for 3 years 6 months 17 days at 4 %.
12. $4,133.24 for 1 year 8 months 3 days at 6 %.
13. $1,671.19 for 4 years 1 month 18 days at 5 %.
14. $1,791.50 for 7 years 3 months 9 days at 6 %.

Find the answers to the nearest number of cents.

741. Find the amounts in the preceding.

6 % METHOD BY DAYS.

Multiply by the number of days and divide by 6,000.

742. 1. What is the interest on $24,000 for 7 days at 6 % ?

$$\$24000 \times \frac{6}{100} \times \frac{7}{360} =$$

$$\$24000 \times \frac{7}{6000} = \$28$$

Explanation.—1. The interest for 1 year is 6 % = .06 = $\frac{6}{100}$. Multiply $24,000 by $\frac{6}{100}$ to find the interest for 1 year.

2. Counting 360 days to the year, 7 days is $\frac{7}{360}$ of a year. Multiply the interest for 1 year by $\frac{7}{360}$ to find the interest for 7 days.

3. In cancelling, it is seen that the interest at 6 % for 1 day is found by multiplying the given sum by $\frac{1}{6000}$, or by dividing by 6,000, the interest for the given number of days being found by multiplying this result by 7, the number of days.

743. To find the interest at 6 % for any number of days, multiply by the number of days and divide by 6,000 (or by *1,000 and by 6).*

744. Find the interest for the given number of days on the following:

24 days.	25 days.	28 days.	30 days.
1. $6,500.	4. $145.	7. $ 9,000.	10. $400.
2. $5,680.	5. $250.	8. $10,500.	11. $880.
3. $8,340.	6. $369.	9. $12,240.	12. $900.

NOTE.—Interest at any other than 6 % may be found as in Art. 739.

745. Find the amounts in the preceding.

EXACT INTEREST.

Subtract $\frac{1}{73}$ from the simple interest.

746. 1. Find the exact interest on $2,000 for 25 days at 4 %.

Explanation.—1. Divide .025 by 6 to find the interest on $1 for 25 days at 6 %. Divide by 3 to find it at 2 %. Subtract to find it at 4 %.

2. Multiply by 2,000 to find the interest on $2,000.

3. This is simple interest, counting 360 days to the year. In exact interest, 365 days are counted to the year. The answer is $\frac{5}{365}$ or $\frac{1}{73}$ too large. To find the exact interest, then, subtract $\frac{1}{73}$ of the simple interest.

6) .025
3) .004$\frac{1}{6}$ Int. on $1 @ 6 %.
.001$\frac{7}{18}$ " " 2 %.
$.002$\frac{7}{9}$ " " 4 %.
2000
73)5.55\frac{5}{9}$
.076+
$5.48. Ans.

Or; 1. Multiply the principal by the rate by the number of days, divided by 365.

2. Work by cancellation.

$$\frac{\overset{400}{\cancel{\$2,000}} \times 4 \times \cancel{25.}}{\cancel{100} \times \cancel{365}_{73}} = \frac{400}{73} = \$5.48. \text{ Ans.}$$

Exact interest is to be found only when specified. It is used, however, by the U. S. government and by England, and by many banks.

2. Find the exact interest on $1,050.60 for 8 mo. 19 da. at 6 %.

3. What is the exact interest on a mortgage for $2,500 for [illegible] da. at 7 %?

[illegible] the exact interest at 5 %, on a note for $5,000 1886, and paid March 17, 1886?

747. What is the exact interest on the following?

4 %, 75 da.	4½ %, 63 da.	6 %, 93 da.	7 %, 115 da.
1. $140.65	4. $590.	7. $ 956.75.	10. $1,263.
2. $620.	5. $375.80.	8. $1,125.	11. $1,781.50.
3. $367.	6. $715.	9. $1,339.	12. $1,540.

748. Find the amounts in the preceding.

PROBLEM II.

749. To find the part that one number is of another.

Rate = Interest ÷ Interest at 1 %.

NOTE.—The amount and the principal being given, the interest may be found by subtraction.

1. At what rate will $12,000 gain $4,218.50 in 5 yr. 4 mo. 27 da.?

The interest on $12,000 for 5 yr. 4 mo. 27 da. at 1 % is $649. $4,218.50 is the interest at as many per cent. as $649 is contained times in $4,218.50.

2. If the interest on $625 for 1 yr. 6 mo. is $42.18¾, what is the rate of interest?

3. $9,000 draws $6 as 6 days' interest. What is the rate?

4. What is the rate of interest, if $590 amounts to $634.25 in 18 months?

5. What is the rate of interest on $500, if the amount for 3 years 6 months is $657.50?

6. If the amount is $1,899, the principal 1,800, and the time 11 months, what is the rate of interest?

750. Find the rate of interest on the following:

Principal.	Time.	Interest.
1. $ 195,	4 years 6 months,	$ 87.75.
2. $ 350,	6 months,	$ 12.25.
3. $ 400,	162 days,	$ 11.70.
4. $1,500,	1 year 2 months 21 days,	$ 73.50.
5. $1,750,	2 year 4 months,	$367.50.

751. Find the rate of interest in the following:

	Amount.	Time.	Principal.
1.	$1,975,	2 years,	$1,121.25.
2.	$1,468.50,	1 year 8 months,	$1,615.35.
3.	$1,365.78,	3 months,	$1,339.00.
4.	$1,985.88,	45 days,	$1,985.88.
5.	$1,275,	10 months,	$1,224.
6.	$1,763.41,	1 year 3 months 17 days,	$1,656.

752. To find the time, the principal, interest, and rate being given.

Time = Interest ÷ Interest for 1 year.

1. If $70 is paid for the use of $500, when the rate of interest is 6 %, for what time is the money lent?

Explanation.—1. Multiply $500 by .06, finding $30 or 1 year's interest.

2. If $30 is one year's interest, $70 is as many years' interest as $30 is contained times in $70, or $2\frac{1}{3}$ years, which equals 2 yr. 4 mo. Ans.

$500 × .06 = $30.00

$30)$70 ($2\frac{1}{3}$

Ans., 2 yr. 4 mo.

2. What time has a note for $1,650 to run, if the interest for that time at 8 % is $429?

3. Mr. Arnold, by lending $2,500 at 7 %, gained $962.50. For what time did he lend the money?

4. In what time will $3,600 gain $930 at 6 %?

5. In what time will $4,500 gain $1,868 at 4 %?

753. Find the time in the following:

	Principal.	Rate.	Interest.		Principal.	Rate.	Interest.
1.	$ 600	6 %	$ 126.00.	6.	$ 800	$4\frac{1}{2}$ %	$ 54.00.
2.	$1,250	4 %	$ 262.50.	7.	$1,500	$6\frac{1}{2}$ %	$ 308.75.
3.	$1,800	8 %	$ 325.20.	8.	$ 450	8 %	$ 58.80.
4.	$4,000	7 %	$ 970.66⅔.	9.	$4,800	10 %	$ 836.00.
5.	$6,000	9 %	$1,453.50.	10.	$7,000	5 %	$1,356.25.

PROBLEM III.

754. To find a number, a part being given.

Principal = Interest ÷ Rate.

NOTE.—The amount may be found by adding the principal and the interest.

1. What is the principal, if the interest for 90 days, at 6 %, is $14.31 ?

The interest on $1 for 90 days at 6 % is $.015 ; $14.31 is the interest on as many dollars as $.015 is contained times in $14.31.

2. If money lent for 7 months at 4½ % earns $210, what is the sum lent ?

3. Money lent for 20 years at 10 % yields $1,000 interest; what was the amount lent ?

4. The interest on an investment for 3 yr. 9 mo. 27 da. at 4 % is $1,472.62½. What was the principal ?

5. What principal will yield an interest of $44.87 for 5 yr. 10 mo. at 5 % ?

755. Find the principal in the following:

	Interest.	Time.	Rate.
1.	$3.15,	108 days,	6 %.
2.	$114,	2 years 3 months,	8 %.
3.	$11.00,	11 months,	4 %.
4.	$13.28,	1 year 7 months 6 days,	5 %.
5.	$39,	4 years 8 months 15 days,	10 %.
6.	$21.59,	1 year 4 months 28 days,	4½ %.
7.	$278.95,	8 years 2 months,	7 %.
8.	$135.72,	2 years 10 months 20 days,	9 %.
9.	$409.37,	11 years 2 months,	6 %.
10.	$144,	128 days,	4½ %.
11.	$4,305,	7 months,	6 %.
12.	$539,	9 years 9 months 18 days,	11 %.

NOTE.—Find the amounts in the preceding examples.

PROBLEM IV.

756. To find a number, the number plus a part being given.

Principal = Amount ÷ Amount of $1.

1. A man lending a sum of money for 1 yr. 8 mo. at 6 % interest, received at the end of the time $220. What was the sum lent?

$1.10 is the amount of $1 for 1 yr. 8 mo. at 6 % ; $220 is the amount of as many dollars as $1.10 is contained times in $220.

2. A note for a certain sum is due in 2 yr. 6 mo. 18 da., with interest, money being worth 7 %. If $3,064.10 is sufficient to pay the note with interest, what is the face of the note?

3. What principal will amount to $1,030.20 in 2 yr. 7 mo. 24 da., at 8 %?

4. In payment of a bill, a merchant offers an interest-bearing note. If the note in 9 months 12 days, at 9 per cent. interest, amounts to $1,284.60, what is the face of the note?

757. Find the principal in the following:

Amount.	Time.	Rate.
1. $660.00,	1 year 8 months,	6 %.
2. $502.00,	1 year 5 months 10 days,	8 %.
3. $1,996.35,	1 year 6 months 21 days,	7 %.
4. $2,286.00,	3 years 6 months 27 days,	4 %.
5. $6,237.50,	2 years 9 months,	9 %.
6. $12,133.33⅓,	4 yr. 3 months 6 days,	5 %.
7. $3,108.00,	9 months 18 days,	4½ %.
73⅓,	1 year 1 month 13 days,	8 %.
,	2 years 4 months 15 days,	6 %.
0,	3 years 6 months 24 days,	7 %.

758. The following is a table of the legal rate per cent. of interest in the different states:

STATES.	%	BY SPECIAL AGREEMENT.	STATES.	%	BY SPECIAL AGREEMENT.	STATES.	%	BY SPECIAL AGREEMENT.
Alabama	8		Kansas	7	12	New Mexico (T.)	6	12
Alaska (Ter.)	—		Kentucky	6	8	North Carolina	6	8
*Arizona (Ter.)	8		Louisiana	5	8	Ohio	6	8
*Arkansas	6		*Maine	6		Oregon	8	12
*California	7		Maryland	6		Pennsylvania	6	‡7
*Colorado	10		*Massachusetts	6		*Rhode Island	6	
Connecticut	6		Michigan	6	10	*South Carolina	7	
Dakota	7	12	Minnesota	7	12	Tennessee	6	
Delaware	6		Mississippi	6	10	Texas	8	12
Dist. of Columbia	6	10	Missouri	6	10	*Utah (Ter.)	10	
*Florida	8		*Montana	10		Vermont	6	
*Georgia	7		Nebraska	7	10	Virginia	6	8
Idaho	10	24	New Hampshire	6		West Virginia	6	
Illinois	6	8	*Nevada	10		Washington	10	
Indiana	6	8	New Jersey	6	†7	Wisconsin	7	10
Iowa	6	10	New York	6		*Wyoming	12	

* Any rate by special agreement. † In certain localities only, and under a proviso. ‡ On balances in the hands of commission merchants, etc.

759. The following is a table used by bankers and others for computing simple interest, the interest on $1, or on £1 being:

	4%	5%	6%	7%		4%	5%	6%	7%
Da.					Da.				
1	.00011	.00013	.00016	.00019	23	.00255	.00319	.00383	.00447
2	.00022	.00027	.00033	.00038	24	.00266	.00333	.00400	.00466
3	.00033	.00041	.00050	.00058	25	.00277	.00347	.00416	.00486
4	.00044	.00055	.00066	.00077	26	.00288	.00361	.00433	.00505
5	.00055	.00069	.00083	.00097	27	.00300	.00375	.00450	.00525
6	.00066	.00083	.00100	.00116	28	.00311	.00388	.00466	.00544
7	.00077	.00097	.00116	.00136	29	.00322	.00402	.00483	.00563
8	.00088	.00111	.00133	.00155	Mo.				
9	.00100	.00125	.00150	.00175	1	.00333	.00416	.00500	.00583
10	.00111	.00138	.00166	.00194	2	.00666	.00833	.01000	.01166
11	.00122	.00152	.00183	.00213	3	.01000	.01250	.01500	.01750
12	.00133	.00166	.00200	.00233	4	.01333	.01666	.02000	.02333
13	.00144	.00180	.00216	.00252	5	.01666	.02083	.02500	.02916
14	.00155	.00194	.00233	.00272	6	.02000	.02500	.03000	.03500
15	.00166	.00208	.00250	.00291	7	.02333	.02916	.03500	.04083
16	.00177	.00222	.00266	.00311	8	.02666	.03333	.04000	.04666
17	.00188	.00236	.00283	.00330	9	.03000	.03750	.04500	.05250
18	.00200	.00250	.00300	.00350	10	.03333	.04166	.05000	.05833
19	.00211	.00263	.00316	.00369	11	.03666	.04583	.05500	.06416
20	.00222	.00277	.00333	.00388	Yr.				
21	.00233	.00291	.00350	.00408	1	.04000	.05000	.06000	.07000
22	.00244	.00305	.00366	.00427	2	.08000	.10000	.12000	.14000

MISCELLANEOUS PROBLEMS.

760. Oral Problems. 1. How much interest must a man pay for the use of $200 for 2 years at 6 %?

2. If $9 is 3 years' interest on $50, what is the rate of interest?

3. What principal will bear $6 interest in 2 years at 5 %?

4. What sum will amount to $42 in 1 year at 5 %?

5. If $400 is lent for 1 year at 6½ %, what amount must be paid to the lender at the expiration of the time?

6. What is the interest of $300 for 3 years 3 months at 4 %?

7. In what time will any sum double itself at 12 %?

8. A man lends $200 at 6 %. For what time does he lend the money, if the interest amounts to $18?

9. If $400 is borrowed for 2 years 6 months at 5 %, what amount must be paid at the end of the time?

10. At what rate will $500 yield $100 in 2½ years?

11. When the interest is $36, the time 2 years, and the rate 3 %, what is the principal?

12. How long will it take $20 to gain $2.25 interest at 5 %?

13. What sum will amount to $31.20 in 1 year at 4 %?

14. Mr. Howard invests $400 at 5 % interest. What is the amount at the end of 5 years?

15. What rate of interest must a man receive for the use of his money to earn $180 on $1,000 in 2 years 3 months?

16. How much money must be lent to gain $90 interest in 1 year 8 months at 9 %?

17. If $70 is loaned for 1 year 6 months at 6 %, how much interest will it earn?

18. A debt with interest for 2 years at 6 % amounted to $336. What was the debt?

19. What is the interest on $100 for 18 days at 6 %?

[illegible] is the amount?

[illegible]n how many days will $50 gain $.10 at 6 %?

761. Written Problems. 1. What is the interest on $95 for 2 yr. 9 mo. 27 da., at 8 %?

2. What is the difference between the interest on $600 for 1 month at 6 % and for 1 month at 6½ %?

3. A buys a farm for $1,100, to be paid in 18 months, with interest at 6 %. What was the amount paid?

4. What will be the gain or loss in interest, if $1,500 drawing interest in New York is lent in Kansas for 115 days?

5. Edward Hill loans $1,200. If he receives $1,000, as interest for 20 years, what rate of interest does he receive?

6. If $800 drawing interest in Utah is invested in Texas for 3 yr. 6 mo. 24 da. at legal rate of interest, what is the loss in interest?

7. To what sum does a note of $120 for 103 days amount, with interest at 6 %?

8. If a note for $324 is drawn for 3 months at 7 %, what is the amount of the note?

9. At what rate will $1,000 double itself in 5 years?

10. What is the exact interest on $322.50 from Feb. 8, 1887, to March 1, 1888, at 6 %?

11. What is the difference between the common and the exact interest on $250 for 3 months at 6 %?

12. If the interest on a sum of money for 1 yr. 3 mo. 18 da. at 5 per cent. is $86.45, what is the sum?

13. At what rate will any sum double itself in 12 years?

14. The amount of principal and interest for 5 yr. 11 mo. 1 da. at 8 % is $1,402.51. What is the principal?

15. The interest paid for the use of $2,100 for 100 days was $26.25. What was the rate of interest?

16. What rate of interest does a loan of $2,500 for 20 years require to double itself?

17. If $180 at 4 % in 3 yr. yields $21.60, what rate will be required *to have the* same amount of interest in one year?

18. A loan of $350 at 12 % yields $60.20. Find the time for which the money is loaned.

19. What is the rate of interest, if the interest on $1,500 for 1 year 10 months 24 days is $128.25 ?

20. If a loan of $900 at 7½ % yields $900 interest, or doubles itself, what is the time of the loan ?

21. How long must $541.21 draw interest at 5 per cent. to amount to $720.25 ?

22. At what rate will $2,000 gain $36 in 81 days ?

23. What is the exact interest on $1,000 from April 17 to August 11 of the same year, at 6 % ?

24. What is the interest at 3 % of $3,000 for 3 yr. 3 mo. 3 da. ?

25. At what rate will $500 gain $4.58⅓ in 2 mo. 6 da. ?

26. If $400 at interest for 10 years amounts to $700, what is the rate ?

27. 18 days' interest on $1,000 is $2. What is the rate ?

28. Find the difference between exact interest on $3,650 for 123 days at 6 %, and interest as ordinarily computed ?

29. Find the interest and the amount on $1,150.60 for 15 months at 9 %.

30. The interest on $60 for 5 yr. 8 mo. 18 da. is $34.30. What is the rate of interest ?

31. What is the interest on $400 from Dec. 19, 1883, to May 13, 1884, at 7 % ?

32. B gave a note of $600 payable in 4 months. What was the rate of interest, if he paid $614 for the note when due ?

33. Find the amount and interest on a note of $250, from April 11, 1885, to March 7, 1886, with interest at 6 %.

34. What was the interest at 6 % on $5,000, in U. S. 5-20 bonds from Aug. 15 to Dec. 20, 1875 ?

·ds were bonds redeemable at any time, at the option of the ıent, between 5 and 20 years from the date of their issue.

35. A loan of $18,000 draws $405 interest every six months. What rate of interest does the money draw ?

36. What principal will yield $53.40 in 1 yr. 1 mo. 3 da., at 8 per cent. ?

37. Find the interest on $1,100 for 27 days at 1 %.

38. The interest and the principal amount to $1,905, in 4 yr. 6 mo. at 6 %. What is the principal ?

39. A loan of $7,000 produces $1,368 at 6 %. What is the time ?

40. The rent of a house valued at $6,000 is $41.66⅔ per month. What rate of interest is the rent ?

COMPOUND INTEREST.

762. To find the compound interest.

1. What is the compound interest on $250 for 2 years 5 months 3 days, at 8 % ?

$250	Amt. 2d yr., $291.60	**Explanation.** —
1.08	1.034	1. Multiply the principal by 1.08 to find the amount at the end of the first year.
$2000	116640	
250	87480	
$270.00 Amt. 1st yr.	291600	
1.08	$301.51440	
21 60	$250.00	2. Multiply $270 by 1.08 to find
270	$ 51.51 Ans.	
$291.60 Amt. 2d yr.		

the amount at the end of the second year.

3. Multiply $291.60 by 1.034 ($.034 being the interest on $1 for 5 mo. 3 da.) to find the final amount.

4. Subtract $250, the first principal, from $301.51, the final amount, to find the compound interest. Ans., $51.51.

763. To find the compound interest, find the amount for each year (or half year, or quarter year, when compounded semi-annually or quarterly), use it as the principal for the next period, and subtract the first principal from the last amount. The remainder is the compound interest.

764. Find the compound interest in the following:

Principal.	Rate.	Time.
1. $200,	3 %,	2 yr. 4 mo. 15 da.
2. $325,	4 %,	4 yr. 3 mo. 10 da.
3. $564,	5 %,	3 yr. 6 mo. 20 da.
4. $836,	6 %,	4 yr. 8 mo. 6 da.
5. $910,	7 %,	2 yr. 6 mo. 18 da.

Principal.	Rate.	Time.
6. $1,250,	5 %,	2 yr. 6 mo.
7. $2,630,	8 %,	1 yr. 8 mo. 15 da.
8. $3,155,	4 %,	2 yr. 6 mo. 10 da.
9. $4,020,	6 %,	1 yr. 4 mo.
10. $5,600,	8 %,	1 yr. 7 mo. 5 da.

NOTE.—Compound in Ex. 6, 7, and 8 *semi-annually;* in 9 and 10 *quarterly.*

765. The following is a table used in calculating compound interest, showing the amount of $1, or of £1, at 2½, 3, 3½, 4, 5, 6, 7, and 8 per cent., compound interest, for any number of years, from 1 to 20:

Years	2½ per ct.	3 per cent.	3½ per ct.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	8 per cent.
1	1.025000	1.030000	1.035000	1.040000	1.050000	1.060000	1.070000	1.080000
2	1.050625	1.060900	1.071225	1.081600	1.102500	1.123600	1.144900	1.166400
3	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016	1.225043	1.259712
4	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477	1.310796	1.360489
5	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226	1.402552	1.469328
6	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519	1.500730	1.586874
7	1.188686	1.229874	1.272279	1.315932	1.407100	1.503630	1.605782	1.713824
8	1.218403	1.266770	1.316809	1.368569	1.477455	1.593848	1.718186	1.850930
9	1.248863	1.304773	1.362897	1.423312	1.551328	1.689479	1.838459	1.999005
10	1.280085	1.343916	1.410599	1.480244	1.628895	1.790848	1.967151	2.158925
11	1.312087	1.384234	1.459970	1.539454	1.710339	1.898299	2.104852	2.331639
12	1.344889	1.425761	1.511069	1.601032	1.795856	2.012197	2.252192	2.518170
13	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928	2.409845	2.719624
14	1.412974	1.512590	1.618695	1.731676	1.979932	2.260904	2.578534	2.937194
15	1.448298	1.557967	1.675349	1.800944	2.078928	2.396558	2.759032	3.172169
16	1.484506	1.604706	1.733986	1.872981	2.182875	2.540352	2.952164	3.425943
17	1.521618	1.652848	1.794676	1.947901	2.292018	2.692773	3.158815	3.700018
18	1.559659	1.702433	1.857489	2.025817	2.406619	2.854339	3.379932	3.996020
19	1.598650	1.753506	1.922501	2.106849	2.526950	3.025600	3.616528	4.315701
20	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136	3.869685	4.660957

BUSINESS FORMS.

766. Business forms are generally printed in blank, and, when used, they have the proper names, amounts, and dates written in places, which, in the following forms, are indicated by dotted lines.

767. RECEIPT ON ACCOUNT.

$ *750.* New York, *Dec. 3,* 188*9.*

Received from *James Johnson,*

Seven hundred fifty dollars on account.

William Perry.

768. RECEIPT IN FULL.

$ *430.* Savannah, *June 5,* 188*9.*

Received from *George Morris,*

Four hundred thirty dollars, in full of acc't to date.

Terence Tomlinson.

769. BANK CHECK.

No. *1527.* St. Louis, *Sept. 18,* 188*9.*

Third National Bank

Pay to the order of *Walter Benson,*

Three hundred thirty-seven $\frac{67}{100}$ **Dollars.**

$*337* $\frac{67}{100}$ *John Putnam.*

770. **PROMISSORY NOTE.**

$ *930.* San Francisco, *Mar. 19,* 189*0.*

One month after date *we* promise to pay to the order of *Taylor & Co.,*

Nine hundred thirty 00/100 **Dollars.**

at the *First National Bank.*

Value received.

No. *38.* Due *April 19/22.* *Ball & Brown.*

771. **DRAFT.**

$ *1,425.* Nashville, *Oct. 14,* 189*0.*

Three days after sight pay to the Order of *Mark Mason,*

Fourteen hundred twenty-five 00/100 **Dollars.**

Value received, and charge to the account of

To *Toole & Talcott,* *Lowell, Mass.* } *Henry Hart.*

772. **BILL OF EXCHANGE.**

Exchange for £ *2000.* Philadelphia, *Nov. 6,* 188*7.*

At Sight of this First of Exchange,
(Second and Third of the same tenor and date unpaid,)

Pay to the order of *Richard Graham.*

Two Thousand Pounds Sterling,

for value received, and charge the same to the account of

To *Jones, Smith & Co.,* *London, Eng.* } M. A. Price & Co.

773. A **promissory note** is a written promise to pay a specified sum of money at a specified time and place.

A *joint note* is one in which two or more jointly make a promise to pay. A *joint and several note* is one in which two or more make a promise for the full amount of which each one is liable.

774. A promissory note is *negotiable* if, without condition, it is made payable to the order of a certain person or if it is made payable to bearer.

A promissory note is sometimes said to be *bankable*, when it is made payable at a bank.

775. Interest is not computed on a note, unless specified in the note, or unless the note is past due. In *interest-bearing* notes, the words, *with interest*, occur in the body of the note.

A note or debt drawing more than legal interest is said to be *usurious*. Each state has laws imposing penalties upon creditors for agreeing to receive usury.

776. The **face** of the note is the sum to be paid, as promised in the note.

777. The **amount** is the face plus the interest.

778. The **maker** or **drawer** of the note is the party that makes the promise by signing the note.

779. The **payee** is the party named in the note, as the party to whom the money is to be paid.

780. An **indorser** is one that signs his name *on the back* of the note, and is thus made security for its payment. Each holder indorses the note when he disposes of it.

The payee is the first indorser.

781. The **holder** of the note is the party that owns it.

782. The **days of grace** are the three days allowed by law or by custom for the payment of a note after it is due.

If a note is not paid on or before the last day of grace, a formal notice *of protest is sent to* its maker and to each indorser.

BANK DISCOUNT.

783. Bank discount is the charge made by a bank for advancing money on a note before it is due ; it is the simple interest on the face of the note from the day of discount to the day of maturity.

784. The **day of maturity** is the last day of grace.

NOTE.—The *term of discount* is the time from the day of discount to the day of maturity, including the three days of grace.

785. The **proceeds** of a note is the face of the note less the discount.

786. If a note is not paid by the maker on or before the last day of grace, a **protest,** or formal notice of the fact is sent to the indorsers.

787. In applying PERCENTAGE to **Bank Discount** the ***face*** of the note is the *base ;* the ***discount*** is the *percentage ;* the ***proceeds*** is the *difference ;* and the **rate of discount** is the rate as in percentage.

NOTE.—In interest-bearing notes the amount is the base.

PROBLEM I.

788. To find a part of a number.

Bank Discount = Face × Rate × Time (plus 3 days).

NOTE.—In interest-bearing notes, multiply the amount by the rate by the time plus three days. The face, or, in interest-bearing notes, the amount minus the bank discount, equals the proceeds.

1. A 30 days' note for $500 was discounted at a bank, on the day it was drawn, at 6 %. What was the bank discount?

The term of discount is 33 days. The bank discount on $500 is 500 × 55 (the bank discount on $1 for 33 days). Ans., $2.75.

2. A merchant has a 60 days' note, drawn July 6, for $1,000 discounted by a bank July 20. The rate of discount being 6 %, how much money does the bank give for the note?

The day of maturity = Sept. 7. The term of discount = number of days from July 20 to Sept. 7 = 49 days. The bank discount = $8.17. The proceeds = $1,000 − $8.17 = $991.83, Ans.

3. A 30 days' $1,500 note, drawn May 10, was discounted on day of date at 7 %. Find the bank discount and proceeds.

PROBLEM II.

789. To find what part one number is of another.

Rate = Bank Discount for 1 year ÷ Face.

1. The bank discount on a 60 days' note for $200, discounted 30 days after date, is $1.10. What is the rate?

The term of discount is 33 days. The bank discount for 1 year = $\frac{\$1.10 \times 360}{33} = \$12$. $12 ÷ $200 = .06, or 6 %.

790. Find the bank discount on the following notes:

	1.	2.	3.
Face	$210.	$405.30	$917.50
Date	May 10	Jan. 26	Sept. 18
Time	3 mo.	60 da.	3 mo.
Rate	7 %	6 %	4½ %
Due	Aug. 10/13	March 27/30	Dec. 18/21
Discounted	May 10	Feb. 25	Oct. 13

	4.	5.	6.
Face	$800.	$1,265.36	$675
Date	June 15	July 31	Nov. 27
Time	6 mo.	90 da.	30 da.
Rate	5 %	8 %	10 %
Due	Dec. 15/18	Oct. 29/Nov. 1	Dec. 28/31
Discounted	Aug. 4	Sept. 9	Dec. 1

791. Find the proceeds of the following notes:

	1.	2.	3.
Face	$512	$390.15	$687.59
Date	Feb. 28	Oct. 9	Feb. 17
Time	2 mo.	4 mo.	75 da.
Rate	6½ %	7½ %	9 %
Due	Apr. 28/May 1	Feb. 9/12	May 3/6
Discounted	Mar. 3	Jan. 1.	Apr. 10

	4.	5.	6.
Face	$745	$1,184	$3,126.
Date	Dec. 23	Apr. 27	Mar. 12
Time	3 mo.	60 da.	90 da.
Rate	4 %	6 %	4 %
Due	Mar. 23/26	June 26/29	June 10/13
Discounted	Jan. 29	Apr. 27	Mar. 31

PROBLEM III.

792. To find a number, a part being given.

Face = Bank Discount ÷ Bank Discount of $1.

Proceeds equals face minus bank discount.

1. A bank charged $12.40 for discounting a 90 days' note at 6 %. What was the face of the note?

The bank discount of $1 for 93 days is $.0155 : $12.40 is the bank discount on as many dollars as $.0155 is contained times in $12.40, or 800. Ans., $800.

2. If $5.50 is charged for discounting a 30 days' note, how much money did the owner of the note receive, the rate of discount being 8 %?

The bank discount of $1 for 33 days at 8 % is $.007$\frac{3}{3}$. The face of the ...uals $5.50 ÷ $.007$\frac{3}{3}$, or $750. The proceeds equals $750 ... or $744.50.

PROBLEM IV.

793. To find a number, the number minus a part being given.

Face* = *Proceeds* ÷ *Proceeds of $1.

Bank discount equals face minus proceeds.

1. The proceeds of a 60 days' note, discounted at a bank at 6 %, was $890.55. What was the face of the note?

The proceeds of $1 for 63 days at 6 % is $.9895 : $890.55 is the proceeds of as many dollars as $.9895 is contained times in $890.55, or 900. Ans., $900.

MISCELLANEOUS PROBLEMS.

794. Written Problems. 1. A note for $1,800, dated June 8, 1881, and payable 60 days after date is discounted at 5½ per cent., on July 5. What is the proceeds?

2. What is the bank discount at 6 per cent. on $340 due in 90 days?

3. What is the face of a note the proceeds of which are $1,491.75, time 30 days, rate of discount 6 %?

4. What is the bank discount at 7 per cent. on $560 due in 60 days?

5. What is the bank discount at 6½ per cent. on $2,000 due in 60 days?

6. A bank discounted a note, giving the owner of the note $592.65. The time of the note was 60 days, and rate of discount, 7 per cent. What was the face of the note?

7. A sixty days' note for $950 was dated and discounted at a bank on the same day at 6 per cent. per annum; required the discount and the proceeds.

8. For how much must a note be drawn so that, when discounted at bank, the proceeds will be $2,938, time *90 days and rate of* discount 8 per cent.?

9. Find the proceeds of the following note, if discounted on its date at a bank at 6 per cent.:

$1,250 GALVESTON, *June* 1, 1887.

Four months after date, I promise to pay Silas Gregg, or order, twelve hundred fifty dollars.

Value received. GRAY CHEEVER.

10. A note for $325 for 3 months at 8 per cent. interest is discounted on date of note. What were the avails of the note?

11. Find the face of a note payable in 30 days, rate of discount being 6 per cent., and proceeds, $497.25.

12. For how much must a note for 60 days be drawn, that the proceeds shall be $1,650.60, at 10 per cent. discount?

13. Find the discount and the proceeds of the following note, discounted at 6 %, Oct. 4:

$560.25. CLEVELAND, O., *July* 1, 1887.

Five months from date I promise to pay to the order of Henry Allen five hundred sixty and $\frac{25}{100}$ dollars.

Value received. WILLIAM ROBERTS.

14. For what sum must a note be drawn to yield $149.12½, time 30 days, rate of discount 6 %?

15. For what sum must a note payable in 60 days, discounted at 8 %, be drawn to yield $123.25 proceeds?

16. What is the time of a note for $400, when the bank charges $4.30 for discounting it at 6 %?

The bank discount on $400 for 1 day equals $\frac{\$400 \times .06 \times 1}{360 \times 360} = \$.06\frac{2}{3}$; $4.30 is the bank discount for as many days as $.06⅔ is contained times in $4.30, or 65. The time is 65 days − 5 days of grace, or 60 days. Ans.

What time is a note for $730 drawn, if a bank discounts it for $5.25, the rate of discount being 5 %?

TRUE DISCOUNT.

795. Discount is a sum deducted from a debt or from a price.

796. The **present worth** of a sum of money payable without interest at a future time is the money which at interest for the given time will amount to that sum.

797. True discount is the difference between the present worth and the face of the debt; it is the simple interest on the present worth of the debt, for the given time.

NOTE.—Bank discount is the interest on the *face*, or *future* worth of a *debt;* that is, it is interest on present worth and interest.

798. Bank discount differs from true discount in including interest on interest, and in charging interest for the three days of grace.

In true discount, interest is not charged for the days of grace.

799. In applying PERCENTAGE to **True Discount,** the ***present worth*** is the *base*, the ***true discount*** is the *percentage*, the ***face*** of the debt is the ***amount,*** and the ***rate of interest*** is the *rate*, as in percentage.

PROBLEM I.

800. To find a part of a number.

True Discount = Present Worth × Rate × Time.

The amount or face of a debt equals the present worth plus the true discount; or, the present worth equals the face minus the true discount.

1. With $700 cash, a merchant discharged a debt due in 1 year 3 months, money being worth 6 %. How much was discounted for present payment of the debt?

The true discount equals $\frac{\overset{7}{\cancel{\$700}} \times \cancel{6} \times 15}{\cancel{100} \times \cancel{12}_{2}} = \$52.50$. Ans.

2. What is the amount of a debt due in 1 year 6 months 24 days, if $960 cash will settle it, money being worth 8 %?

The true discount equals $\frac{\$960 \times 8 \times 18.8}{100 \times 12} = \$120.32$. The amount of the debt equals $960 + $120.32, or $1,080.32.

PROBLEM II.

801. To find what part one number is of another.

Rate = True Discount for one year ÷ Present Worth.

1. If a firm deducts $96 for prepayment of a debt of $1,296, due in 1 year 4 months, what rate of discount does it charge?

$96 is the true discount on $1,296 for $1\frac{1}{3}$, or $\frac{4}{3}$ years. The true discount for 1 year is $96 ÷ $\frac{4}{3}$, or $72. The rate of discount is $72 ÷ $1,200, the present worth. Ans., 6 %.

PROBLEM III.

802. To find a number, a part being given.

Present Worth = Face ÷ Amount of $1.

The face is the amount of the present worth.

1. How much cash is needed to pay a debt of $2,718.75, due in 1 year 3 months, money being worth 7 %?

$1 of the present worth amounts to $1.0875 in 1 yr. 3 mo. at 7 %. There are as many dollars of present worth in $2,718.75, as $1.0875 is contained times in $2,718.75, or 2,500. Ans., $2,500.

2. If a man pays cash for a debt of $3,142.50, due in 6 months 10 days, how much is discounted, money being worth 9 %?

The present worth, $3,142.50 ÷ $1.0475 equals $3,000. The true discount equals $3,142.50 − $3,000, or $142.50. Ans.

3. What is the present worth of $1,500 due in 2 years 6 months, when money is worth 6 %?

PROBLEM IV.

803. To find a number, the number plus a part being given.

Present Worth = True Discount + Interest on $1.

1. For making cash payment of a debt due in 2 years 8 months, a merchant has $240 deducted from his bill. How much did he pay, money being worth 6 %?

The interest on $1 for $2\frac{2}{3}$ years equals $\$1 \times \frac{.02}{.06} \times 8 = \$.16$. If $.16 is deducted as the interest on $1 of present worth, $240 is deducted for as many dollars of present worth as $.16 is contained times in $240, or 1,500. Ans., $1,500.

2. If $210 is deducted, for present payment, from a debt due in 1 year 6 months, what is the present value of the debt, money being worth 7 %?

The true discount on $1 for $1\frac{1}{2}$ years equals $\$1 \times .07 \times \frac{3}{2} = \$.105$. $210 is the true discount on as many dollars as $.105 is contained times in $210, or 2,000. The face of the debt equals $2,000 + $210 or $2,210.

804. Find the present worth and the true discount on the following:

1. $540, due in 1 yr. 6 mo., at 6 %.
2. $763.75, due in 2 yr. 6 mo., at 7 %.
3. $461, due in 3 mo. 20 da., at 8 %.
4. $1,594.50, due in 8 mo. 12 da., at 9 %.
5. $893.76, due in 9 mo. 18 da., at $\frac{2}{3}$ % a month.

805. Find the face when the present worth, time, and rate of discount are as follows:

1. Present worth, $945; time, 3 yr. 7 mo. 8 da.; rate, 5 %.
2. Present worth, $1,200; time, 1 yr. 3 mo. 24 da.; rate, $7\frac{1}{2}$ %.
3. Present worth, $1,512; time, 75 da.; rate $8\frac{1}{2}$ %.
4. Present worth, $387; time, 2 yr. 2 mo., 19 da.; rate, $\frac{1}{3}$ % *a month.*

806. Find the time when the present worth, face, and rate are as follows:

1. Present worth, $500; face, $545; rate, 6 %.
2. Present worth, $630; face, $694.40; rate, 8 %.
3. Present worth, $900; face, $1,058.90; rate, 7 %.
4. Present worth, $1,800; face, $2,177.25; rate, 5 %.

MISCELLANEOUS PROBLEMS.

807. Written Problems. 1. Which is more profitable to the coal dealer, and how much more, to sell coal at $6.75 cash or $7.50 on 6 months' time, money being worth 10 %?

2. Mr. Hayes owes $636, due in 9 months. How much should be deducted for present payment, money being worth 8 %?

3. Bought $824 worth of merchandise on 4 months' time and sold for $875 cash. How much was gained, money being worth 9 %?

4. Mr. Edwards owes $810 due in 3 months, $930 due in 8 months, and $1,575 due in 12 months. How much does he require to make present payment of the three debts, money being worth 5 %?

5. What is the difference between the bank and the true discount of $2,755, due in 1 yr. 8 mo. 12 da., at 6 %?

6. A man, owing a bill of $2,174 on time, liquidates it by a cash payment of $2,000. Money being worth 6 %, for what time was the bill incurred?

7. If $3,540 was accepted as present payment of a note for $4,118.20, how long had the note to run, money being worth 7 %?

8. A merchant owed $4,761.40. If $565.60 was deducted [illegible] payment, how much time had he in which to pay [illegible]oney being worth 8 %?

9. What is the face of a debt due in 1 yr. 3 mo., without interest, for which $3,000 in cash is accepted, money being worth 5 %?

10. A merchant pays with $2,880 a bill due in 8 mo. 20 da., without interest. What is the face of the bill, money being worth 7 %?

11. For what time is a note for $4,560 drawn, if its present worth, when discounted at a bank at 6 %, is $4,000?

The interest on $4,000 for 1 year at 6 % is $240. The term of discount equals the true discount, $560 ÷ $240, or 2⅓ years. Ans., 2 years 4 months.

12. If $560 is deducted for present payment of a note for $6,560, money being worth 8 %, for what time was the note drawn?

13. How long has a debt of $2,439.20 to run, if $2,400 cash will pay it, money being worth 6 %?

PARTIAL PAYMENTS.

808. Partial payments are part payments of a note or of a debt.

809. The creditor is allowed interest not only on debts bearing interest, but on debts past due; the debtor is allowed interest on payments made before due. The subject of PARTIAL PAYMENTS adjusts the interest between creditor and debtor.

NOTE.—Partial payments discharge first the interest due and then the principal. A payment not equal to the interest due is reserved, and united with the next payment.

810. Indorsements are the acknowledgments of partial payments written *on the back* of the note, or other written obligation.

811. United States Rule.—1. Compute the interest on the principal to the time of the first payment, and, if this payment equals or exceeds the interest then due, add the interest to the principal, and from the sum subtract the payment; the remainder will form a new principal, with which proceed as before.

2. If the payment is less than the interest, compute the interest on the principal to the time when the sum of the payments shall first equal or exceed the interest due; add the interest to the principal, and from the sum subtract the sum of the payments, and treat the remainder as a new principal.

812. Written Problems. 1. Find what was due May 1, 1888, on the following note:

$1,000. PHILADELPHIA, *Jan.* 1, 1886.

For value received, I promise to pay Ralph Rogers, or order, One Thousand Dollars on demand, with interest at 6 per cent.

WILLIAM MASON.

The above note was indorsed as follows:

July 1, 1886, $200; Sept. 16, 1886, $150; Apr. 22, 1887, $20; June 1, 1887, $230; Dec. 13, 1887, $100.

Explanation.

Yr. Mo. Da.	Intervals of Time.	Payments.	Interest.	Principals.
1886 – 1 – 1	mo. da. mo.			$ 1,000
1886 – 7 – 1	6 – 0 = 6	$200	$30.	$ 830
1886 – 9 – 16	2 – 15 = 2.5	$150	$10.375	$690.375
1887 – 4 – 22	7 – 6 = 7.2	$ 20	$24.853	
1887 – 6 – 1	1 – 9 = 1.3	$230	$ 4.457	$469.715
1887 – 12 – 13	6 – 12 = 6.4	$100	$15.031	$384.746
1888 – 5 – 1	4 – 18 = 4.6		$ 8.849	Ans.

Find first all the intervals of time.

The work may be written as follows:

$1,000	**1st Prin.**	690.375	**3d Prin.**	469.715	**4th Prin.**
.03		.036		.032	
30.00	Int.	4142250		939430	
1000.		2071125		1409145	
1030.00	Amt.	*24.853500	Int.	15.031	Int.
200.	Pay.	690.375		469.715	
830.00	**2d Prin.**	.006½		484.746	Amt.
.012½		345187		100.	Pay.
41500		4142250		384.746	**5th Prin.**
996000		4.487437	Int.	.023	
10.37500	Int.	24.853		1154238	
830.		690.375		769492	
840.375	Amt.	719.715	Amt.	8.849158	Int.
150.	Pay.	250.	Pay.	384.746	
690.375	**3d Prin.**	469.715	**4th Prin.**	$393.595	Ans.

* Payment less than interest. Compute interest on same principal to time of next payment. $20 + $230 exceeds $24.853 + $4.487.

813. Merchants' Rule.—1. Find the amount of the principal, from the time interest commences to the time of settlement.

2. Find the amount of each payment, from the time it was made to the time of settlement.

3. Subtract the sum of the amounts of the payments from the amount of the principal; the remainder will be the sum due.

814. Written Problems. 1. What is due on the following note according to the Merchants' Rule, at the time of settlement, Jan. 1, 1886?

$560. BROOKLYN, N. Y., *Jan.* 1, 1885.

For value received, I promise to pay Frank Mitchell, or order, Five Hundred and Sixty Dollars, on demand, with interest, at 6 per cent. RICHARD KENDALL.

Payments indorsed as follows:—Feb. 15, $50; June 1, *$100; Sept. 15, $100.*

The following form is used under the Merchants' Rule:

Explanation.

Principal.		$560
Interest on the same for one year.		$ 33.60
Amount of principal.		$593.60
First payment, Feb. 15, 1885.	$50.00	
Interest on same to Jan. 1, 1886.	2.63	
Second payment, June 1, 1885.	100.00	
Interest on same to Jan. 1, 1886.	3.50	
Third payment, Sept. 15, 1885.	100.00	
Interest on the same to Jan. 1, 1886.	1.77	
		257.90
Balance due at date of settlement.		$335.70

NOTE.—Work the following problems by either rule:

2. Find what was due Dec. 1, 1884, on the following note:

$180. MONTGOMERY, ALA., *June* 1, 1884.

Six months after date, I promise to pay to the order of James Smith, One Hundred-eighty Dollars with interest, at the Second National Bank. HENRY BROWN.

Value received.

Indorsed: July 1, 1884, $25; Aug. 1, $25; Sept. 1, $25; Oct. 1, $75.

3. Find what was due Jan. 1, 1887, on the following note:

$350. NEW YORK CITY, *Jan.* 1, 1886.

One year after date, we jointly and severally promise to pay George Gray, or order, Three Hundred-fifty Dollars with interest. SILAS HAYES.

Value received. ROLAND JONES.

Indorsed: Mar. 1, 1886, $50; June 1, 1886, $75; Sept. 1, 1886, $100; Nov. 1, 1886, $100.

4. What was due July 15, 1884, on the following note, interest being 6 % ?

$1000 CLEVELAND, O., *July* 15, 1881.

Three years after date, I promise to pay to Hiram Blank, or order, One Thousand Dollars, with interest.

Value received. SAMUEL FARNHAM.

Indorsed : Jan. 1, 1882, $500 ; July 1, 1882, $350 ; July 1, 1883, $25 ; Jan. 1, 1884, $80.

5. A note of $700, dated August 5, 1881, was indorsed as follows : Oct. 1, 1881, $150 ; Jan. 3, 1882, $200 ; March 15, 1882, $125 ; July 5, 1882, $100 ; Sept. 16, 1882, $75. What was due October 1, 1882, interest at 6 % ?

6. A note of $175, dated Jan. 20, 1884, indorsed as follows : March 11, 1884, $50 ; May 4, 1884, $25 ; August 19, 1884, $60 ; Nov. 25, 1884, $15. What was due Jan. 1, 1885, interest at 7 % ?

7. Hiram Stone held a note of $500 against George Brown, dated April 21, 1887, interest at 8 %, indorsed as follows : May 1, 1887, $200 ; June 1, 1887, $175 ; July 1, 1887, $100. What was due July 21, 1887 ?

8. A mortgage of $4,900 held against Franklin Beaver, dated Oct. 21, 1885, interest at 5 %, was indorsed as follows : June 1, 1886, $1,000 ; Nov. 11, 1886, $500 ; May 13, 1887, $300 ; Jan. 15, 1888, $1,450 ; July 25, 1888, $1,250. What was due Oct. 21, 1888 ?

9. On a debt of $340, dated May 14, 1886, payments were made as follows : June 5, 1886, $75 ; June 20, 1886, $50 ; July 9, 1886, $100 ; July 25, $50. What was due Aug. 1, 1886, with interest at 6 % ?

10. Ernest Pierce borrowed $1,000 Sept. 13, 1885, from Louis Stevens, giving a note payable on demand, with interest at 7 %. Jan. 1, 1886, a payment of $550 was made and the *balance included in* a new note. What was the face ?

EQUATION OF PAYMENTS.

815. Equation of payments is the process of finding the average time for the payment of several debts due at different times, without loss of interest to creditor or to debtor.

816. The **equated time** is the date at which one payment will equitably discharge the several debts.

817. The **term of credit** of a debt is the time a debt has to run.

818. The **average term of credit** of several debts is the average of the terms of credit of the different debts.

819. Debts beginning at the same date.

1. A owes B $212, due one month hence; $224, due in two months; $236, due in three months. What is the equated time for the payment of the whole, with interest at 6 %?

$212 for 1 month equals the use of $1 for 212 months.
$224 for 2 months equals the use of $1 for 448 months.
$236 for 3 months equals the use of $1 for 708 months.
$672 — 1,368 months.

As A is entitled to the use of $1 for 1,368 months, he is entitled to the use of $672 for $\frac{1}{672}$ of 1,368 months, 2 mo. 1 da. Ans.

820. Payments at different times on a given debt.

1. Mr. Rogers bought a farm worth $9,300 on 18 months' time. He paid $1,500 down; $500 at the end of 6 months; $900 at the end of 12 months, at the same time giving a note for the balance. What was the equitable time for which to draw the note?

18 months before due he paid $1,500, which equals the use of $1 for 27,000 months.
12 " " " " " 500, " " " " " $1 for 6,000 "
6 " " " " " 900, " " " " " $1 for 5,400 "
2,900 — 38,400 "

As his payments were equivalent to the payment of $1 for 38,400 months before it was due, he was entitled to the use of the balance, (9,300–$2,900) for $\frac{1}{6400}$ of 38,400 months or 6 months, after nths.

821. Debts beginning at different dates.

1. Henry Marvin bought 3 bills of groceries as follows: Sept. 5, 1888, $375 on 30 days' time; Sept. 30, 1888, $255 on 30 days' time; and Oct. 10, 1888, $570 on 60 days' time. What is the equated time of payment for the whole?

Sept. 5 + 30 days = Oct. 5	$375 × 0 =	0	Days for which
Sept. 30 + 30 " = Oct. 30	255 × 25 =	6,370	he was entitled
Oct. 10 + 60 " = Dec. 9	570 × 65 =	37,050	to the use of $1
	$1,200	43,800	after Oct. 5.

Oct. 5 is the earliest date when any one of the bills was due. He was entitled to the use of $1 for 43,800 days, or to the use of $1,200, the whole debt, for $\frac{1}{1200}$ of 43,800 days, or 36 days, after Oct. 5. Oct. 5 + 36 days = Nov. 10, Ans.

MISCELLANEOUS PROBLEMS.

822. Written Problems. 1. What is the equated time of payment of 4 bills due as follows: Jan. 1, $350; March 15, $275; April 10, $225; June 5, $150?

2. William Dean owes $2,000; $750 is due in 30 days, $600 in 60 days, and the balance in 90 days. What is the average time of payment?

3. A bought of Lowell, Bliss & Co. three bills of goods: Nov. 12, $250 on 90 days; Dec. 7, $180 on 60 days; Jan. 2, $170 on 30 days. At what date shall a note for the whole amount be made payable?

4. Travis & Bros. bought three bills on 3 months' time as follows: Jan. 6, $945; Feb. 2, $650; Mar. 14, $705. They gave a note in settlement. When was the note payable?

5. Mr. Johnson buys property for $11,000. He pays $5,000 down, $4,000 in 1 year, and the balance in 3 years. What is the average time of credit?

6. Mr. Packer bought a piano for $750. He paid $250 down, the balance to be divided into four equal annual payments. *What was the* average time of the payment?

7. A hardware dealer sold the following bills: Feb. 18, $150 on 60 da.; Feb. 26, $260 on 90 da.; Mar. 6, $200 on 30 da. At what date is the whole amount due?

8. Frank Stilson owes John Parker $2,700, due in 4 months. He paid $500 at the end of 1 month; $800 at the end of 2 months; $700 at the end of 3 months, giving a note at the same time for the balance. For what time must the note be drawn?

9. The following bills of mdse. were bought on 60 days' credit: Apr. 1, 1886, $1,200; May 15, 1886, $800; June 4, $1,000; July 10, $500. At what date shall a note for the whole amount be made payable?

10. Goods to the following amounts were sold: Jan. 27, 1885, $415 @ 60 da.; Feb. 17, 1885, $380 @ 30 da.; Mar. 22, 1885, $600 @ 90 da.; Apr. 12, 1885, $255 @ 30 da. What sum will settle the account, without loss of interest to either party, April 1, 1885, when discounted at 6 %?

11. Merchandise to the following amounts was bought: Sept. 5, 1887, $690.25 @ 6 mo.; Sept. 19, 1887, $587.50 @ 5 mo.; Oct. 1, 1887, $549.45 @ 3 mo.; Nov. 30, 1887, $475.65 @ 4 mo.; Dec. 4, 1887, $197.15 @ 2 mo. What is the present worth of a note, dated Jan. 1, 1888, in payment of the whole account, discounted at 7 %?

12. A bought real estate for $8,500 on 3 years' time. He paid $2,000 down; $1,000 at the end of six months; $1,200 at the end of 1 year; $1,500 at the end of 2 years, giving a note for the balance. What would be the equitable time of the note?

13. A merchant owed a wholesale house in 1887 as follows: $350, due Jan. 18; $745, due Feb. 15; $895, due [illegible] $176, due March 17. He settled the account Feb. [illegible] 4 months' note. What was the face of the note, [illegible] worth 5 %?

AVERAGING ACCOUNTS.

823. Averaging accounts is the process of finding the equated time for the payment of the balance of an account, or of finding the cash balance due at any stated time.

824. An **account** is a record of the debits and credits in any business transaction.

Debits are recorded items of debt or of values received; credits are recorded items of values given.

825. Debts and payments at different times.

1. Find the equated time for the payment of the balance of the following account:

Dr. JAMES REYNOLDS. Cr.

1886.				1886.		
June 10.	To Mdse. @ 3 mo.	$828		Sept. 12.	By Cash,	$800
July 15.	" " @ 2 mo.	280				
Aug. 20.	" " @ 4 mo.	576				

Explanation.

June 10 + 3 mo. = Sept. 10.	$828 × 0 =	0	Sept. 12.	$800 × 2 = 1600
July 15 + 2 mo. = Sept. 15.	280 × 5 =	1400		
Aug. 20 + 4 mo. = Dec. 20.	576 × 101 =	58176		
	$1684 =	59576		
	800	1600		
	884	)57976 = 66		

1. Add the term of credit to the date of purchase of each item.
2. Take the earliest date thus found on either side of the account as the point of reckoning, and multiply each item by the number of days intervening between that date and the date of the item.
3. The balance of the products, divided by the balance of the account, 66 days, is the average time. The two balances being on the same side, the time must be added to Sept. 10, which gives Nov. 15 as the equated time.

NOTE.—If it is assumed that the whole debt is cancelled Sept. 10, the debtor is entitled to the interest of $280 for 5 days and of $828 for 101 days; and the creditor, to the interest of $800 for 2 days.

2. Required the cash balance of the following account Nov. 25, 1886, interest at 6 %:

Dr. HENRY C. WALKER. Cr.

1886.					1886.		
Aug. 18	To Mdse.	@ 90 days	$2500 00		Sept. 20	By Cash,	$1000 00
Sept. 20	" "	@ 60 days	500 00		Oct. 10	" "	500 00
Oct. 1	" "	@ 30 days	475 00		Nov. 5	" "	275 00
Nov. 5	" "	@ 30 days	335 00				

Explanation.—1. The earliest date on either side, after adding the terms of credit, is Sept. 20. The equated time for paying the balance of the account, $2,035, is 96 days from Sept. 20, or Dec. 25, 1886.

2. The cash balance on Nov. 25, 1886, is the present worth of $2,035 for the difference of time between Dec. 25 and Nov. 25, or 30 days. $2,035 ÷ $1.005 = 2,024.875. Ans., $2,024.88.

1. Ray Arnold bought goods as follows:

1885, March 20,	To Mdse.	@ 2 mo.	. . . . .	$263.48
" April 3,	" "	@ 3 mo.	. . . . .	$411.42
" April 25,	" "	@ 1 mo.	. . . . .	$150.10
" May 31,	" "	@ 2 mo.	. . . . .	$175.00

The payments were as follows:

1885, April 1,	By Cash	. . . . .	$175.00
" April 19,	" "	. . . . .	$250.00
" May 5,	" Draft, 1 mo.	. .	$300.00
" June 10,	" Cash	. . . . .	$175.00

When did the balance become due?

NOTES.—1. Reckon days of grace (Art. 782) on notes and drafts.

he balance of interest is in favor of the debtor, his time is d beyond the date taken; but if the balance is in favor of ditor, the cash balance must be paid at a date previous.

2. The following is the account of Robert Gordon:

1886, Aug. 24,	To Mdse.	@ 90 da.,	. . . . .	$1,900.00
" Sept. 8,	" "	@ 60 da.,	. . . . .	$1,200.00
" Oct. 20,	" "	@ 30 da.,	. . . . .	700.00
" Nov. 15,	" "	@ 60 da.,	. . . . .	$1,000.00

Credits are:

1886, Aug. 30,	By Cash,	. . . .	$1,000.00
" Sept. 28,	" Draft,	30 da., .	$1,500.00
" Nov. 1,	" "	30 da., .	500.00
" Dec. 1,	" Cash,	. . . .	$1,500.00

What is the balance and when is it due?

3. Find the equated time of the following account:

Dr. CHARLES MATTHEWS & Co. Cr.

1887.			1887.		
June 10	To Mdse., 60 da.	$400	July 15	By Cash	$300
June 23	" " 30 da.	$250	Aug. 27	" Note 30 da.	$150
July 7	" " 30 da.	$170	Aug. 30	" Note 60 "	$200

4. What is the equated time of the following account and the cash balance Jan. 1, 1886, interest at 6 %?

Dr. HARVEY & BOWEN. Cr.

1885.				1885.			
July 16	To Mdse., 4 mo.	$ 520	00	Aug. 1	By Mdse., 4 mo.	$ 300	00
Aug. 31	" " 3 mo.	375	00	Oct. 1	" Cash,	200	00
Sept. 7	" " Cash	1000	00	Dec. 1	" Draft, 10 da.	250	00
Oct. 15	" " 3 mo.	935	00	" 18	" Note, 30 da.	1000	00
Nov. 21	" " Cash	500	00				
Dec. 14	" " 2 mo.	170	00				

NOTE.—To find the cash balance: When the date of settlement is earlier than the equated date found for paying the balance, find the present *worth of the* balance for the time between the *when it is later,* find the amount of the balance with in

EXCHANGE.

326. *Exchange* is a method of paying money at a distance by means of drafts, or bills of exchange.

327. Domestic exchange is exchange between places in the same country.

328. Foreign exchange is exchange between different countries.

329. A draft or a bill of exchange is a written order for the payment of money at a distance.

Drafts are usually used in *inland* or *domestic exchange;* and bills of exchange in *foreign exchange.*

330. Bills of exchange are usually drawn in triplicate, forming a set of exchange, the copies being sent for security by different conveyances; if one of the sets is paid, the others are cancelled.

NOTE.—For the form of a draft, or of a bill of exchange, see page 285.

331. The *drawer* is the party that makes the order; the **drawee**, the party on whom the order is made; the **payee**, the party to whom the money is to be paid.

332. A **sight draft** is payable at sight.

Days of grace are usually allowed.

333. A **time draft** is payable at a specified time.

334. An **acceptance** is the consent to pay, written across the face of a draft by the drawee.

The word "Accepted," the date, and the signature are usually written.

335. A **letter of credit** is a letter addressed by one bank to its correspondents in other countries, directing them to pay [illegible] ned in the letter any amount not exceeding a

836. The **par of exchange** is the legal or established relation of the units of value of different countries.

The *intrinsic* par depends on the relative weight and purity of coins; the *commercial* par depends on the relations of commerce between the two countries.

837. The **rate** or **course of exchange** is the market value of the monetary unit of the bill of exchange.

The monetary denominations of the country on which the bill is drawn are used in making the bill.

838. If the balance of trade between the U. S. and any foreign country is in favor of the foreign country, bills of exchange on that country are *in demand* in the U. S., are therefore bought in the U. S. *at a premium;* or, *at a discount,* if the reverse is the case.

839. To find the cost of a draft.

$$Cost = Face \times Cost\ of\ \$1.$$

In time drafts, the bank discount is subtracted from the cost of $1.

1. What is the cost of a sight draft on St. Louis for $1,500, at $1\frac{1}{2}$ % premium ?

The rate of premium being $1\frac{1}{2}$ %, the course of exchange is $1.01\frac{1}{2}$. The cost of $1 is $1.01\frac{1}{2}$. $1,500 cost 1,500 × $1.01\frac{1}{2}$, or $1,522.50.

2. What will be cost of a 30 days' draft on New Orleans for $1,800, at $\frac{1}{2}$ % discount ?

The cost of $1 is $.99\frac{1}{2}$ ($1 less the $\frac{1}{2}$ % discount) — $.0055 (the discount for 33 days), or $.9895. The cost of 1,800 is $1,800 × .9895, or $1,781.10. Ans.

3. The face of a draft on Baltimore is $2,500, the discount at 1 per cent. What is the cost of the draft ?

4. A merchant in Baltimore wishes to remit $1,134 to Chicago, premium at $\frac{3}{4}$ per cent. What is the cost of the bill of exchange ?

5. A merchant in New Orleans purchases a draft of $1,000 on New York, exchange being at 1½ per cent. discount. What does he pay for the draft?

6. What must be paid in Philadelphia for a draft of $3,000, payable 60 days after sight on Chicago, exchange being at a premium of ¾ per cent., the rate of interest being 6 per cent.?

7. A merchant in Cincinnati wishes to pay $2,500 in Boston, exchange on Boston ½ per cent. discount, interest 6 %, time 30 days. What is the cost of the draft?

840. To find the face of a draft.

Face = Cost ÷ Cost of $1.

In time drafts, subtract the bank discount from the cost of $1.

1. How large a draft at ¼ % discount can be bought for $1,995?

The rate of discount being ¼ %, the cost of $1 is $.99¾ : $1,995 is the cost of as many dollars as $.9975 is contained times in $1,995 or 2,000. Ans., $2,000.

2. How large a 30 days' draft can be bought for $1,312.35, exchange at 1½ premium, and interest at 6 per cent.?

The cost of $1 is $1.01½ ($1 plus the premium) − $.0055 (the discount for 33 days), or $1.0095 : $1,312.35 is the cost of as many dollars as $1.0095 is contained times in $1,312.35, or 1,300. Ans., $1,300.

841. Find the cost of each of the following drafts:

Sight Drafts.		Time Drafts.			
Face.	Rate of Exchange.	Face.	Rate.	Time.	Int.
1. $5,000	1½ % premium.	7. $1,124,	1 % premium,	60 da.	7 %
2. $1,860	⅝ % discount.	8. $4,720,	½ % discount,	30 da.	6 %
3. $3,650	¾ % discount.	9. $3,195,	¾ % premium,	30 da.	6 %
	½ % premium.	10. $2,376,	1½ % discount,	60 da.	6 %
	2 % premium.	11. $1,200,	⅝ % premium,	30 da.	7 %
	1 % discount.	12. $1,000,	2 % discount,	90 da.	6 %

842. Find the face of each of the following:

Sight Drafts.			Time Drafts.			
Cost.	Rate of Exchange.		Cost.	Rate.	Time.	Int.
1. $1,791	½ % discount.		7. $2,019.00,	1½ % premium,	30 da.	6 %.
2. $4,692	2 % premium.		8. $4,897.50,	1 % discount,	60 da.	6 %.
3. $2,437.50	2½ % discount.		9. $ 593.15,	½ % discount,	30 da.	7 %.
4. $4,069.25	¾ % discount.		10. $1,204.30,	1 % premium,	30 da.	7 %.
5. $1,408.75	⅝ % premium.		11. $ 747.75,	¾ % premium,	60 da.	6 %.
6. $1,624	1½ % premium.		12. $2,350.80,	1½ % discount,	30 da.	6 %.

843. To find the rate:

$$Rate = \left.\begin{matrix} Premium, \\ Discount \end{matrix}\right\} \text{ or } \div Face.$$

In time drafts, the *premium* equals the cost minus the face plus the bank discount; or, the *discount* equals the face minus the cost plus the bank discount.

1. If $403 is the cost of a draft on Chicago for $400, what is the rate of premium?

$403 − $400 = $3, the premium on $400. The rate of premium equals $3 ÷ $400, or .00¾. Ans., ¾ % premium.

2. What is the rate of discount, when a 60 days' draft for $1,200 cost $1,182.90, the rate of bank discount being 6 %?

$1,200 − $1,182.90 = $17.10, the discount + the discount for 63 days. $17.10 − $12.60 (the discount for 63 days) = $4.50. The rate of discount equals $4.50 ÷ $1,200, or .00375. Ans., ⅜ %.

MISCELLANEOUS PROBLEMS.

844. Written Problems. 1. A merchant wishes to remit $3,422 to New York. If exchange on New York is at 1½ % premium, what will be the cost of the bill?

2. A Louisville merchant wishing to send $5,000 to Baltimore buys a bill of exchange on Baltimore at 1 % discount. How much must he pay for the bill?

3. What is the face of a draft that costs $4,060, when exchange *is at 1½ % premium*?

4. If a draft for $3,000 costs $3,048.75, what is the rate of exchange ?

5. A draft on Chicago cost $6,418.75, when exchange was at $1\frac{1}{4}$ % discount. What was the face of the draft ?

6. What must be paid in Baltimore for a 30 days' draft on St. Louis, for $6,000, the rate of exchange being $1.00\frac{3}{4}$?

7. Exchange being at $1\frac{1}{8}$ % discount, what is the cost of a 60 days' draft for $8,000 ?

8. How large a draft can be bought for $7,031.50 at 90 days, exchange being at 2 % premium ?

9. What is the rate of exchange when a 30 days' draft for $4,000 can be bought for $4,033 ?

10. Find the face of a time draft for 60 days, when $1,988.67 is paid for it, exchange at $\frac{1}{2}$ %, and discount at 6 % per annum.

FOREIGN EXCHANGE.

845. To find the cost of a bill of exchange.

$$\textit{Cost} = \textit{Face} \begin{cases} \times \textit{ Value of the Foreign Unit in U. S. Money.} \\ \div \textit{ Value of \$1 in Foreign Money.} \end{cases}$$

If there is brokerage it is estimated on and added to the cost.

1. What is the cost of a bill of exchange on London for £350. 15s., at $485\frac{1}{2}$, brokerage $\frac{1}{4}$ % ?

£350. 15s. equal £350.75. If one pound is worth $4.85\frac{1}{2}$, £350.75 are worth 350.75 × $4.85\frac{1}{2}$, or $1,702.89. Counting the brokerage, each dollar costs $1.00\frac{1}{4}$, and $1,702.89 costs 1,702.89 × $1.00\frac{1}{4}$, or $1,707.15. Ans.

2. Find the cost of a bill of exchange on Paris for 1,044 francs at 5.22, brokerage $\frac{1}{2}$ %.

If [illegible] cost $1, 1,044 francs will cost as many dollars as 5.22 is [illegible]s in 1,044, or 200. Containing the brokerage, each [illegible] $00\frac{1}{2}$, and $200 will cost 200 × 1.00\frac{1}{2}$, or $201. Ans.

846. To find the face of a bill of exchange.

$$Face = Cost \begin{cases} \div \textit{ Value of Foreign Unit in U. S. Money.} \\ \times \textit{ Value of \$1 in Foreign Money.} \end{cases}$$

1. Find the face of a bill on London that costs $585.63, when exchange is 4.86.

If $4.86 is the cost of one pound, $585.63 is the cost of as many pounds as $4.86 is contained times in $585.63, or 120½. Ans., £120. 10s.

2. What is the face of a bill on Berlin that costs $2,955, when the rate of exchange is 98½ ?

If 4 marks can be bought for $.98½, one mark can be bought for ¼ of $.98½, or $.24⅝. As many marks can be bought for $2,955 as $.24⅝ is contained times in $2,955, or 12,000. Ans., 12,000 marks.

3. What is the face of a bill on Paris that cost $1,800, when the rate of exchange was 5.19 ?

If 5.19 francs can be bought for $1, 1,800 × 5.19 francs, or 9,342 francs, can be bought for $1,800.

847. To find the rate of foreign exchange.

$$Rate = \begin{cases} \textit{Cost} \div \textit{Face} \text{ (when Value of Foreign Unit is Given).} \\ \textit{Face} \div \textit{Cost} \text{ (when Value of \$1 is Given).} \end{cases}$$

If there is brokerage, first divide the cost by $1 + rate of brokerage, and then proceed as above.

1. If a bill of exchange on London for £200.12s. costs $972.91, what is the rate of exchange ?

£200. 12s. = £200.6. If the cost of exchange for £200.6 is $972.91, the cost of exchange for one pound is $972.91 ÷ 200.6, or $4.85. Ans., 4.85.

2. If $200.50 is the cost of a bill of exchange on Paris for 1,036 francs, when brokerage is ¼ %, what is the rate ?

$200.50 equals the cost of exchange plus the brokerage, or $200.50 = 100 % + ¼ %. $200.50 ÷ 1.0025 = $200, the cost of exchange. If $200 will buy 1,036 francs, $1 will buy as many francs as 200 is contained *times in 1,036* francs, or 5.18 francs. Ans.

MISCELLANEOUS PROBLEMS.

848. Written Problems. 1. What is the value in U. S. money of a bill on London for £115. 11s. 3d. at the actual par, brokerage $\frac{1}{2}$ %?

2. Find the value in English money of $1,800, exchange at $4.8665.

3. What is the cost of a bill of exchange on Liverpool for £275. 12s. 6d., exchange at $4.87, gold at 110 in U. S. currency?

4. Mr. Elliott sent a draft to Birmingham, Eng., for $1,860 in gold. Exchange was $4.86. What was the face of the bill?

5. What is the value of a bill on Paris for 2,000 francs, exchange at 5.15, brokerage $\frac{1}{4}$ %?

6. What is the face of a draft on Paris that can be bought for $1,500, exchange at 5.16?

7. Harvey & Co. send $946.89 to Hamburg. What is the face of the draft, exchange at 94$\frac{1}{2}$?

8. What is the value of a bill of exchange on Bremen for 2,500 marks, exchange at 94$\frac{1}{2}$?

9. What was the cost in U. S. currency of 2,000 marks, when gold was at 110?

10. What is the value in marks of $3,000 in greenbacks, gold at 110, exchange at 94$\frac{1}{2}$?

11. The cost of a draft on Manchester for £280 is $1,363.60. What is the rate of exchange?

12. What is the cost of a bill of exchange on London for £200, exchange $4.87 and brokerage at $\frac{1}{4}$%?

13. What is the rate of exchange, if the face of a bill for 25,965 francs costs $5,000?

14. The face of a draft for 2,400 marks costs $567. What is the rate of exchange?

ange being 20$\frac{1}{2}$ marks to £1 sterling, what is the aft in marks that cost £50?

PROPORTION.

849. Causes and effects are said to be proportional; that is, if the cause is twice as great the effect should be twice as great, etc.

850. Causes may be labor, money invested, money spent, etc.; the corresponding effects are work done, money gained or lost, goods bought, etc.

851. The relation between two causes or between two effects may be expressed by common fractions; thus, $\frac{\$18}{\$6}$ may express the relation between two causes, the one being three times the other; $\frac{9 \text{ hats}}{3 \text{ hats}}$ may express the relation between two effects, the \$18 purchasing 9 hats and the \$6, 3 hats, each hat costing \$2.

852. The relation between the causes is always the same as the relation between the effects; thus $\frac{\$18}{\$6} = \frac{9 \text{ hats}}{3 \text{ hats}}$, each fraction being equal to 3; that is, the first cause is three times the second cause, and the first effect is three times the second effect.

853. A **ratio** is the relation between two numbers of the same kind; it is found by dividing the first number by the second; thus, $\frac{\$18}{\$6}$, written also, \$18 ÷ \$6, or \$18 : \$6; it equals 3, and is read, the ratio of \$18 to \$6.

854. A **proportion** is an equality of ratios; thus, $\frac{\$18}{\$6} = \frac{9 \text{ hats}}{3 \text{ hats}}$; written also, \$18 ÷ \$6 = 9 hats ÷ 3 hats; or, \$18 : \$6 : : 9 hats : 3 hats; or, 18 : 6 : : 9 : 3.

The last form is the usual form; it is, however, simply an abbreviation of the first form; it is read 18 *is to* 6 *as* 9 *is to* 3.

855. Proportion is an application of COMMON FRACTIONS, the common fraction expressing a ratio, and an equality of common fractions expressing a proportion.

NOTE.—The numerators and denominators are called *proportionals*.

856. The **antecedents** are the first and the third terms of a proportion.

NOTE.—The antecedents are the numerators of the fractions representing the ratios.

857. The **consequents** are the second and the fourth terms of a proportion.

NOTE.—The consequents are the denominators of the fractions representing the ratios.

858. The **means** are the second and the third terms of the proportion.

859. The **extremes** are the first and the fourth terms of the proportion.

860. A proportion being an application of common fractions, the principles applied to both are the same.

861. Illustrate the PRINCIPLES OF PROPORTION by using the following forms:

$$18 : 24 :: 12 : 16; \text{ or, } \frac{18}{24} = \frac{12}{16}.$$

I. *Multiplying or dividing an antecedent produces a like effect on the value of a ratio.*

II. *Multiplying or dividing a consequent produces an opposite effect on the value of a ratio.*

III. *Multiplying or dividing both antecedent and consequent by the same number produces no effect on the value of a ratio*

IV. *The product of the extremes is equal to the product of the means.*

NOTE.—Principle IV., thus: Reducing the fractions to a common denominator, $\frac{18 \times 16}{24 \times 16} = \frac{12 \times 24}{16 \times 24}$. Multiplying both fractions by 24×16, the denominators, gives $18 \times 16 = 12 \times 24$, the product of the extremes equal to the product of the means.

862. Proportion is subdivided as follows: Simple Proportion, Compound Proportion, Inverse Proportion, and Distributive Proportion.

NOTE.—Every problem in proportion contains a *question* and a *condition*, each of which contains two quantities, one in the question being unknown.

SIMPLE PROPORTION.

863. Simple proportion is a proportion in which each term consists of one number.

864. 1. If 15 barrels of flour cost \$75, what is the cost of 12 barrels?

1st cause is to 2d cause as 1st effect is to 2d effect.

x : \$75 : : 12 : 15

1st cause x	$\frac{5}{\cancel{75}}$	2d cause.
2d effect $\cancel{15}$	12	1st effect.

\$60. Ans.

Explanation.—1. Take the unknown term as the first term and call it x.

2. x represents a cause, as it is to buy 12 barrels of flour as an effect; the other cause is \$75 and the effect, 15 barrels bought.

3. Arrange the terms in their order: 1st cause : 2d cause : : 1st effect : 2d effect.

4. As the product of the extremes is equal to the product of the means, $15 \times x = 12 \times 75$; x therefore equals $12 \times 75 \div 15$, or 60. Ans., \$60.

NOTE.—As the first term then equals the second term times the third term divided by the fourth term, the third term may be written under the second, and the fourth at the left, with a vertical line between them, thus indicating the multiplication and the division, which may be abbreviated by cancellation.

2. How many barrels of flour can be bought for \$36, if 17 barrels cost \$68?

1st effect is to 2d effect as 1st cause is to 2d cause.

x : 17 : : \$36 : \$68

1st effect, x	$\cancel{17}$	2d effect.
2d cause, $\cancel{68}$	$\cancel{36}$ 9	1st cause.

3. For \$49, how many hats can be bought, if 2 of the same cost \$14?

865. Rules for Proportion.—I. 1. Write x as the first term to represent the unknown quantity in the question, the corresponding quantity in the condition being the second term; write the known quantity in the question as the third term, the corresponding quantity in the condition being the fourth term.

2. Multiply the second term by the third term and divide the product by the fourth term. The quotient is the first term or answer.

The fourth term may be cancelled with the second or the third term.

Or, II. 1. Write the causes on opposite sides of the vertical line, and also the effects, writing the first cause and the second effect on one side, and the second cause and the first effect on the other side.

2. The product of the two known quantities on one side divided by the known quantity on the other side, gives the unknown quantity.

NOTE 1.—Represent the unknown quantity by x, and make it the first term; it may, however, be the first, second, third, or fourth term.

NOTE 2.—If either term is a fraction, the numerator is treated as the term, the denominator being written on the other side of the line.

NOTE 3.—Cancellation is applied, whenever possible, between the numbers on opposite sides of the line.

866. Written Problems. 1. If 12 yards of cloth cost \$75, what is the cost of 8 yards at the same rate?

2. If 12 bushels of wheat cost \$48, how many bushels can be bought for \$216?

3. What is the cost of 90 gallons of wine, if 495 gallons cost \$1,980?

4. If 24 acres 3 roods of land rent for \$210, what is the rent of 36 acres 3 roods 20 perches, at the same rate?

Reduce both areas to square rods, or perches.

5. How much silk can be bought for $1,025, if 51 yards cost $425 ?

6. If a pole 30 feet high casts a shadow of 16 feet, what is the height of a church steeple whose shadow is 64 feet ?

7. If 180 bushels of oats feed 40 horses for 12 days, how long will 450 bushels feed them ?

8. A bushel of wheat weighs 60 lb. What is the price of 4,272 lb. of wheat, at $1.50 a bushel ?

9. A bushel of Indian corn is 56 lb. What is the price of 3,080 lb. of corn, at $.80 a bushel ?

10. If .25 of a gallon of wine cost $.40, what is the cost of 1.7 gallon ?

11. If $59\frac{1}{2}$ yards of cloth can be bought for $\$14\frac{7}{8}$, how much of the same cloth can be bought for $\$31\frac{1}{4}$?

12. If $6\frac{2}{3}$ yards of muslin cost $6.67, what will cost $4\frac{3}{8}$ yards of the same muslin ?

13. If $500 is sufficient to pay a debt of $524.375 due a year hence, how much will be sufficient to pay a debt of $5,000 due a year hence, at the same rate ?

Principal is proportionate to interest, but not to amount.

COMPOUND PROPORTION.

867. A **compound proportion** is a proportion in which each of two or more of the terms is the product of two or more numbers.

868. The term compounded may be a cause or an effect; thus, it may be men, working so many days, of so many hours each, in which the cause is hours' work, found by multiplying the number of men by the number of days, by the number of hours ; or, it may be the work done, as a ditch or a wall, with the length, breadth, and depth, or height giv
in *which the effect is* the product of the three dimensior

869. Rule for Compound Proportion.—1. Consider each term as a single number, even when it is an indicated product of several numbers.

2. Proceed as in simple proportion.

NOTE.—The unknown factor is represented by x.

870. Written Problems. 1. If 12 men build a wall 60 ft. long, 4 ft. thick, and 20 ft. high, in 24 days, working 12 hours a day, how many men will it take to build a wall 100 ft. long, 3 ft. thick, 12 ft. high, in 18 days, of 8 hours each?

Explanation.—1. Let x represent the unknown quantity. We have the proportion, the 1st cause, x men in 18 days of 8 hours each is to the 2d cause, 12 men in 24 days of 12 hours each, as the effect of the 1st cause, a wall 100 ft. by 3 ft. by 12 ft., is to the effect of the 2d cause, a wall 60 ft. by 4 ft. by 20 ft. This is reduced to the simple proportion, $144x : 3{,}456 : : 3{,}600 : 4{,}800$. Making the product of the means equal to the product of the extremes, the value of x is found to be 18. Ans., 18 men.

1st c.	2d c.	1st e.	2d e.
x	12	100	60
18	: 24	: : 3	: 4
8	12	12	20

$144x : 3{,}456 : : 3{,}600 : 4{,}800$

$691{,}200x = 12{,}441{,}600.$

$x = 18$ men. Ans.

Or,

1st c. { x | ~~100~~ 5 } 2d c.
{ ~~18~~ 6 | ~~3~~ 1 }
{ ~~8~~ 4 | ~~12~~ 3 }

2d c. { ~~60~~ 5 | ~~12~~ 3 } 1st c.
{ ~~4~~ 1 | ~~24~~ 6 }
{ ~~20~~ 5 | ~~12~~ 3 }

$x = 18$

2. If 2 loads of hay will serve 3 horses for 4 weeks, how long will 5 loads serve 6 horses, at the same rate?

3. If the 16-cent loaf of bread weighs 48 oz. when wheat is at $1.62 per bushel, what should be the price of wheat when the 12-cent loaf weighs $32\frac{2}{3}$ oz.?

4. If a man can travel 360 miles in 12 days of 8 hours each, how many hours a day must he walk, at the same rate, to complete 450 miles in 20 days?

[illegible] is paid for the use of $500 for 12 days, at the [illegible]at should be paid for the use of $350 for 9 days?

6. If 14 men mow 168 acres in 12 days of $8\frac{1}{4}$ hours each, how much can 20 men mow in 11 days of $7\frac{4}{5}$ hours each ?

7. If 5 women can finish a certain quantity of work in $10\frac{5}{8}$ days of $9\frac{3}{5}$ hours each, how long will it take 3 women to do twice the same work, counting 10 hours to the day ?

8. If 20 men can perform a piece of work in 12 days, how many men will it require to perform another piece of work, 3 times as great, in $\frac{1}{5}$ of the time ?

9. If 8 men can dig a trench 100 ft. long, 3 ft. broad, and 4 ft. 6 in. deep, in 9 days, how many will be required to dig a trench 80 ft. long, 5 ft. broad, and 2 feet deep, in $5\frac{1}{3}$ days ?

10. If 6 iron bars, each 4 ft. long, 3 in. broad, and 2 in. thick, weigh 288 lb., how much will 15 weigh, each $6\frac{1}{2}$ ft. long, 4 in. broad, and 3 in. thick ?

INVERSE PROPORTION.*

871. Inverse proportion is a proportion in which the two causes or the two effects are the same ; the other quantities are then equal and their factors vary *inversely ;* that is, as one increases the other decreases.

Thus : If 15 men in 25 days can build a bridge, how long will it take 20 men to build the bridge ? The effects being the same, the causes are equal, and the number of men increases, as the time decreases.

872. Rules for Inverse Proportion.—I. State the problem as in compound proportion, writing a letter as the constant quantity, and cancelling it as any other quantity.

Or, II. 1. The constant cause or effect is unwritten, being considered as cancelled ; the products representing the other quantities are written as equal quantities on opposite sides of the vertical line.

2. The known product divided by the given factors of the unknown product is the answer.

* Inverse and Distributive Proportion may be omitted or reserved for review after General Analysis.

873. Written Problems. 1. If 10 men build a wall in 4 days, how long will it take 6 men to build the same wall?

Explanation.—1. The 1st cause is 6 men in x days; the second cause, 10 men in 4 days. The effect in each case is the wall. The compound proportion may be reduced to the simple proportion $6\ x : 40 : : w : w$. As the 3d term is equal to the 4th, the 1st is equal to the 2d; that is, $6\ x = 40$, and $x = 6\frac{2}{3}$ days. Ans.

1st c.	2d c.	1st e.	2d e.
x	4		
6	10	: :	$w : w$
$6x$	40	: :	$w : w$

$x = 6\frac{2}{3}$ days.

1st c.	2d c.
x	~~4~~ 2
~~3 6~~	10
w	w
$3x = 20$	
$x = 6\frac{2}{3}$	

2. If 16 men in 9 hours can dig a ditch, how many men can dig the same ditch in 6 hours?

3. 100 men built a bridge in 5 weeks. How many men would it have taken to build it in 2 weeks?

4. How long will it take 300 men to pave a street, if 200 men can pave it in 60 days?

5. If 80 men build a house in 40 days of 10 hours each, how many days of 8 hours each will be required by 50 men?

6. A man bought 30 barrels of flour at $8 per barrel. How many barrels, at $6, can he buy for the same amount?

7. A man digs a ditch 14 feet long, 10 feet wide, and 6 feet deep, in 6 days. How deep a ditch, 12 feet long, and 10 feet wide, can he dig in the same time?

8. A borrowed from B $876.25 for 102 days, and afterward returned the favor by lending the sum of $1,051.50. For what time should he lend it to balance the favor?

9. A coach-wheel 7 ft. 8 in. in circumference revolves 3,762 times in performing a certain journey. What is the circumference of a wheel which makes only 3,496 revolutions in the same distance?

10. How many dozen pairs of gloves, at $.50 a pair, should be given for 24 dozen pairs of stockings, at $1.25 a pair?

DISTRIBUTIVE PROPORTION.

874. Distributive proportion is a proportion in which a given number (cause or effect) is divided into parts having a given ratio; the given number is then taken as a cause, its effect being represented by the sum of the numbers that form the ratio.

Thus: Divide \$500 into 2 parts that bear to each other the ratio of 2 : 3. Here \$500 is taken as a cause and 5 (2+3) represents its effect. x : \$500 : : 2 : 5; and, x : \$500 : : 3 : 5.

875. Rule for Distributive Proportion.—Form a separate proportion for each part, comparing the unknown part with the whole, as the number representing it in the given ratio is compared with the sum of the numbers in the ratio.

876. Written Problems. 1. Divide \$24 between John and Henry so that John's share shall be to Henry's as 3 to 5.

x : \$24 : : 3 : 8
$8x = \$72$. $x = \$9$

x : \$24 : : 5 : 8
$8x = \$120$ $x = 15$

$x \mid \overset{3}{\cancel{\$24}}$ $x \mid \overset{3}{\cancel{\$24}}$
$\cancel{8} \mid 3$ $\cancel{8} \mid 5$
$x = \$9$ $x = \$15$

Explanation.—1. \$24 is represented by 8, the sum of the numbers in the ratio. x, representing John's share, is to \$24, the whole sum, as 3, which represents John's share, is to 8.

2. In like manner, x, representing Henry's share, is to \$24 as 5, Henry's share in the ratio, is to 8. Answer, $x = \$9$, John's share; $x = \$15$, Henry's share.

2. A farm of 325 acres is divided between two brothers. Their shares are to each other as 5 is to 8. How many acres does each brother receive?

3. A man, having 1,250 head of sheep, sells part of them. The number he sells is to the number he has left as 2 is to 3. How many did he sell, and how many had he left?

4. A and B start a business in partnership, A investing \$9,000. Their losses in one year, when equitably divided, *are \$600 for A and \$1,000* for B. How much was B's capital?

MISCELLANEOUS PROBLEMS.

877. Written Problems. 1. What is the height of a steeple whose shadow was 170 feet 8 inches, at the same time that the shadow of a rod 6 feet 6 inches high was 5 feet 4 inches?

Horizontal shadows are proportional to perpendicular heights.

2. If 3,000 copies of a book of 11 sheets require 66 reams of paper, how much paper will be required for 5,000 copies of a book of $12\frac{1}{2}$ sheets?

3. 9 men build a wall in $16\frac{3}{7}$ days. How many men would be required to build the same in $24\frac{9}{14}$ days?

4. Divide 72 into three parts which shall be to each other as 1, 2, and 3.

5. What is the elevation per 100 feet in a road rising 462 feet in one mile and a half?

6. If 7 men earn \$45.72 in $10\frac{1}{2}$ days, what sum, at the same rate, will pay the wages of 28 men for $31\frac{1}{2}$ days?

7. A piece of iron is beaten into a sheet $\frac{3}{8}$ of an inch thick, and covering 17 square feet. What would be its surface, if its thickness were decreased to $\frac{3}{5}$ of an inch?

8. A father divides \$9,500 between his three sons, so that their shares are to each other as 4, 6, and 9. How much does each one receive?

9. If 18,656 bricks are required to build a wall 176 yards long, how many are required for a wall of the same height and thickness, and 369 feet long?

10. If \$100 gain \$80 in 15 months, what sum will gain \$120 in 3 months, at the same rate?

11. A coach goes a certain distance in 24 hours, at the rate of 9 miles an hour. In what time would the distance be travelled by rail, at the rate of 32 miles an hour?

12. The ages of three sisters are to each other as 3, 4, and 5. The sum of their ages is 65. What is the age of each?

PARTNERSHIP.

878. Partnership is the association of two or more persons for the transaction of business, and for the sharing of profits and losses in the business.

Such an association may be called a *firm*, a *company*, a *house*, etc.; and their name is the firm *style*.

879. An **active partner** is one of the persons associated in business, and having a voice in the management.

880. A **silent partner** is a partner who takes no part in the management, and whose liability is limited to his investment.

A silent partner is sometimes called a *special* partner.

881. The **liabilities** are the debts of a firm.

882. The **assets** are the property of a firm.

883. Profits and losses are distributed by distributive proportion, the share of profit or of loss being proportionate to the share of capital.

884. Capital is that which is invested in the business; it may be money, securities, other property, or services.

The private property of each active or general partner may become security for firm liabilities after the firm assets have been exhausted.

885. Rule for Partnership.—Form a distributive proportion for each partner, the terms being the entire capital, each partner's capital, the entire gain or loss, and each partner's gain or loss.

NOTE.—Where the partners have invested their capital for different times, take instead of the partners' capital the product of the capital by the time, and instead of the entire capital the sum of the products of capital by time.

886. Written Problems. 1. A and B engaged in business as partners. A invested \$9,000, and B \$7,000. They gained \$2,400. What was each man's share of the gain?

Explanation. —1. A's share of the gain, or x, is to \$2,400, the whole gain, as \$9,000 is to \$16,000.

x : \$2,400 : : \$9,000 : \$16,000
16,000 x = \$21,600,000
x = \$1,350

x : \$2,400 : : \$7,000 : \$16,000
16,000 x = \$16,800,000
x = \$1,050

Or,

x	\$2,400
16,000	9,000

16,000 x = \$21,600,000
x = \$1,350

x	\$2,400
16,000	7,000

16,000 x = \$16,800,000
x = \$1,050

2. B's share of the gain, or x, is to \$2,400 as \$7,000, B's capital, is to \$16,000, the whole capital.

3. By dividing the product of the means, in each case, by the given extreme, we have as an answer, \$1,350 for A, and \$1,050 for B.

2. A and B form a partnership. A puts in \$800, and at the end of 3 months B puts in \$1,000. At the end of 6 months from the time at which B puts in his capital, the profits to be divided are \$330. What should each receive?

Explanation. —1. A has \$800 invested for 9 months, which is the same as \$7,200 for 1 month. B. has \$1,000 invested for 6 months, or \$6,000 for 1 month.

A, \$800 × 9 = \$7,200
B, \$1,000 × 6 = \$6,000
x : \$330 : : \$7,200 : \$13,200
13,200 x = \$2,376,000
x = \$180

x : \$330 : : \$6,000 : \$13,200
13,200 x = \$1,980,000
x = \$150

Or,

x	\$330
13,200	7,200

13,200 x = \$2,376,000
x = \$180

x	\$330
13,200	6,000

13,200 x = \$1,980,000
x = \$150

2. A's gain, x, is to \$330, the whole gain, as \$7,200, representing A's investment, is to \$13,200, representing the entire investment.

3. B's gain, x, is to \$330, the whole gain, as \$6,000, representing B's investment, is to \$13,200, the whole investment.

4. Dividing the product of the means, in each proportion, by the extremes, A, \$180; B, \$150. Ans.

3. A, B, and C form a joint capital in trade, of which A contributes $2,500; B, $3,250; and C, $3,500. At the end of a year the profits are $2,775. What is each man's share?

4. A, B, and C rent a pasture for $250. A puts in 3 times as many sheep as B, and B twice as many as C. How much should each pay?

5. A, B, and C rent a pasture together for $90 per annum. A put in 6 oxen for 4 months, B put in 10 oxen for 3 months, and C put in 3 oxen for 12 months. How much of the rent should each person pay?

6. A and B form a joint capital for conducting a business. A invests £300, and B, £500. At the end of a year they have gained £187. What is the share of each in the profits?

7. A ship's cargo is valued at $21,600. Of this, A owns $9,600, B $7,200, and C $4,800. To save the cargo in a storm, goods to the value of $9,000 are thrown overboard. How much of this loss must A, B, and C each sustain?

8. A, B, and C form a joint capital in business for 1 year. A puts in $600; B, $1,200; C, $1,800; the profit to be divided at the end of the year is $1,709.04. What is each person's share?

9. Three merchants trade together with a capital of which A contributes $3,000 for 6 months, B, $2,500 for 9 months; and C, $2,000 for 12 months. The whole gain being $2,042.50, what was each man's gain?

10. A, B, C, and D unite their capital in the proportion of 5, 7, 9, and 11. At the end of 6 months B doubles his capital, and at the end of 12 months they divide a profit of $42,600. What should each man receive?

11. A, B, and C join their capitals, which are in the proportion of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. At the end of 4 months A withdraws $\frac{1}{2}$ of his capital, and at the end of 9 months more
their *profits, $1,420.* What should each man re

GENERAL ANALYSIS.

887. General Analysis is the explanation of the solution of any problem by reference to unity.

888. Oral Problems. 1. If 6 men do a piece of work in 4 days, how long will it take 8 men?

If 6 men do the work in 4 days, it will take one man 6 times 4 days, or 24 days. 8 men will do the work in $\frac{1}{8}$ of 24 days, or 3 days. Ans.

2. If 5 men, working 8 hours a day, can build a wall in 3 days, how long will it take 6 men, working 5 hours a day, to build the same wall?

The work of 5 men for 3 days of 8 hours each equals the work of one man for $5 \times 3 \times 8$ hours, or 120 hours. If one man can do the work in 120 hours, 6 men can do it in $\frac{1}{6}$ of 120 hours, or 20 hours. If 5 hours equals 1 day, 20 hours equals as many days as 5 is contained times in 20, or 4. Ans., 4 days.

3. John lost $\frac{1}{4}$ of his money and spent $\frac{1}{3}$ of the remainder, when he had $.25 left. How much money had he at first?

4. Mrs. Ellis paid $15 for 12 yards of silk. How much would 20 yards have cost her?

5. Three boys together have $.80. The second has 3 times as much as the first, and the third has 4 times as much as the second. How much money has each boy?

6. If 3 men require 4 days of 7 hours each to perform a certain work, how many men can perform the same work in 7 days of 6 hours each?

7. If $\frac{1}{4}$ of a ton of coal costs $1.375, how much will 3 tons cost at the same rate?

rles and Henry together have 28 marbles. For every rles has, Henry has 4. How many marbles has each

9. The cost of 4 pounds of cheese is equal to the cost of 3 dozen eggs. How many pounds of cheese cost as much as 9 dozen eggs?

10. If a man can travel 12 miles 160 rods in 3 hours, how far can he travel in 6 hours?

11. One man does a piece of work in 8 days, and another in 12 days. How many days will it take them to do the work together?

12. What number is that which, when multiplied by the difference between $\frac{3}{4}$ and $\frac{1}{8}$ of itself, gives 40?

13. What is the cost of 3 gallons of milk, when 2 quarts 1 pint cost \$.20?

14. A and B work 5 days of 6 hours each. How many hours a day for 8 days must C work to equal the labor of both?

15. The cost to a grocer of 4 pounds 8 ounces of tea was \$1.80. How much did he gain on 3 pounds, if he sold it at \$.60 a pound?

889. Written Problems. 1. If 80 men build a bridge in 24 days, how many men can build the bridge in $\frac{1}{3}$ of the time?

$\frac{1}{3}$ of 24 da. = 8 da.

$\frac{24 \times 80 \text{ men}}{8} = 240$ men.

$\frac{1}{3}$ of the time equals 8 days. If 80 men build the bridge in 24 days, one man can build it in 80 × 24 days, or 1,920 days. To build it in 8 days, it will take as many men as 8 is contained times in 1,920, or 240. Ans., 240 men.

2. A can build a fence in 12 days, and B can build the same fence in 8 days. How long will it take them, working together, to build $\frac{1}{3}$ of the fence?

$\frac{1}{12} + \frac{1}{8} = \frac{5}{24}$.

$\frac{1}{3} \div \frac{5}{24} = \frac{8}{5} = 1\frac{3}{5}$ da.

If A can build the fence in 12 days, he can build $\frac{1}{12}$ of it in 1 day. B can build $\frac{1}{8}$ in 1 day. Together they can build the sum of $\frac{1}{12}$ and $\frac{1}{8}$, or $\frac{5}{24}$. If they can build $\frac{5}{24}$ of the fence in 1 day, they can build $\frac{1}{3}$ of it in as many days as $\frac{5}{24}$ is contained times in $\frac{1}{3}$—that is, $\frac{1}{3} \div \frac{5}{24}$—which *equals* $1\frac{3}{5}$ days.

890. Rule for General Analysis.—From the condition of the problem, find the value of one unit; and then find the value of the required number of units or the required part of a unit.

NOTE.—Good practice may be found in solving the problems under proportion by analysis.

891. Written Problems. 1. A man sold his horse and carriage for $675. He received $3\frac{1}{2}$ times as much for the horse as for the carriage. How much did he receive for the horse?

For both horse and carriage he received $(3\frac{1}{2} + 1 =) 4\frac{1}{2}$ times the value of the carriage. Therefore, $3\frac{1}{2} \times (\$675 \div 4\frac{1}{2}) = ?$

2. A drover had a large herd of cattle. He lost $\frac{1}{8}$ of them during a storm; he sold $\frac{2}{3}$ of the remainder, and had left 910 head of cattle. How many head of cattle had he at first?

910 is $\frac{1}{3}$ of $(3 \times 910 =) 2,730$, remainder after the storm. This was $\frac{7}{8}$ of the entire herd; therefore, $\frac{8}{7} \times 2,730 = ?$

3. A and B start together in business. A puts $5,000 in the business, and B puts in $9,000. At the end of a year, B's share of the profits was $1,980. What was A's share?

4. A man sells 56 shares of R.R. stock at 75 per share and invests the proceeds in stock at 105. How many shares of the latter does he buy?

56 shares sold for $4,200. $105 purchases 1 share; therefore, $\$4,200 \div \$105 = ?$

5. If a piece of ground 120 rods by 180 rods sells for $16,200, what will be the value, at the same rate, of a piece of ground containing $\frac{3}{4}$ of a square mile?

6. 276 spiders will produce as much silk as 23 silk-worms. ny spiders will be required to produce as much as 100 ıs?

7. A man bequeaths his property to his wife, son and daughter. To his wife he leaves $2\frac{1}{2}$ times as much as to his son; and to his daughter he leaves $\frac{1}{2}$ as much as to the son. The wife receives $15,000 more than the son. How much does each receive?

8. If 18 horses cost in keeping $30 a week, what will 90 horses cost per week?

9. If 30 ounces of gold are worth $525, what will be the value of 18 ounces?

10. If 19 oxen cost $1,330, and 2 oxen are worth 14 sheep, what is the price of each sheep?

11. If 8 lb. of coffee cost $2.50, what is the cost of 100 lb.? How many pounds can be bought for $6.25?

12. If the cost of 42 acres of land is $330, what is the cost of 91 acres of the same land? How many acres of it can be bought for $605?

13. Rations for ten men for 24 days will last 15 men how many days?

14. If 50 workmen can do a piece of work in 12 days, how many days will it take 75 workmen to do the same work?

15. If 3 men can mow 8 acres of wheat in 2 days, how long will it take 5 men to mow 20 acres, at the same rate?

16. If 40 bushels of corn feed 12 horses 37 days, how many days will 195 bushels of corn feed 9 horses?

17. A man travels 90 miles in 3 days of 8 hours each, in how many days of 6 hours can he travel 540 miles?

18. If 2 loads of hay will serve 3 horses for 4 weeks, how long will 5 loads serve 6 horses, at the same rate?

19. If a man earns $30 in 15 days of 8 hours each, in how many days of 10 hours each can he earn $60 at the same rate?

20. If £17. 16s. pay 8 laborers for 18 days, how many laborers, *at the same rate*, will £13. 7s. pay for 12 days?

INVOLUTION.

892. A **power** is a number formed by taking a factor one or more times.

893. The powers of a number are called the first, second, third, etc., power, according as the factor is used one, two, three, etc., times.

894. A **square** is the second power of a number; thus, 9 is the *square* of 3.

895. A **cube** is the third power of a number; thus, 27 is the *cube* of 3.

896. The exponent of a power is a small figure written at the right and above a number to denote the degree of the power; it shows how many times the number is used as a factor.

In 5^2, 7^3, 5^3, 12^3, 15^7, 2, 3, and 7 are the *exponents*, indicating the 2d, 3d, and 7th powers.

897. Involution is the process of finding any power of a number.

898. Rules for Involution.—I. To find any power of a number, multiply the number by itself, using it as many times as a factor as there are units in the exponent of the power.

II. To indicate the product of two or more powers of the *same number*, simply add the exponents.

III. To indicate the quotient of one power of a number divided by another power of the *same number*, subtract the exponent of the divisor from the exponent of the dividend.

Thus, $5^3 \times 5^4 = 5^7$, for $5^3 = 5 \times 5 \times 5$, and, $5^4 = 5 \times 5 \times 5 \times 5$, the product being $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$ or 5^7; and, $5^7 \div 5^3 =$ or $5^7 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$ and $5^3 = 5 \times 5 \times 5$, and ng the one by the other gives $5 \times 5 \times 5 \times 5$, or 5^4.

899. Find the following powers:

1. 27.3^2	6. 1.0101^3	11. $(11\frac{1}{11})^4$	16. 15^5
2. 81^2	7. $.99^3$	12. 25.5^4	17. 3.2^5
3. 1.01^2	8. $.000089^2$	13. $.001^4$	18. $.089^5$
4. $.0009^2$	9. 55.005^3	14. $.01^4$	19. $.0001^5$
5. $(\frac{3}{4}\frac{2}{5})^2$	10. $(\frac{1}{14}\frac{3}{4})^3$	15. $.1^4$	20. 1.09^5

900. Find the value of the following:

1. $.01^3 \times .9^2 = ?$	5. $.01^3 \div 1$	9. $8.4^3 \div (\frac{3}{4})^3$
2. $2.1^2 \times (5\frac{1}{2})^3 = ?$	6. $.1^4 \div 1^4 = ?$	10. $5^5 \div .625$
3. $9.3^4 \times (\frac{1}{2})^5 = ?$	7. $2.2^3 \times .01^2 = ?$	11. $4.^6 \div 2.4^3$
4. $12 \div .20736$	8. $(1\frac{1}{4})^4 \times (\frac{5}{4})^3$	12. $10.5^2 \times .2205$

EVOLUTION.

901. Evolution is the process of finding the *equal* factors of a number.

902. The **root** of a number is one of its equal factors.

903. The root of a number is called the second, third, fourth, etc., root, according as it is one of its two, three, four, etc., equal factors.

904. The **square root** of a number is one of its two equal factors.

5 is the *square root* of 25.

905. The **cube root** of a number is one of its three equal factors.

3 is the *cube root* of 27.

906. The **radical sign** is $\surd$; it indicates that some root of the number before which it is placed is to be found.

Thus, $\surd 25$ indicates the square root of 25; $\sqrt[3]{125}$ indicates the cube root of 125.

907. The **index** of a root is a small figure written before and above the radical sign to indicate what root is to be found.

In $\sqrt[3]{1728}$, $\sqrt[4]{625}$, 3 and 4 are the indices; and indicate respectively *that the 3d and the* 4th root are to be found.

908. Illustrate the PRINCIPLES OF EVOLUTION by using the four following forms:

$1 \times 1 =$	$1 \times 1 \times 1 =$	$1 \times 1 \times 1 \times 1 =$
$9 \times 9 =$	$9 \times 9 \times 9 =$	$9 \times 9 \times 9 \times 9 =$
$99 \times 99 =$	$99 \times 99 \times 99 =$	$99 \times 99 \times 99 \times 99 =$
$(t + u)^2 = t^2 + 2tu + u^2$	$(t + u)^3 = t^3 + 3t^2u + 3tu^2 + u^3$	

I. *Every place in a square root may require two places in the second power.*

II. *Every place in a cube root may require three places in the third power.*

III. *The square of a number containing tens is equal to the square of the tens plus twice the product of the tens by the units plus the square of the units.*

IV. *The cube of a number containing tens and units equals the cube of the tens plus three times the square of the tens multiplied by the units, plus three times the tens multiplied by the square of the units, plus the cube of the units.*

SQUARE ROOT.

909. Inductive Exercises:

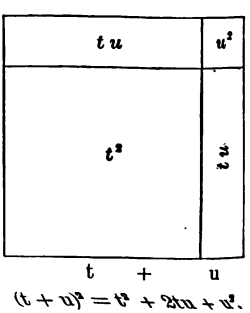
$1^2 = 1.$	$10^2 = 100.$	$100^2 = 10{,}000.$
$9^2 = 81.$	$99^2 = 9{,}801.$	$999^2 = 998{,}001.$

From the preceding, it is evident that for each place in a number there may be two places in the square of the number.

1. What is the square of 25 ?

$25 =$ tens + units.
$\underline{25}$
$\overline{25} =$ sq. of units.
$10 =$ units $\times$ tens.
$10 =$ units $\times$ tens.
$4 =$ sq. of tens.
$= $ sq. of 25.

$t + u = 25$
$\underline{t + u}$
$tu + u^2$
$t^2 + tu$
$t^2 + \overline{2tu} + u^2 =$
$t^2 + (2t + u)\,u.$



$(t + u)^2 = t^2 + 2tu + u^2.$

2. What is the square root of 625 ?

Explanation.—1. Beginning at the right, separate the figures into *periods* of two places each, as two figures of the power correspond to one in the root.

```
 . .
625)25
 4
45|225
  |225
```

2. The first period must contain the square of the tens. The greatest square in 6 is 4. Write its square root, 2, in the answer. Subtract the square of 2 from 6 and bring down the next period.

3. It is evident that the remainder equals twice the product of the tens by the units, plus the square of the units ($2tu + u^2$), which equals twice the tens plus the units, all multiplied by the units $(2t + u) \times u$. If, then, twice the tens be used as a trial divisor the quotient may be the units. The complete divisor, however, is twice the tens plus the units.

4. Multiply twice the tens plus the units, that is, the complete divisor by the units, and subtract. Answer, 25.

910. Rule for Square Root.—1. Separate the given number in both directions from the decimal point into periods of two places each.

2. Find the square root of the greatest square in the first period at the left, and place it in the answer as the first figure in the root. Subtract the square of this number from the first period and bring down the next period.

3. Use the remainder thus found, except its lowest order, as a trial dividend, and twice the part of the root already found, as a trial divisor. Write the quotient in the root and also at the right of the trial divisor. Multiply the completed divisor by the part of the root just found and subtract the product from the entire dividend.

4. If any product is greater than the dividend, too large a number has been taken for that order in the root.

5. If a trial divisor exceeds the trial dividend, place a cipher in the root and also at the right of the trial divisor; then bring down the next period and proceed as before.

911. Find the square root:

1. 9,216.	5. $\frac{289}{361}$.	9. .00010201.	13. 14,496.16.
2. 22,801.	6. $\frac{529}{676}$.	10. .0000015625.	14. 1,00020001.
3. 15,625.	7. $1\frac{369}{529}$.	11. .9801.	15. $287\frac{401}{625}$.
4. 62,500.	8. $7\frac{1}{9}$.	12. 1.4641.	16. 1.72265625.

912. Find the value of the following:

1. $\sqrt{.20736} + \sqrt{20736} = ?$	4. $\sqrt{207\frac{9}{16}} \div \sqrt{.45\frac{1}{4}^2} = ?$
2. $(1.7)^4 \div \sqrt{2.89^4} = ?$	5. $\sqrt{1\frac{9}{16}} \div \sqrt{1\frac{9}{16}} = ?$
3. $1.28 - \sqrt{.262144^3} = ?$	6. $\sqrt{\frac{5}{4}^2} \div \sqrt{\frac{5}{4}^2} = ?$

NOTE.—$\frac{1}{2}$, as an exponent, indicates square root.

CUBE ROOT.

913. Inductive Exercises:

$1^3 = 1.$	$10^3 = 1{,}000.$	$100^3 = 1{,}000{,}000.$
$9^3 = 729.$	$99^3 = 970{,}299.$	$993^3 = 996{,}002{,}999.$

914. From the preceding it is evident that for each place in a number, there may be three places in its cube.

1. What is the cube of 25 ?

$25 =$ tens + units.

25

$625 = t^2 + 2tu + u^2$

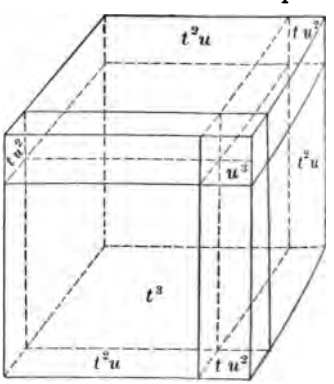
$25 = t + u.$

$3125 = t^2u + 2tu^2 + u^3$

$1250 = t^3 + 2t^2u + tu^2$

$15{,}625 = t^3 + 3t^2u + 3tu^2 + u^3$

$= t^3 + (3t^2 + 3tu + u^2)\,u$



$(t + u)(t + u) = t^2 + 2tu + u^2$; $(t + u)^2(t + u) = t^3 + 3t^2u + 3tu^2 + u^3$

$= t^3 + (3t^2 + 3tu + u^2)u$

NOTE.—The t^3 having been removed, the units may be found by dividing the remainder of the given number by the quantity in the par-

. The quantity in the parenthesis is called the divisor, or ete divisor ; it consists of three terms. The first term, d the *trial* divisor.

2. What is the cube root of 15,625 ?

Explanation.—1. Separate the figures into periods of three places each, beginning at the decimal point.

	15,625	25
	8	
1,200	7,625	
300		
25		
1,525	7,625	

2. The largest cube in the first period (t^3) is evidently 8, and 2 is its cube root. Write 2 in the root, subtract 8 from 15, and bring down the next period.

3. It is evident from the preceding, that the units (u) may be found by dividing the remainder of the given number, and that the divisor consists of three parts, the trial divisor being three times the square of the root already found $3t^2$. Dividing 7,625 by $(3 \times 2^2 =)$ 1,200, gives 5 (6 having been found too large).

4. Complete the divisor by adding to 1,200 ($3t^2$) three times the product of the tens by the units $(3tu =)$ $(3 \times 20 \times 5 =)$, 300, and the square of the units, $(u^2 =)$ 25. Multiply the completed divisor by 5 and subtract. If there is no remainder the exact root has been found.

915. Rule for Cube Root.—1. Separate the given number in both directions from the decimal point into periods of three places each.

2. Write the cube root of the greatest cube in the first period as the first figure in the root ; subtract the cube from the first period and bring down the next period.

3. Use the remainder as the dividend, and the square of the root already found, multiplied by 300, as a trial divisor. Write the quotient at the right in the root.

4. Complete the trial divisor by adding to it the product of the part of the root already found multiplied by the part just found and by 30, and the square of the root just found.

5. Multiply the complete divisor by the part of the root just found, and then subtract and proceed as before.

6. If any product is greater than the remainder, too large a number has been taken for that part of the root.

7. If any trial divisor is greater than the dividend, write a cipher in the root, and also in the trial divisor ; then bring *down the next period* and proceed as before.

916. Find the cube root:

1. 166,375.
2. 704,969.
3. 421,875.
4. 27,270,901.
5. 1,259,712.
6. $\frac{1331}{2744}$.
7. $\frac{175616}{195313}$.
8. $17\frac{1}{343}$.
9. $2{,}032\frac{8}{27}$.
10. $1{,}027\frac{624}{1331}$.
11. 6.859.
12. .006561.
13. .035937.
14. 3.442951.
15. 9.391821.

917. Find the value of the following:

1. $\sqrt[3]{4.4} + \sqrt[3]{1.5} = ?$
2. $\sqrt[3]{\frac{1728}{4913}} + \sqrt[3]{\frac{1000}{39304}} = ?$
3. $\sqrt[3]{.1331} + \sqrt[3]{.729} = ?$
4. $\sqrt[3]{1{,}522.756}$.
5. $\sqrt{248\frac{1}{16}}$.
6. $\sqrt{169} - 121^{\frac{1}{2}} = ?$
7. $\frac{20\frac{1}{4}}{\sqrt{9^2}} \times \frac{9^2}{\sqrt{20\frac{1}{4}^2}} = ?$
8. $\sqrt[3]{9.469 - .029791} = ?$
9. $\overline{1951.375 + (3048.625)^{\frac{1}{3}}} \div \sqrt[3]{.000125}$
10. $4^{\frac{1}{2}} + \sqrt[3]{64} + 729^{\frac{1}{2}} \times \sqrt[3]{.064}$
11. $\sqrt[3]{1{,}418{,}481}$.
12. $\sqrt{20.25}$.
13. $\sqrt{289^2} + 289^{\frac{1}{2}} = ?$
14. $82.81 + .0729 \times \frac{2}{3}$ of $\sqrt{2.16} = ?$

$\frac{1}{2}$ as an exponent is the same as 3 as an index; if there is no index over the radical sign, 2 is understood to be the index.

918. To find the fourth (2×2) root, find the square root of the square root; to find the sixth (2×3) root, find the square root of the cube root; to find the eighth (2×4) root, find the square root of the fourth root.

919. Find the following roots:

1. $\sqrt[4]{334.1696}$
2. $\sqrt[4]{.00000081}$
3. $\sqrt[3]{1.030301}$
4. $\sqrt[4]{.00000001}$
5. $\sqrt[6]{.000064}$
6. $\sqrt[8]{2.30463981}$
7. $\sqrt[6]{.000000000001}$
8. $\sqrt[6]{191{,}102{,}976}$
9. $\sqrt[8]{390625}$
10. $\sqrt[8]{.00000256}$
11. $\sqrt[6]{.16777216}$
12. $\sqrt[4]{2687.3856}$

Practical applications of square and cube root will be found in Mensuration.

MENSURATION.

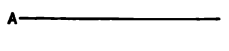
920. Mensuration treats of the measurement of angles, lines, surfaces, and solids.

921. A **line** is that which has length only.

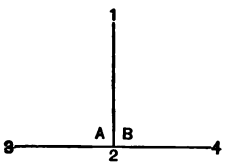
922. An **angle** is the difference in direction of two straight lines that meet.

923. A **surface** is that which has length and breadth only.

924. A **solid** is that which has length, breadth, and thickness.

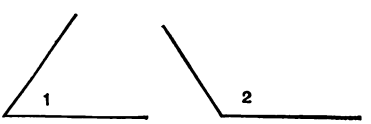


925. Lines that have the same direction, being everywhere equally distant, are **parallel lines**; as A and B.



926. If one straight line meets another straight line so as to make the two angles equal, each angle is called a **right angle**, and the one line is said to be **perpendicular** to the other line.

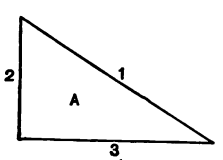
Thus, A and B are right angles and $\overline{1\,2}$ is perpendicular to $\overline{3\,4}$.



927. An angle less than a right angle is an **acute angle**; an angle greater than a right angle is an **obtuse angle.**

1 is an *acute* angle and 2 is an *obtuse* angle.

928. A **triangle** is a surface bounded by three straight lines.



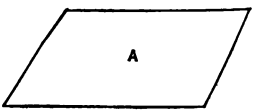
929. A *right-angled triangle* is a triangle one of whose angles is a right angle.

A is a *right-angled* triangle.

930. The **hypothenuse** is the side opposite the right angle; the side on which the triangle stands is the **base**; the other side is called the perpendicular, or the altitude.

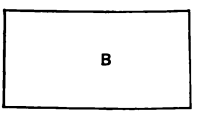
931. A **quadrilateral** is a four-sided figure.

932. A **parallelogram** is a quadrilateral with its opposite sides parallel.



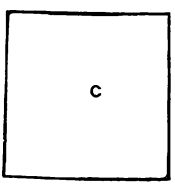
The figure A is a *parallelogram.*

933. A **rectangle** is a parallelogram, each of whose angles is a right angle.



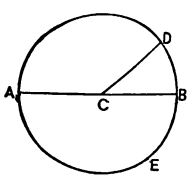
The figure B is a rectangle.

934. A **square** is a rectangle with all of its sides equal.



The figure C is a square.

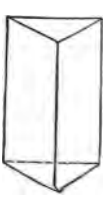
935. A **circle** is a plane figure bounded by a curved line, all the points of which are equally distant from a point within called the centre.



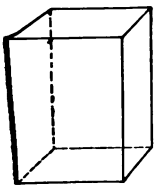
936. The line that bounds a circle is the **circumference**; any straight line from the centre to the circumference is a **radius**; any straight line drawn from circumference to circumference through the centre is a **diameter.**

A D B E is the *circumference;* C, the *centre;* A B, a *diameter;* and C D, a *radius.*

937. A **prism** is a solid with two ends or bases equal and parallel; a prism is triangular or square, as the base is a triangle or a square.

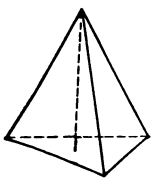


re represents a triangular prism.

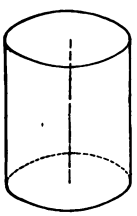


938. A **parallelopiped** is a prism, whose bases are parallelograms.

The figure represents a right (upright) parallelopiped.

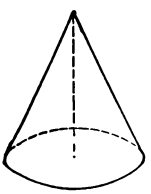


939. A **pyramid** is a solid with one base, and with triangles for its faces. The vertices of the triangles meet at a common point called the *apex*. If the line connecting the apex with the centre of the base is perpendicular to the base, the pyramid is a **right pyramid.**



940. A **cylinder** is a solid with two equal and parallel circles for bases and a curved surface. The curved surface is called its **convex surface.**

The figure represents a right cylinder.



941. A **cone** has one circular base and a curved surface terminating in a point called the apex.

The figure represents a right cone.

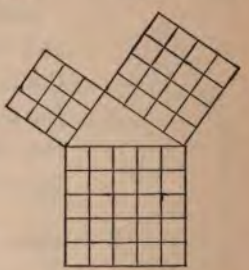
942. The **altitude** of a prism, cylinder, pyramid, or cone, is the perpendicular distance between its two ends, or its base and apex.

943. A **sphere** or globe is a solid having a curved surface, all the points of which are equally distant from a point within called the centre. The **radius** is any straight line from the centre to the surface. The **diameter** is any straight line passing through the centre and extending from surface to surface.

LINES.

944. Rules.—I. The hypothenuse of a right-angled triangle equals the square root of the sum of the squares of the two sides.

II. A side of a right-angled triangle equals the square root of the difference of the squares of the hypothenuse and the other side.



III. The circumference of a circle equals 3.1416– times the diameter; or 6.2832– times the radius.

Circumference is nearly 3¼ times the diameter.

IV. In similar figures corresponding lines are proportional.

945. Written Problems. 1. Required the hypothenuse of a right-angled triangle whose sides are 24 and 32 feet.

2. Find the hypothenuse of a right-angled triangle whose sides are 2 inches and .99 of an inch.

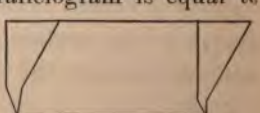
3. If a ladder 103.44 feet long be placed so as to reach a window 40 feet high on one side of a street, and a window 60 feet high on the other side, what is the breadth of the street?

4. Find the circumference of a circle whose radius is 6.3662 yards.

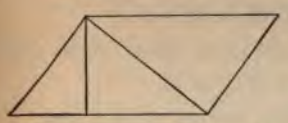
5. If the diameter of the earth is 7,912 miles, what is the length of a meter, or one ten-millionth part of a quarter of the circumference?

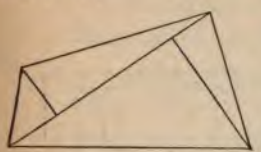
SURFACES.

946. Rules.—I. The area of a parallelogram is equal to the product of one side by the perpendicular distance between it and the other side.



II. The area of a triangle equals half of the product of either side by the perpendicular to it from the opposite vertex; **or,** from half the sum of the three sides subtract each side separately; multiply together the half sum and the three remainders; and the square root of the product is the area.





III. The area of any quadrilateral equals half the product of either diagonal by the sum of the perpendiculars to it from the opposite angles.

IV. The area of a circle equals 3.1416– times the square of the radius.

V. The convex surface of a cylinder equals the circumference of the base by the altitude.

VI. The convex surface of a right cone equals the circumference of the base by half the slant height.

VII. The surface of a sphere equals 3.1416– times the square of the diameter.

VIII. Similar areas are proportional to the squares of the corresponding lines.

947. Written Problems. 1. What is the area of a parallelogram, one of whose sides is 37 feet, and its distance from the opposite side 10 feet 6 inches?

2. Find the area of a triangle whose base is 40 feet, and altitude 15 feet.

3. If the sides of a triangle are 16.6, 18.32, and 28.6 feet, what is its area?

4. If the diagonal of a quadrilateral is 498 yards, and the perpendiculars let fall upon it from the opposite angles are 10.8 and 18.8 yards, what is the area?

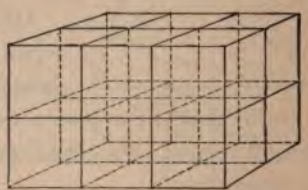
5. *Find the area of a* circle whose radius is 2 feet.

6. What is the convex surface of a cylinder, the radius of whose base is 20 feet, and whose altitude is 50 feet?

7. Find the convex surface of a right cone whose slant side is 50 feet, and the diameter of its base 8 ft. 6 in.

8. Find the surface of a sphere whose radius is 54.5 feet.

SOLIDS.



948. Rules.—I. The volume of a prism or of a cylinder equals the area of the base multiplied by the altitude.

II. The volume of a pyramid or of a cone equals the area of the base multiplied by one-third of the altitude.

III. The volume of a sphere equals 4.1888 times the cube of the radius.

IV. Similar solids are proportional to the cubes of any corresponding lines.

949. Written Problems. 1. Each edge of the base of a square prism is 34 inches and its height is $12\frac{5}{12}$ feet. Find its contents.

2. Find the solid contents of a cylinder whose altitude is 12 feet, and the diameter of its base 15 feet.

3. The sides of the base of a triangular pyramid are 3, 4, and 5 feet, and the altitude of the pyramid is 6 feet. Find its solid contents.

4. Find the solid contents of a cone whose altitude is 27 feet, and the diameter of the base is 5 feet.

5. What is the volume of a sphere whose diameter is 12 ft.?

6. The diagonal distance across a square is 500 feet. What *is the distance* around the square; and how many acres does *its surface* contain?

MISCELLANEOUS PROBLEMS.

950. Written Problems. 1. Two churches stand on opposite sides of a street. The height of the tower of one of them is 100 feet, and a straight line drawn from the top of this tower to the top and to the bottom of the tower of the opposite church is, in each case, 125 feet; what is the width of the street and the height of the opposite tower?

2. What is the entire surface of a cylinder, whose height is 15 feet and whose diameter is 8 feet? What are its solid contents?

3. How many bushels of wheat would a rectangular bin 20 feet long, 8 feet wide, and 10 feet deep, hold?

4. How many gallons of water can be contained in a cylindrical cistern, 8 feet in diameter and 10 feet deep?

5. How many square inches are there in the surface of a cube whose solid contents are 2,744 cubic inches?

6. Given a conical mound, 15 feet in diameter at the base, and having a slant height of 25 feet; to find (1) the height of the mound, (2) the number of square feet in its surface, and (3) the cubic feet in its solid contents; also (4) the length of the side of an equivalent mass in the form of a cube; and (5) the height of an equivalent pyramid whose base is 15 feet square.

7. Find the volume of a cylindrical marble column whose diameter is 15 inches and length 15 feet.

8. Find the diameter and number of acres of surface in a circular pond one mile in circumference.

9. Find the number of square feet of plastering in a room 20 feet long, 15 feet wide, and 12 feet high, in which there are three doors, 8 by $3\frac{1}{2}$ feet; two windows, extending to the floor, each 10 by 3 feet; a fire-frame, 4 by 5 feet; the base board being 10 inches wide.

MISCELLANEOUS PROBLEMS.

951. Written Problems. 1. A house is insured for $\frac{3}{4}$ of its value at $\frac{1}{2}$ %. If the premium was \$27, what was the value of the house?

2. For how much must a house worth \$12,000 be insured at $\frac{1}{4}$ % to cover $\frac{2}{3}$ of its value and the premium?

3. A man sold 550 barrels of flour at \$7 per barrel. The entire cost of the flour, including the cost of insurance at 2 %, was \$3,646.50. How much did he gain on each barrel?

4. An insurance company lost \$5,593 on a house destroyed by fire. For how much was the house insured, if the rate of insurance was $\frac{1}{8}$ %?

5. What is the duty on 600 yards of carpet, 24 inches wide, invoiced at 10s. per yard, when there is a specific duty of \$.25 on the square yard, and an ad valorem duty of 40 %?

6. The duty on 40 yards of silk at 24 % was \$12. For how much per yard was the silk bought?

7. The duty on 50 pieces of carpet, each containing 100 yards $\frac{3}{4}$ of a yard wide was \$2,200. There was an ad valorem duty of 35 %, and a specific duty of \$.12 per square yard. What did it cost per yard?

8. If \$276 duty was paid for wine on which a leakage of 8 % was allowed, how many gallons were there, the duty being \$1$\frac{1}{2}$ a gallon?

9. What rate of tax must be assessed upon \$5,100,000 worth of property to pay \$39,984 for the paving of a street, and to pay 2 % for collection?

10. After collecting a bill, an agent remitted \$798 to the *principal*, reserving a commission of 5 %. What was the *amount collected*?

11. A certain town is taxed $12,626.60. There are 825 voters in the town, taxed at the rate of $1 each. What is the rate of tax if $2,500,400 worth of real estate and $450,000 worth of personal property are taxed?

12. A owes B $441, due 1 year hence; $345, due in 3 years, and $220, due in 2 years. Find the equated time of payment of these debts.

13. Find the equated time of payment of $100, due in 4 months, and $200, due in 6 months.

14. What is the equated time of payment of a debt of $1,600, $\frac{1}{4}$ of which is due in 6 months, $\frac{1}{5}$ in 5 months, and the remainder in 4 months?

15. A owes B a debt of $6,000, due in 4 months; he pays $2,000 at the end of one month, and $1,500 at the end of two months. When is the remainder due?

16. A owes B $416, due in 6 months; and $432, due in 12 months. What is the average time of payment?

17. $716\frac{19}{100}$. BALTIMORE, *Feb.* 16, 1889.

On demand, I promise to pay Joseph Johnson, or order, seven hundred and sixteen and $\frac{19}{100}$ dollars, with interest at 6 per cent.

Value received. EDWARD MASON.

Indorsed: Received, May 3, 1889, $324.10; Sept. 9, 1889, $180.03; Dec. 15, 1889, $63.50. What is due April 3, 1890?

18. $800. MOBILE, ALA., *April* 19, 1888.

For value received I promise to pay T. B. Maury, or order, on demand, eight hundred dollars, with interest.

EDWIN C. LEE.

Indorsed: Oct. 21, 1888, $50; July 2, 1889, $10; May 4, 1890, $20. What is due April 19, 1891, at 8 per cent.?

19. $7,000. NEW YORK, *Jan.* 1, 1885.

Three months after date, I promise to pay Peter Carter, or order, seven thousand dollars, with interest.

Value received. JAMES E. JOY.

Indorsed: May 3, 1886, $400; Aug. 8, 1887, $70; Sept. 9, 1888, $120; Oct. 7, 1889, $950. What is due Jan. 1, 1890, interest at 7 per cent.?

20. $2,500. PITTSBURGH, *Aug.* 8, 1884.

Six months after date, I promise to pay Amos B. Grier, or order, two thousand five hundred dollars, with interest.

Value received. HENRY PETERSON.

Indorsed: Nov. 1, 1885, $400; Dec. 14, 1886, $100; July 6, 1887, $50; Oct. 21, 1888, $750; April 18, 1889, $500. What remained due July 1, 1889, interest at 6 per cent.?

21. Average the following account:

DR. JAMES SMITH. CR.

1889.				1889.		
June 10	To Mdse., 3 mos.	$827 50		Sept. 12	By Cash.....	$800 00
July 15	" " 2 mos.	80 00				
Aug. 20	" " 4 mos.	75 75				

22. Find the equated time for the payment of the amount of an account having the following items:

May	10.	Mdse. on 60 days	$350
"	30.	" " 3 mos.	425
June	25.	" " 60 days	250
July	20.	" " 30 days	300

23. The width of a room is $\frac{2}{3}$ of the length, and the room requires 32 yards of carpet, $\frac{3}{4}$ of a yard wide. What are the dimensions of the room?

What is the value of $\sqrt{\frac{6561}{9801}}$?

25. When is the amount of the following account due? If not settled till Nov. 1, what amount should be paid, interest being at 6 per cent.?

1889. M. A. JOHNSON to CHAS. CLAY, *Dr.*

June	6.	For Mdse. on	3 mos.	$675
July	3.	" " "	60 days	500
"	15.	" " "	90 days	875
Aug.	20.	" " "	2 mos.	250

26. Find when the balance of the following account is due:

DR. SAMUEL MASON. CR.

1890.			1890.		
Aug. 20	To Mdse. 30 da.	$262 50	Oct. 1	By Cash.....	$200 00
Sept. 15	" " 60 "	300 00	Oct. 20	" "	275 00
Oct. 25	" " 90 "	810 40			
Nov. 1	" " 30 "	906 75			

27. Find the equated time for the payment of the balance of the following account:

DR. EDWARD THOMSON. CR.

1890.			1890.		
Jan. 1	To Mdse. . .	$800	Feb. 10	By Cash.....	$500
Feb. 20	" " . .	750	Mar. 5	" "	400
Mar. 30	" " . .	275	April 1	" "	300
April 5	" " . .	346			

28. A piece of work can be done in a day of 11½ hours by 2 men, by 5 women, or by 12 boys. In what time could it be done by 1 man, 2 women, and 3 boys together?

29. How much money must be invested in 5 % stock at *4 % premium to yield* a quarterly income of $375?

30. A can do a piece of work in 12 days, working 10 hours a day; B can do it in 15 days, working 6 hours a day; C can do it in 9 days, working 8 hours a day. In what number of hours can A, B, and C, working together, do the work?

31. A cistern which is full can be emptied by one pipe in 25 seconds, and filled by another in 45 seconds. If the first pipe is turned on for 10 seconds, and then the second also, in what time will the cistern be emptied?

32. Required a number of which the half, third, fourth, and two-sevenths added together make 575.

33. A horse is tied to a post by a rope long enough to allow him to graze over a circular piece of ground containing half an acre. What is the length of the rope?

34. The gable ends of a house are each 32 feet wide, and the perpendicular height of the ridge-pole above the eaves is 10 feet. How many feet of boards will it take to cover both gable ends?

35. In an arithmetical series the extremes are 2 and 26, and the number of terms is 7. What is the sum of all the terms?

NOTE.—See Appendix for Progressions.

36. A has $10, B has as much as A and half as much as C, and C has as much as A and B together. What sum has each?

37. What is the area of a right-angled triangle, the hypothenuse being 64 feet, and one side 28 feet?

38. A sphere is 16 inches in diameter. What is the diameter of a sphere $\frac{3}{4}$ as large?

39. A person, being asked the time of day, replied that the time past noon was equal to $\frac{9}{15}$ of the time to midnight. What ime?

staff 5 feet 6 inches long cast a shadow of 7 feet, height of a steeple whose shadow is 140 feet?

41. What is the L. C. M. of $6\frac{1}{2}$ and $7\frac{1}{4}$?

42. What is the G. C. D. of $\frac{2}{3}$, $\frac{1}{5}$, and $\frac{3}{16}$?

43. The diameter of a circle is 4 feet. What is the diameter of a circle whose area is 4 times as large?

44. Mrs. D's age 32 years after her marriage is 48 years. $\frac{4}{5}$ of her husband's; what fraction was her age of his at marriage?

45. A man who owns $\frac{5}{8}$ of a ship sells $\frac{4}{5}$ of his share for $\$9,546.66\frac{2}{3}$. At this rate, what is the value of the whole ship, and what part of the ship does he still own?

46. How many acres are there in a rectangular farm 24 rods wide and $\frac{1}{4}$ of a mile long?

47. What will be the cost of 5 planks 21 ft. long, 8 in. wide, and $2\frac{1}{4}$ in. thick, at \$36 per M.?

48. What is the value of 6 piles of four-foot wood, 36 ft. long and $5\frac{1}{4}$ ft. high, at $\$4\frac{2}{3}$ per cord?

49. A man sold property at 30 % gain. The purchaser sold it at 10 % loss, receiving $\$8,219\frac{1}{4}$. What was the cost to the first man?

50. A man paid \$128 for a horse. He sold it at a gain of $31\frac{1}{4}$ %, making a reduction from the asking price of $12\frac{1}{2}$ %. What was the asking price?

51. Paid \$42.36 premium on a house worth \$4,236 and insured for $\frac{2}{3}$ of its value. What was the rate of insurance?

52. What must be the face of a 3 months' note which, when discounted at a bank at 9 %, yields \$351.63?

53. A lends B \$225 for 8 months. B afterward lends A \$150. How long may A keep the money to balance the use of the money he loaned B?

54. How many bushels of grain will a bin contain, if it is 10 feet long, 6 feet wide, and 5 feet deep?

55. What are the largest lots, each containing the same number of acres, into which three farms, containing *respectively 48, 64, and 96 acres*, can be divided?

56. How many yards of muslin $\frac{3}{4}$ of a yard wide will it take to cover the convex surface of a cylinder 8 feet in diameter and 14 feet high?

57. How many cords of wood are there in a pile 24 feet long, 12 feet high, and 6 feet thick?

58. What rate of interest is paid when a note payable in 30 days is discounted at a bank at 6 %?

59. What is the depth of a cubical cistern that will hold 300 barrels of water?

60. A man bought 16 bushels of wheat at $\$.87\frac{1}{2}$ a bushel, 24 bushels at \$1.25 a bushel, and 30 bushels at $\$1.66\frac{2}{3}$ a bushel. What was the average cost per bushel?

61. A plank contains 15 board feet. If it is 10 inches wide and 2 inches thick, how long is it?

62. What rate of interest does a bank receive for discounting a note payable in one year, the rate of discount being 6 %?

63. If it cost \$128 to enclose a square field of 40 acres, how much would it cost to enclose a rectangular field of the same area, whose length is 4 times its breadth?

64. What are the dimensions of a cubical pile of wood that contains 200 cords?

65. Two men travelling toward each other were $53\frac{1}{2}$ miles apart. When they met one had travelled $5\frac{1}{2}$ miles less than the other. How far did each travel?

66. What was the face of a 30 days' note which, when discounted at a bank at 6 %, yielded \$855.27?

67. What is the entire area of a cubical block containing 216,000 cubic inches?

68. Multiply six-tenths by six-thousandths and divide the uct by nine hundred thousand. What is the result?

A rectangular field 10 rods wide and containing $2\frac{1}{4}$ is to be enclosed by a close board fence 8 feet high. will be the cost of the fence at \$10.50 per M.?

70. How many feet square is a cistern which is 9 feet deep, if it costs $18 to excavate it at $1.50 per cubic yard ?

71. What is the depth of a cubical bin that will hold 500 bushels of grain ?

72. What is the present worth of $4,573.50, due in 3 months 8 days, money being worth 6 % ?

73. How many barrels of water will a cistern 7 feet by 8 feet by 12 feet contain ?

74. What is the exact interest of $3,250 from Jan. 15, 1888, to April 3 of the same year ?

75. Find the area of a walk 8 feet wide, that surrounds a square lot containing .225 of an acre.

76. What is the cost of a sight draft on St. Louis for $1,500, when exchange is at $2\frac{1}{2}$ per cent. premium ?

77. What is the face of a bill on London that can be bought for $2,906.42, when exchange is at $4.84 ?

78. A lot containing $\frac{3}{4}$ of an acre is 15 rods long. How long is a straight walk between the opposite corners ?

79. The sum of two numbers is 180. What are the numbers, if one number is $\frac{1}{3}$ of the other ?

80. Divide eighty thousand by eight-thousandths, and multiply the quotient by 95-hundredths. What is the result ?

81. A can do a piece of work in 8 days. B can do the same work in 14 days. How long will it take them working together ?

82. John Wright bought of William Purdy of St. Louis, August 5, 1886, 8 tons of nut coal at $5.50 per ton, and 4 tons of stove coal at $5.25 per ton ; Sept. 13, 1886, he bought 2 cords of wood at $4.50 per cord. Make out and receipt Mr. Wright's bill.

83. A tree 60 feet high was broken 25 feet from the bottom. How far from the foot of the tree did its top strike the *ground?*

84. What is the diameter of a cylindrical cistern 10 feet deep containing 282,744 cubic feet ?

85. How many gallons will a cylindrical cistern hold that is 5 feet in diameter and 8 feet deep ?

86. How many square feet are there in the walls of a room 20 feet long, 15 feet wide, and 16 feet high, without deducting for doors and windows ?

87. If 25 % is gained by selling 250,000 feet of lumber at $18.75 per M., how much would have been gained or lost, if it had been sold for $16.25 per M. ?

88. What is the diagonal distance of a rectangular field containing 4 acres, if the length of the field is 8 chains ?

89. How many barrels of water will a cubical cistern hold, if it is 7 feet deep ?

90. If $15,000 is gained on $50,000 worth of goods, on how many dollars' worth of goods will $3,500 be gained ?

91. If the difference between $\frac{1}{3}$ and $\frac{2}{5}$ of a number is 133, what is the number ?

92. The retail price of cloth is 25 % above cost, and the wholesale price is 16 % less than the retail price. Is there gain or loss on the wholesale price, and how much ?

93. A ladder rests against a house 22 feet from the pavement. What is the length of the ladder, if the distance from the house to the base of the ladder is 12 feet ?

94. Money being worth 8 %, which is better for the purchaser, and how much better, to pay $1,900 cash, or $2,080 in 6 months' time ?

95. A book that cost $5 was sold at a gain of 25 %. What was the marking price, if a deduction of $33\frac{1}{3}$ % was made ?

96. What per cent. is gained on an article marked $15 and old at 20 % and 10 % off, if the cost was $8 ?

is the cost of a bill of exchange in Paris for
hen the course of exchange is 5.16 ?

98. Find the second term of the proportion $32.18 : x :: 51\frac{3}{8} : 4{,}257$, and write the answer in words.

99. A and B become partners in business. A puts in \$3,000. 4 months later B puts in \$5,000; and 3 months after B, C puts in \$4,000. The profits at the end of the year are \$3,000. What is each man's share?

100. \$12,500 is divided among three brothers whose shares are to each other as 5, 7, and 8. What is each one's share?

101. What is the diagonal distance from a lower corner of a room to the opposite upper corner, if the room is 15 feet long, 14 feet wide, and 12 feet high?

102. Find the entire surface of a square pyramid, whose altitude is 84 feet and whose base is 26 feet square.

103. What are the cubical contents of a sphere 12 inches in diameter?

104. The product of three factors is 252,720. If two of the factors are 65 and 48, what is the third?

105. How many yards are there in the smallest piece of ribbon which can be divided into an exact number of pieces containing $2\frac{1}{4}$ yards, $1\frac{1}{8}$ yards, or $3\frac{1}{3}$ yards?

106. A number diminished by .625 of $\frac{3}{5}$ of itself is equal to 91. What is the number?

107. $\frac{3}{5}$ of A's money equals $\frac{4}{5}$ of B's money. A has \$300 less than B. How much money has each?

108. Into how many square lots of the largest possible area can a rectangular field 160 rods long and 80 rods wide be divided?

109. Find the last term of a geometrical progression, in which the first term is 4, the number of terms 8, and the ratio of increase 2.

110. A merchant sold goods which were marked \$1,150 so as to gain 15 % after making a deduction of 20 %. How much *did the goods cost him*?

111. What principal will amount to $1,510.65 in 1 year 8 months 12 days, at 7 %?

112. Find the difference between the interest and true discount of $680 for 2 years 4 months at 6 %.

113. Find the difference between the simple and the compound interest of $1,200 for 3 years 3 months at 8 %.

114. What are the bank discount and proceeds of a 60 days' note for $920, discounted at a bank at 6 %?

115. If 12 men working 9 hours a day can do a piece of work in 5 days, how many men can do a piece of work three times as large in 6 days, working 10 hours a day?

116. Three men use a pasture in common. The first puts in 4 horses for 2 months; the second, 3 cows for 4 months; and the third, 6 cows for 3 months. If the rent is $60, and if each horse eats $\frac{1}{4}$ more than a cow, what is each man's share of the rent?

117. Two men start from the same point; one travels east at the rate of 4 miles an hour, and the other travels south at the rate of 3 miles an hour. How far apart will they be at the end of 6 hours?

118. The time past noon equals $\frac{3}{5}$ of the time to midnight. What is the time of day?

119. How many tons of ice can be taken from a 2 acre pond, when the ice in 4 inches thick, the ice weighing 60 pounds to the cubic foot?

120. How many tons of ice can be obtained from a circular pond 80 feet in diameter, when the ice is $2\frac{1}{2}$ inches thick, the ice weighing 60 pounds to the cubic foot?

121. An agent buys 8,500 barrels of flour at $7.25 per barrel and pays $245 for freight. How much money was sent to him, if his commission was 4 %?

97. How much does a bank pay for a note of $1,800, dated *1,548 franth*, due June 9th, and discounted April 15th at 6 %?

123. What per cent. is gained or lost by buying coal at $6.15 per ton on 3 months' time and selling the same day at $6.25 cash, money being worth 10 %?

124. 4½ per cent. bonds yield 6 per cent. on the money invested. At what rate of discount were they bought?

125. A commission merchant sells $120 worth of butter for a farmer, charging 3 per cent. commission. With the proceeds he buys groceries charging 4 % for buying. What is his commission for selling, his commission for buying, and the cost of the groceries?

126. What is the gain per cent. on an article, when $\frac{4}{5}$ of the selling price is 4 % less than the cost?

127. Two times the time past midnight is equal to $\frac{2}{3}$ of the time till noon. What time is it?

128. For what sum must a 3 months' note be drawn so that, when discounted at a bank at 6 %, the proceeds will pay for 75 barrels of flour at $8.50 per barrel?

129. At $.20 per gallon, what is the value of the oil in a cubical vessel 10 feet high?

130. Sold $\frac{1}{2}$ of a farm for what $\frac{3}{4}$ of it cost. What per cent. was gained?

131. What is the face of a 90 days' draft in New York, if the cost is $5,877, at $\frac{1}{2}$ per cent. discount?

132. Which is the better investment, and how much better, 5 per cent. bonds at 104, or 4 per cent. bonds at 96?

133. A and B can do a piece of work in $3\frac{1}{3}$ days; B and C can do it in 6 days; A and C can do it in $3\frac{3}{4}$ days. How long will it take them working together, and in what time can each man do the work alone?

APPENDIX.

METRIC SYSTEM.

952. The **metric system** is the French system of weights and measures.

953. In the metric system the basis is the one *ten-millionth* part of a quadrant (quarter) of the earth's meridian.

954. A **meter**, the unit of the metric system, is 39.37 inches (nearly 1.1 yards), or the one *ten-millionth* part of a quadrant of the earth's meridian.

NOTE.—The metric system has been authorized by the U. S. and adopted by nearly all the nations of Europe. It is used in the U. S. coast survey.

955. The advantage of the metric system consists in its extensive application of the decimal system. The unit in each table is taken from the meter, and the scale of each table is ten. The names of the fractional orders in each table have the prefixes deci- (tenths), centi- (hundredths), milli- (thousandths), before the name of the unit. The names of the integral orders have the prefixes deka- (tens), hekto- (hundreds), kilo- (thousands), or myria- (tens of thousands), before the name of the unit, as in the following

Table.

	Integral.				Decimal Orders.			
	Kilo-	Hekto-	Deka-	Unit.	Deci-	Centi-	Milli-	
Long measure,	.	.	.	**Meter,**	.	.	.	39.37 in.
Square	.	.	.	**Are,**	.	.	.	1 sq. Dm.
		.	.	**Liter,**	.	.	.	1 cu. dm.
		.	.	**Gram,**	.	.	.	1 cu. cm.

956. The preceding units are used as follows:

1. **Meter** ($1\frac{1}{10}$ yd.) is used for short distances.
   Kilometer, for long distances.
2. **Are** ($\frac{1}{40}$ A.) is used for land.
   Square meter, for small areas.
   Square kilometer, for territories.
3. **Liter** (1.06 qt.) is used for liquids.
   Hektoliter, for grains, roots, etc.
   Cubic meter, for ordinary solids.
   Stere (cu. m.), for wood.
4. **Gram** ($\frac{1}{30}$ oz.), precious metals, prescriptions, etc.
   Kilo (kilogram, nearly $2\frac{1}{5}$ lb.), ordinary articles.
   Metric ton (nearly $1\frac{1}{10}$ tons), heavy weights.

957. The name of any unit may be changed by moving the decimal point; thus,

Length.	Surface.
1. 1,234 m. = 1.234 Km.	6. 12 ares =.12 hektares.
2. .5 Km. = 500 m.	7. .185 centares =1,850 sq. cm.
3. 100 mm. = .1 m.	8. .41875 sq. Km. =41,875 are.
4. .25 Dm. = 25. dm.	9. 25,000 sq. mm. =2.5 sq. dm.
5. 345 Hm. = 3.45 Mm.	10. 18,275 sq. cm. =1.8275 Dm.

Volume.	Capacity.
11. 764 dst. = 7.64 steres.	13. 500 liters = 5 Kl.
12. .00005 cu. m. = 50 cu. cm.	14. .001 Dl. = .01 liters.

958. Long Measure.

10 millimeters, mm.	make	1 centimeter, cm.
10 centimeters	"	1 decimeter, dm.
10 decimeters	"	1 meter, m.
10 meters	"	1 dekameter, Dm.
10 dekameters	"	1 hektometer, Hm
10 hektometers	"	1 kilometer, Km.
10 kilometers	"	1 myriameter,

959. **Square Measure.**

100 centameters (ca.), or sq. meters make 1 are, a.
100 ares . . . . . " 1 hektare, Ha.

Also,

100 sq. millimeters (sq. mm.) make . . 1 square centimeter, sq. cm.
100 square centimeters. . . " . . 1 square decimeter, sq. dm.
100 square decimeters. . . " . . 1 square meter, sq. m. or centare.
100 square meters. . . . . " . . 1 square dekameter, sq. Dm. or are.
100 square dekameters. . . " . . 1 sq. hektometer, sq. Hm. or hektare.
100 square hektometers. . . " . . 1 square kilometer, sq. Km.

Cubic Measure. (Capacity and Volume.)

960. **Volume.**

1,000 cubic millimeters (cu. mm.) make . . 1 cubic centimeter, cu. cm.
1,000 cubic centimeters . . . " . . 1 cubic decimeter, cu. dm.
1,000 cubic decimeters . . . " . . 1 cubic meter, cu. m. or stere.

Also (for wood),

10 decisteres dst. . . . . make . . . . . 1 stere, st.

961. **Capacity.**

10 milliliters, ml. . . . . make . . . . . 1 centiliter cl.
10 centiliters . . . . . " . . . . . 1 deciliter, dl.
10 deciliters . . . . . " . . . . . 1 liter, l.
10 liters . . . . . " . . . . . 1 dekaliter, Dl.
10 dekaliters . . . . . " . . . . . 1 hektoliter, Hl.
10 hektoliters . . . . . " . . . . . 1 kiloliter, Kl.

962. **Weights.**

10 milligrams, mg. . . . make . . . 1 centigram, cg.
10 centigrams . . . . . " . . . 1 decigram dg.
10 decigrams . . . . . " . . . 1 gram, g.
10 grams . . . . . " . . . 1 dekagram, Dg.
10 dekagrams . . . . . " . . . 1 hektogram, Hg.
10 hektograms . . . . . " . . . 1 kilogram, Kg. or **Kilo,** K
10 kilograms . . . . . " . . . 1 myriagram, Mg.
10 myriagrams . . . . " . . . 1 quintal, Q.
10 quintals . . . . . " . . . 1 metric ton, T.

963. To Reduce the units of the Metric System to U. S. Weights and Measures:

Length.

1. 1 **meter** = 39.37 + in. or 3 ft. $3\frac{3}{8}$ in.

Surface.

2. 1 **are** or sq. dekameter = 119.6 sq. yd. or 3.954 sq. rd.

Volume.

3. 1 **cu. meter** or stere = 35.316 + cu. ft. or 1.308 cu. yd.

Capacity.

4. 1 **liter** = 61.022 cu. in., or 1.0567 liquid qts., or .908 dry qts.

Weight.

5. 1 **gram** = 15.432 grains, or 1 cu. cm., or 1 ml. of water.
6. 1 kilogram or **kilo** = 2.204 lbs. Avoir.

964. To Reduce U. S. Weights and Measures to units of the Metric System:

Length.

1. 1 inch = 2.54 centimeters.
2. 1 foot = 3.048 decimeters.
3. 1 rod = 5.029 meters.
4. 1 mile = 1.6093 kilometers.

Surface.

6. 1 sq. yard = .8361 square meter.
6. 1 sq. rod = .2529 of an are.
7. 1 acre = .4047 hektare.
8. 1 sq. mile = 2.59 sq. kilometers.

Volume.

9. 1 cubic yard = .746 cu. meter.
10. 1 cord = 3.625 steres or cubic meters.

Capacity.

11. 1 liquid qt. = .9465 liter.
12. 1 gall. = 3.785 liters.
13. 1 dry qt. = 1.101 liters.
14. *1 bu. = .3524 hektoliter.*

Weight.

15. 1 lb. Troy = .373248 kilo.
16. 1 oz. Avoir. = 28.35 grams.
17. 1 lb. Avoir. = .4536 kilo.
18. 1 T. = .9072 metric tons.

965. Written Problems. 1. A bought 500 meters of black silk at $1.75 per meter, and sold it at $1.80 per yard. How much did he gain?

2. A grocer buys a ton of sugar at 22 cents a kilo. How much does he pay for the ton?

3. Mr. Nelson buys a farm containing 80.94 hektares at $60 an acre. What is the value of the farm?

4. How many liters of water equal one metric ton?

5. How many grams are equal to one kilometer of water?

6. How many cubic millimeters are there in one metric ton?

7. How many grams are there in one cubit meter?

8. A field is 1 hektometer wide and 4 hektometers long. How many acres are there in the field?

9. If a hogshead of wine costs $400, what does one liter cost?

10. What is the cost of 30 meters and 5 decimeters of silk, at 10 francs per meter?

11. A farmer bought 404.64 acres of land at $100 an hektare. What was the cost?

12. If a silver half dollar weighs 12½ grams, how many half dollars will weigh 10 kilos?

13. How much will 2,000 nickels weigh if each nickel weighs 5 grams? Express the answer in kilos.

14. What is the weight in lb. Troy of 4,800 letters, if they average 30 grams a letter?

15. How many steres are there in a pile of wood 45 ft. long, 4 ft. wide, and 7 ft. high?

16. A field containing 25 acres, produces 63 tons of hay; how many kilos, is this per hectare?

17. A traveller walks at the rate of 4 kilometers an hour. How many miles will he go in 5 days, walking 9 hours each day?

ny gallons of water will a cistern hold, which
;. 2 meters wide, and 1½ meters deep?

PROGRESSIONS.

ARITHMETICAL PROGRESSION.

966. An **arithmetical progression** is a series of numbers increasing or decreasing by a common *difference.* Thus, 1, 3, 5, 7, 9, etc., is an *increasing arithmetical progression;* and 19, 15, 11, 7, etc., is a *decreasing arithmetical progression.*

967. In an arithmetical progression, the principal quantities are the *first term,* the *last term,* the common *difference,* the *number* of terms, and the *sum* of the series of terms.

The abbreviations for the above are respectively a, l, d, n, and s.

968. To find the last term of an arithmetical series.

	1st term.	2d.	3d.	4th.	last term.
Progression	= 5,	9,	13,	17,	21.

	1st.	1st + d.	1st + 2d.	1st + 3d.	1st + 4d.
Or	= 5,	5 + 4,	$5 + \overline{2 \times 4}$,	$5 + \overline{3 \times 4}$,	$5 + \overline{4 \times 4}$.

From the preceding, it is evident that any term or the last term equals the first term plus the number of terms minus one multiplied by the difference.

$$l = a \pm (n - 1) \times d.$$

NOTE.—Subtract $(n - 1) \times d$ from a in a decreasing progression.

969. To find the sum of an arithmetical series.

Sum = $5 + (5 + \overline{1 \times 4}) + (5 + \overline{2 \times 4}) + (5 + \overline{3 \times 4}) + (5 + \overline{4 \times 4})$.

or, in inverse order,

Sum = $(5 + \overline{4 \times 4}) + (5 + \overline{3 \times 4}) + (5 + \overline{2 \times 4}) + (5 + \overline{1 \times 4}) + 5$.

$2s = (a+l) + (a+l) + (a+l) + (a+l) + (a+l)$.

If the progression, written in the reverse order, is added to the progression written in the natural order, it is evident that twice the sum equals the sum of the first and last terms taken as many times as there are terms; that is $2s = n \times (a + l)$, or,

$$s = n \times \frac{(a + l)}{2}.$$

970. To find any one of the five quantities.

1. $l = a + (n - 1) \times d$; therefore, 2. $a = l - (n - 1) \times d$; and

3. $d = \frac{l - a}{n - 1}$; and, 4. $n = \frac{l - a}{d} + 1$.

5. $s = n \times \frac{(a + l)}{2}$; therefore, 6. $n = \frac{2s}{a + l}$

and, 7. $a = \frac{2s}{n} - l$; and, 8. $l = \frac{2s}{n} - a$.

971. To find any one of the five quantities in an arithmetical progression, substitute known quantities for the corresponding letters in the appropriate equation.

972. Written Problems. 1. The first term of an increasing A. P. is 7, the common difference is 4, and the number of terms 10. What is the last term?

2. The first term of a decreasing A. P. is 100, and the common difference is 2. What is the 17th term?

3. If a man lay up \$1 the first week of the year, \$3 the second week, \$5 the third week, and so on to the close of the year, how much will he lay up the last week of the year?

4. The first term of an increasing A. P. is 10, and the common difference is 5. What is the 100th term?

5. The first term of an A. P. is 1, the last term is 100, and t[illegible] terms is 34. Find the common difference.

[illegible]xtremes of an A. P. are 2 and 52, and the [illegible]s 11. Find the common difference.

7. The first term of an A. P. is $104\frac{2}{3}$, the last 8, and the number of terms 59. Find the common difference and the 55th term.

8. The two extremes being 10 and 70, and the common difference 3, what is the number of terms?

9. The extremes are 2 and 53, and the common difference 3. What is the number of terms?

10. The first term of an A. P. is 3, the last 139, and the number of terms 35. Find the sum of the terms.

11. The first term is $\frac{2}{3}$, the last $2\frac{2}{3}$, and the number of terms 7. Find the sum of the terms.

12. The first term is $\frac{7}{12}$, the last $2\frac{1}{2}$, and the number of terms 24. Find the sum of the terms.

13. Find the sum of 20 terms of the series, 1, 3, 5, 7, 9, &c.

14. Find the sum of 20 terms in each of the following series:—

1. 100, 110, 120, etc.
2. 100, 97, 94, etc.
3. $\frac{1}{2}$, $\frac{3}{4}$, 1, etc.
4. 13, $12\frac{2}{3}$, $12\frac{1}{3}$, etc.

15. Find the number of terms and the sum of the terms of an A. P. whose first term is 1, last term 70, and common difference 3.

16. If a laborer were hired for a year, to receive a farthing the first day, a half-penny the second, three farthings the third, and so on, excluding Sundays, what would he receive for the 25th week? and what would his wages amount to for the year?

17. How far does a person travel in gathering up 200 stones placed in a straight line at intervals of two feet from each other, supposing that he fetches each stone singly, and deposits it in a basket which is in the same line with the stones, 20 yards distant from the nearest stone, and that he starts *from the basket?*

GEOMETRICAL PROGRESSION.

973. A **geometrical progression** is a series of numbers increasing or decreasing by a common *ratio*; that is, by a common multiplier, or divisor. Thus, 2, 6, 18, 54, etc., is an increasing geometrical progression; and 1,728, 144, 12, etc., is a decreasing geometrical progression.

974. In geometrical progressions, the principal quantities are the *first term*, the *last term*, the common *ratio*, the *number* of terms, and the *sum*.

First term, a; last term, l; ratio, r; number of terms, n; sum, s.

975. Three of the principal quantities being given, others may be found by substitution in the following equations:

$$1.\ l = a \times r^{n-1};\quad 2.\ a = \frac{l}{r^{n-1}};\quad 3.\ r = \sqrt[n-1]{\frac{l}{a}};\quad 4.\ s = \frac{r \times l - a}{r - 1}$$

NOTE.—The equation for l is found as follows:

Progression, $= \underset{a}{5},\ \underset{a \times r}{20},\ \underset{a \times r^2}{80},\ \underset{a \times r^3}{320},\ \underset{a \times r^4}{1280}.$

Or, $5,\ 5 \times 4,\ 5 \times 4 \times 4,\ 5 \times 4 \times 4 \times 4,\ 5 \times 4 \times 4 \times 4 \times 4$, etc.

It follows, therefore, that $l = a \times r^{n-1}$.

NOTE.—2. The equation for s is found as follows:

$$\begin{aligned} 4 \times \text{Sum} &= \quad 20 + 80 + 320 + 1280 + 5120 = r \times s. \\ 1 \times \text{Sum} &= 5 + 20 + 80 + 320 + 1280 \quad = 1 \times s \\ 3 \times \text{Sum} &= (r - 1) \times s = 5120 - 5 = r \times l - a. \\ \text{Sum} &= \quad 5115 \div 3 = 1705. \end{aligned}$$

It follows, therefore, that $s = \dfrac{r \times l - a}{r - 1}$.

976. Written Problems. 1. Let 3 be the first term, 2 the common ratio, and 9 the number of terms, to find the last term.

2. Find the 8th term in the geometrical progression 1, 3, 9, 27, &c.

3. The first term of a geometrical progression is 2, the the number of terms 13. Required the last

4. What is the amount of $500 for 4 years, at 5 per cent. per annum, compound interest?

SOLUTION.—$500 = first term. $1.05 = amount of $1 for one year at 5 per cent. = common ratio. 4 years + 1 = 5 = number of terms.

$$(1.05)^4 = 1.21550625$$
$$500 = \text{principal}$$
$$607.75312500 = \text{amount} = a \times r^{n-1}$$

5. What would $1,000 amount to in 5 years, allowing compound interest at 5 per cent.?

6. The two extremes of a G. P. are 2 and 4,096, and the number of terms 12: find the common ratio.

7. The first term of a G. P. is 4, the last 1,024, and the common ratio 2. Find the sum of the series.

8. Find the sum of the series 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, etc., to infinity.

NOTE.—In a *decreasing* series, $s = \frac{a - rl}{1 - r}$, and when the series is extended to infinity, rl becomes practically 0, and $s = \frac{a}{1 - r}$.

9. Find the value of the circulating decimal .3, that is, the sum of the G. P. $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, etc., to infinity.

10. Find the 7th term of the G. P. $3\frac{1}{2}$, 7, 14, etc.

11. When $a = 5$, $l = 1{,}280$, and $n = 5$, find r.

12. When $r = 4$, $n = 10$, and $l = 131{,}072$, find a and s.

13. A gentleman sells 8 horses: for the first he agrees to receive $1, for the second $3, for the third $9, and so on in G. P. What is the total sum for which he sells the horses?

14. Find the sum, to infinity, of 1, $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, etc.

15. A farmer sowed a peck of wheat, and used the whole produce for seed the following year, the produce of this second year again for seed the third year, and the produce of this again for the fourth year. He sells his whole stock after this harvest, and finds he has 12,656$\frac{1}{4}$ bushels to sell. Supposing the increase to have been always in proportion to the seed sown, what was the annual increase?

STANDARD TIME.

977. Standard time is the system by which the North American continent is divided into belts extending north and south, and each 15° wide, the time being considered the same throughout each belt, and the time in successive belts varying one hour.

978. Solar time is the time kept by the sun, midday occurring at a given place when the sun reaches the meridian of that place. Thus, there is a different time for every meridian.

979. To avoid the necessary confusion caused to travellers and others, by the "difference of time," the standard time was adopted, the meridians 75°, 90°, 105°, and 120° west from Greenwich being selected as the **standard meridians**, each one of which gives its time to all places in a belt 15° wide, extending $7\frac{1}{2}$° either side of it.

The sun appears to travel 15° westward each hour, and therefore, the standard meridians are 15° apart.

980. The **time belts** in the U. S. are the Eastern, the Central, the Mountain, and the Pacific.

NOTE.—In a few places, for convenience of railroads, the Central and the Mountain Belts extend into adjacent belts.

981. The standard meridians are that of 75° for the Eastern Belt; 90° for the Central Belt; 105° for the Mountain Belt; and, 120° for the Pacific Belt.

The standard meridian is the central line or meridian in the belt.

982. The standard time is 1 hour earlier in the day, in each successive belt towards the west; and one hour later in the day, in each successive belt towards the east.

Thus, when it is 5 P. M. in the Eastern Belt, it is 4 P. M. in the Central B., in the Mountain Belt, and 2 P. M., in the Pacific Belt.

ANSWERS.

179. **2.** 67 T. 13 cwt. 15 lb. **3.** 125 T. 17 cwt. 30 lb. **4.** £490. 14s. 2 far. **184.** **3.** 4 lb. 10 oz. 6 pwt. 12 gr. **4.** 5 lb. 3 ʒ. 6 ʒ. 2 ℈. 4 gr. **5.** 2 lb. 7 ʒ. 19 gr. **6.** £5. 15s. 11d. **7.** 4 T. 18 cwt. 2 qr. 23 lb. 15 oz. **8.** 2 lb. 6 oz. 17 pwt. 5 gr. **186.** **1.** 51 lb. 2 ℥. 6 ʒ. 2 ℈. **2.** 23 lb. 4 oz. 8 pwt. 20 gr. **3.** 31 T. 15 cwt. 2 qr. 10 lb. **4.** £263. 6s. 7d. 1 far. **5.** £344. 2s. 2d. **6.** 80 T. 14 cwt. 2 qr. 16 lb. **7.** 29 lb. 8 oz. 15 pwt. 17 gr. **8.** 22 lb. 7 ʒ. 9 gr. **187.** **1.** £50. 10s. 10d. 3 far. **2.** 24 lb. 10 oz. 18 pwt. 13 gr. **3.** 7 T. 9 cwt. 3 qr. 15 lb. **4.** 1 lb. 1 ℥. 1 ʒ. 2 ℈. 18 gr. **5.** 7 T. 18 cwt. 3 qr. 16 lb. **6.** 2 lb. 7 ʒ. 17 gr. **7.** 1 lb. 1 oz. 18 pwt. 23 gr. **8.** £100. 1 far. **189.** **1.** $3,575. **2.** $2,314. **3.** $262.50. **4.** $79.20. **5.** $12. **6.** $5.58½. **7.** 6 doz. **8.** $7.50. **9.** $500. **10.** $252. **11.** $1,459.92. **12.** $1,525. **13.** $580. **14.** £200. **15.** 111,600 gr. **16.** 58,080 gr. **194.** **2.** 249 bu. 7 qt. **3.** 713 gal. 2 qt. 1 pt. **4.** 818 bu. 6 qt. **5.** 2,777 gal. 1 qt. **6.** 1,936 lb. 10 oz. 10 pwt. **7.** 2,820 T. 19 cwt. 25 lb. **8.** 1,691 lb. 7 ℥. 4 ʒ. **9.** 49 bu. 7 qt. **210.** **1.** 149 2′. **2.** 51 hr. 58 min. 45 sec. **3.** 1,554 bu. 1 pk. 6 qt. **4.** 5,043 gal. 3 qt. **5.** 3,448 T. 2 cwt. 50 lb. **6.** 812 lb. 5 oz. 5 pwt. **7.** 1,045 lb. 7 ℥. 1 ʒ. **8.** £980. 7s. 6d. **211.** **1.** 2° 19′ 12½″. **2.** 1 hr. 22 min. 3 sec. **3.** 3 bu. 7 qt. 1½ pt. **4.** 5 gal. ½ pt. **5.** 14 cwt. 47 lb. **6.** 2 lb. 1 oz. 17½ pwt. **7.** 1 lb. 4 ℥. 4 ʒ. 2 ℈. **8.** £4. 15s. 3d. **215.** **1.** $54,262.50. **2.** $411 gain. **3.** 3 gal. 3 qt. 1 pt. **4.** $32.40. **5.** 6½ barrels. **6.** 421 boxes. **7.** 8 gallons. **8.** $33.15. **9.** 2 lb. 8 oz. 11 pwt. **218.** **1.** 108 inches. **2.** 84 inches. **3.** 144 inches. **4.** 144 inches. **5.** 594 inches. **6.** 126,720 inches. **7.** 120 inches. **8.** 540 inches. **9.** 316,980 inches. **10.** 266 feet. **11.** 16,336 feet. **12.** 26,415 feet. **13.** 2,062½ yards. **14.** 638 yards. **15.** 880,000 yards. **16.** 11⅛ yards. **17.** 106⅔ yards. **18.** 17⅜⅜ yards. **19.** 44,000 yards. **20.** 66⅔ yards. **21.** 825 yards. **22.** $4\frac{27}{72}$ miles. **23.** $14\frac{2}{4}$ miles. **24.** 34⅜ miles. **224.** **1.** 504 sq. ft. **2.** 400 sq. ft. **3.** 160 sq. ft. **4.** 781¼ acres **5.** 78,400,000 acres. **228.** **1.** 512,000 cu. in. **2.** 990 cu. in. **3.** 248,832 cu in. **230.** **1.** $90.00. **2.** $129.00. **3.** $109.00. **4.** $37.20. **5.** $31,666.66⅔. **6.** $800,000.00. **7.** 40 years. **8.** $39,628. **9.** 15 pitchers. **10.** 1,280 farms. **11.** 40 feet. **12.** $4.34½. **13.** 392 loaves. **14.** $3.68. **15.** $29. **16.** $93.75. **17.** $33.60. **18.** 71 lb. 2 ℥. 1 ʒ. 1 ℈. **19.** 115 A. 1 R. 28 P. **238.** **1.** 400. **2.** 625. **3.** 1,600. **4.** 2,500. **5.** 6,400. **6.** 10,000. **7.** 14,400. **8.** 15,625. **9.** 219,961. **10.** 956,484. **11.** 2,903,616. **12.** 5,612,161. **239.** **1.** 8,000. **2.** 15,625. **3.** 64,000. **4.** 125,000. **5.** 1,000,000. **6.** 1,953,125. **7.** 16,194,277. **8.** 28,652,616. **9.** 104,487,111. **10.** 218,167,208. **11.** 935,441,352. **12.** 1,076,890,625. **267.** **3.** 16. **4.** 3¾. **5.** 7½. **6.** 5. **7.** 8. **8.** 9. **269.** **1.** 1,035 days. **2.** $750. **3.** $33⅓. **4.** 65,400 ft. **5.** 36 bl. **6.** $.74¾. **7.** 75 bl. **8.** 14,700 lb. **9.** $50. **10.** 525 head. **11.** 1½ sacks. **12.** 140 days. **13.** 1,600 doz. **14.** 11 pieces. **15.** 225 acres. **16.** 13⅓ lots. **276.** **1.** 11 acres. **2.** 216 tons in each. **3.** 5 yd. **4.** 5 bl. **5.** 36 acres. **6.** $336. **7.** 12 bl. **8.** 25. **279.** **1.** 112. **2.** 25. **3.** 143. **4.** 377. **5.** 18. **6.** 7. **7.** 77. **8.** 133. **9.** 49. **10.** 3. **11.** 4. **12.** 17. **280.** **1.** 4 yards. **2.** 5 pounds. **3.** 14 gallons. **4.** 5 acres. **5.** 20 yards. **6.** 2 yards. **7.** 5 barrels. **287.**

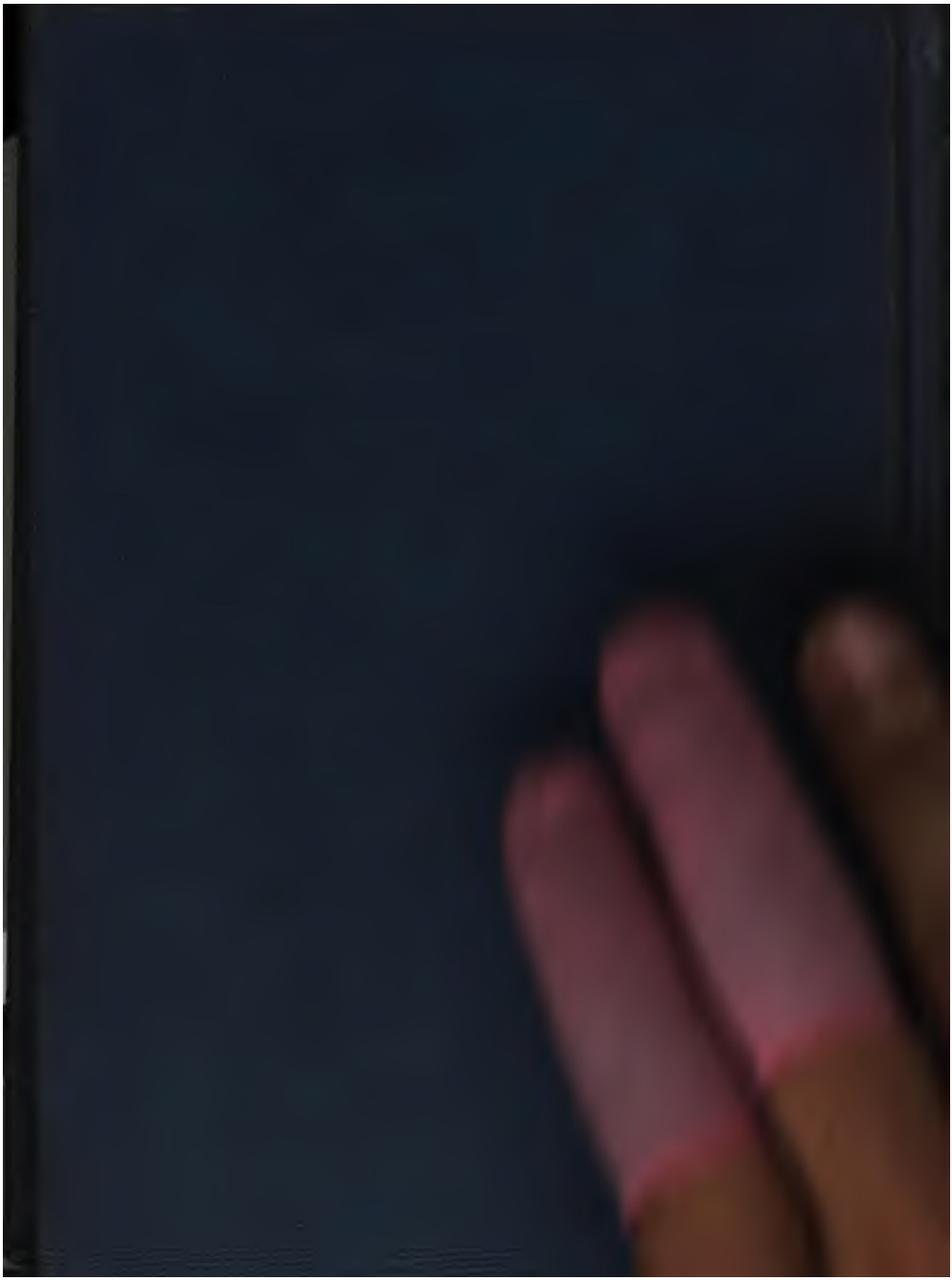
1. 120. 2. 60. 3. 180. 4. 280. 5. 180. 6. 840. 7. 210. 8. 252. 9. 800. 10. 600. 11. 11,700. 12. 2,730. 13. 29,631. 14. 35,700. 15. 227,500. **288.** 1. $5.04. 2. $.90. 3. 144 feet. 4. 60 feet. 5. 528 seconds. 6. 60 yards. 7. 1,386 melons. 8. 1,050 gallons. 9. 900 seconds. **337.** 3. 106$\frac{3}{4}$. 4. 244$\frac{3}{4}$. 5. 150$\frac{1}{2}$. 6. 81$\frac{5}{8}$. **342.** 3. 194\frac{1}{2}$. 4. $252. 5. 163$\frac{5}{8}$. **344.** 1. $10,372$\frac{3}{10}$. 2. $14,940$\frac{3}{4}$. 3. $13,411. 4. $14,121$\frac{1}{8}$. 5. $13,448$\frac{3}{8}$. 6. $8,230$\frac{9}{10}$. 7. $6,769$\frac{3}{4}$. 8. $7,982$\frac{1}{8}$. 9. $7,643$\frac{1}{4}$. 10. $5,429$\frac{5}{6}$. **346.** 1. $226,089$\frac{1}{12}$. 2. $128,239$\frac{1}{2}$. 3. $129,660$\frac{3}{4}$. 4. $113,894$\frac{7}{12}$. 5. 189,893$\frac{5}{8}$. 6. 128,595$\frac{5}{6}$. 7. 134,280$\frac{5}{6}$. 8. 301,547$\frac{1}{2}$. **348.** 1. 1,436$\frac{49}{60}$ acres. 2. $2,565$\frac{1}{10}$. 3. $3,986$\frac{1}{10}$. 4. $1,945. 5. $9,228$\frac{1}{8}$. 6. $13,835$\frac{23}{24}$. 7. 783$\frac{1}{2}$ acres. 8. 8,977$\frac{1}{8}$ tons. 9. $16,846$\frac{1}{4}$. 10. $11,411$\frac{1}{8}$. 11. $14,788$\frac{1}{2}$. 12. $9,923. 13. 59,617$\frac{7}{12}$. 14. 32,807$\frac{1}{2}$. **354.** 2. 44\frac{1}{16}$. 3. 7\frac{1}{12}$. 4. 133\frac{1}{4}$. 5. 26\frac{1}{4}$. 6. 199\frac{3}{4}$. **358.** 1. 14$\frac{3}{8}$. 2. 74\frac{7}{8}$. 3. 7\frac{7}{10}$. 4. 10\frac{1}{2}$. 5. 42\frac{1}{16}$. 6. 150\frac{9}{20}$. 7. 150$\frac{3}{4}$. 8. 138$\frac{33}{64}$. 9. 140$\frac{1}{8}$. 10. 60$\frac{1}{4}$. 11. 809$\frac{1}{20}$. 12. 176$\frac{1}{2}$$\frac{9}{4}$. **360.** 1. 288\frac{5}{6}$. 2. 1,711$\frac{1}{4}$$\frac{3}{4}$. 3. $3,648$\frac{5}{12}$. 4. $3,558$\frac{1}{2}$. 5. $1,986$\frac{3}{10}$. 6. 81.37\frac{1}{2}$. 7. 78.28\frac{1}{2}$. 8. 13.67\frac{3}{4}$. 9. 229.28\frac{3}{4}$. 10. 824.89\frac{3}{8}$. 11. 45,010$\frac{5}{8}$. 12. 922$\frac{1}{4}$$\frac{1}{8}$. 13. 12,769$\frac{1}{12}$. 14. 10,009$\frac{1}{2}$. 15. 8,567$\frac{5}{6}$. **363.** 1. 432\frac{3}{4}$. 2. 11,369$\frac{3}{8}$ tons. 3. 4\frac{1}{8}$. 4. 217\frac{1}{20}$. 5. 12\frac{1}{5}$. 6. 1\frac{1}{20}$. 7. 4\frac{1}{16}$. 8. 230$\frac{1}{20}$ acres. 9. $$\frac{1}{24}$. 10. 16,732$\frac{9}{20}$ pounds. 11. $$\frac{5}{12}$. 12. 70\frac{1}{20}$. 13. 142$\frac{1}{12}$. 14. 294$\frac{37}{40}$. **365.** 1. $\frac{1}{36}$. 2. 353\frac{1}{24}$. 3. 240\frac{9}{20}$. 4. 8\frac{7}{20}$. 5. $3,760$\frac{1}{10}$. 6. 486$\frac{7}{16}$ lb. 7. $17. 8. $1,453$\frac{1}{8}$. 9. 122$\frac{3}{32}$ acres. 10. 142\frac{3}{8}$. 11. 66$\frac{1}{20}$ gallons. 12. 4\frac{1}{2}$. **375.** 4. 281\frac{1}{4}$. 5. 87\frac{1}{2}$. 6. $375. 7. 34$\frac{1}{2}$ miles. **376.** 1. $167.00. 2. 292.50. 3. $460.30. 4. 159.98\frac{1}{2}$. 5. $672.40. 6. $312.81. 7. 518.21\frac{1}{4}$. 8. $612.45. 9. $565.80. 10. 689.91\frac{3}{4}$. 11. 180.37\frac{3}{8}$. 12. 569.47\frac{1}{8}$. 13. 524.07\frac{1}{8}$. 14. 168.64\frac{3}{4}$. 15. 250.42\frac{3}{8}$. 16. 619.23\frac{3}{4}$. 17. $700.07. 18. 402.82\frac{3}{8}$. 19. 353.53\frac{1}{2}$. 20. 441.91\frac{7}{8}$. **377.** 1. $1,500. 2. $24,375. 3. $60,000. 4. 906$\frac{1}{2}$ acres. **379.** 1. $360. 2. 937\frac{1}{2}$. 3. 1,820 acres. 4. $11,242. 5. $490. 6. 3,201 pounds. 7. $4. 8. $833,333$\frac{1}{3}$. 9. 131,621 soldiers. 10. 2,105 poplars. 11. $2,128.14. 12. $23,375. **385.** 2. 337\frac{1}{2}$. 3. 916\frac{3}{4}$. 5. $35,466$\frac{3}{4}$. 6. 65,740 feet. 7. 2,202 miles. 8. $75. 9. 3,827. 10. $121,805$\frac{5}{6}$. **386.** 1. $320.64. 2. 819.94\frac{5}{6}$. 3. $242.40. 4. $202. 5. $267.52. 6. $175.38. 7. $955.34. 8. $1,791.91. 9. $2,631.81. 10. $3,426.94. 11. $151.05. 12. $2,547.33. 13. $1,807.94$\frac{1}{2}$. 14. $2,216.73. 15. $2,723.46. 16. $1,162.90. 17. $2,757.03. 18. $4,734.25. 19. $2,322.32. 20. $2,148.03. **387.** 2. 19$\frac{1}{24}$ miles. 3. $12. 4. 6\frac{1}{4}$. 5. 33\frac{3}{4}$. 6. 5\frac{17}{20}$. **388.** 1. $6. 2. $8. 3. 5\frac{1}{2}$. 4. 24\frac{1}{8}$. 5. 16\frac{1}{4}$. 6. 13\frac{7}{8}$. 7. 16\frac{19}{24}$. 8. 9\frac{1}{10}$. 9. 15\frac{3}{8}$. 10. 9\frac{3}{14}$. 11. 14\frac{7}{10}$. 12. 20\frac{1}{4}$. **389.** 3. $1,190. 4. $1,396$\frac{7}{8}$. 5. $11,082$\frac{3}{8}$. 6. $26,208$\frac{3}{4}$. 7. 48,318$\frac{3}{10}$. 8. $2,253. 9. $33,937$\frac{1}{2}$. 10. $980. 11. $1,600. 12. 184\frac{1}{12}$. 13. $7,125. 14. 1,039,126$\frac{5}{12}$. 15. $1,496,688$\frac{5}{6}$. 16. $87,679$\frac{3}{4}$$\frac{1}{2}$. **390.** 1. 3,112$\frac{1}{2}$. 2. 14,993$\frac{3}{8}$. 3. 12,265. 4. 5,199. 5. 10,701$\frac{3}{4}$. 6. 18,875. 7. 5,564$\frac{5}{8}$. 8. 31,890$\frac{5}{6}$. 9. 9,424. 10. 10,054. 11. 7,571$\frac{1}{8}$. 12. 15,954$\frac{3}{4}$. 13. 51,987$\frac{9}{14}$. 14. 37,623$\frac{7}{12}$. 15. 41,681$\frac{7}{24}$. 16. 62,949$\frac{1}{2}$$\frac{7}{6}$. 17. 396,981$\frac{1}{4}$. 18. 135,812$\frac{7}{12}$. 19. 381,392$\frac{1}{3}$$\frac{9}{2}$. 20. 479,407$\frac{7}{8}$. **392.** 1. $9,142$\frac{1}{2}$$\frac{1}{4}$. 2. 184\frac{1}{4}$. 3. 24\frac{7}{24}$. 4. 72,120,000 pounds. 5. 12,214$\frac{1}{2}$. 6. $3,182$\frac{1}{4}$. 7. 300$\frac{7}{8}$ bushels; 338\frac{3}{64}$. 8. $23,749$\frac{1}{20}$. 9. $171,773$\frac{1}{12}$. 10. $52.52. **399.** 1. 90 days. 2. 50 baskets. 3. $\frac{2}{25}$ times. 4. 5 pounds; 7 pounds. 5. [illegible] vall. 6. 160 dozens. 7. Once. 8. 72 pieces. 9. $\frac{3}{4}$ of a $\frac{3}{8}$. 11. $\frac{1}{16}$ of a yard. 12. $.06$\frac{1}{4}$. 13. 147 loaves. 14.

40 boxes. **15.** $\frac{1}{10}$ of an inch. **16.** 10 hours. **17.** $\frac{9}{10}$ times. **18.** 57 times. **19.** $1\frac{1}{6}$ hours. **20.** $1\frac{1}{2}$ minutes. **21.** 240 yards. **22.** $\$\frac{6}{60}$. **23.** $1\frac{7}{8}$ minutes. **400. 1.** $\frac{9}{10}$ of a yard. **2.** $1\frac{1}{2}$ weeks. **3.** $\frac{3}{4}$ of a week. **4.** 75 suits. **5.** 80 bushels. **6.** 11 yards. **7.** 160 lots. **8.** $\$1\frac{1}{12}$. **9.** $\$.68\frac{3}{10}$. **10.** $5\frac{1}{4}$. **11.** $16\frac{1}{2}$ miles. **401. 3.** $38\frac{6}{10}$ bales. **4.** $13\frac{1}{3}$ patterns. **5.** $\$.02\frac{1}{12}$. **6.** $\$.03\frac{1}{2}$. **7.** $\$42\frac{9}{42}\frac{3}{16}$. **8.** $\$1,283\frac{3}{4}$. **402. 1.** $257\frac{9}{40}$. **2.** $392\frac{8}{15}$. **3.** $785\frac{7}{12}$. **4.** $727\frac{33}{80}$. **5.** $961\frac{23}{84}$. **6.** $2\frac{6}{13}\frac{8}{17}$. **7.** $1\frac{28}{88}\frac{45}{52}$. **8.** $2\frac{1}{46}\frac{15}{75}$. **9.** $2\frac{5}{11}\frac{8}{73}$. **10.** $1\frac{38}{42}\frac{07}{91}$. **11.** $77\frac{88}{847}$. **12.** $5\frac{1}{11}\frac{37}{29}\frac{45}{36}$. **13.** $371\frac{17}{23}\frac{75}{0}$. **14.** $439\frac{13}{49}\frac{1}{96}$. **15.** $370\frac{1}{13}\frac{4}{44}\frac{12}{8}$. **16.** $176\frac{4}{5}$. **17.** $148\frac{1}{13}\frac{28}{8}$. **18.** $224\frac{59}{23}\frac{0}{0}$. **19.** $126\frac{2}{27}\frac{28}{1}$. **20.** $139\frac{1}{2}\frac{02}{84}$. **403. 1.** $21\frac{3}{7}$. **2.** $\frac{5}{27}$. **3.** $1\frac{5}{72}$. **4.** 3. **5.** $3\frac{4}{7}$. **6.** $\frac{62}{64}\frac{4}{1}$. **7.** $\frac{12}{13}$. **8.** $9\frac{29}{60}$. **9.** $\frac{48}{83}$. **10.** $\frac{6}{97}\frac{6}{64}$. **11.** $\frac{58}{87}\frac{9}{2}$. **12.** $\frac{8}{12}\frac{11}{34}$. **404. 1.** 8 pieces. **2.** 15 hours. **3.** $5\frac{1}{4}$ yards. **4.** $5. **5.** 4 jackets. **6.** $5\frac{1}{2}$ weeks. **7.** 42 ties. **8.** 3 sons. **9.** 13 barrels. **10.** $2\frac{1}{4}$. **11.** 77 yards. **12.** $2\frac{3}{4}$. **13.** 12 dresses. **14.** $54\frac{1}{2}$ weeks. **405. 3.** $36\frac{1}{6}$ years. **4.** $560. **5.** 3,000 miles. **407. 1.** 28. **2.** 45. **3.** 22. **4.** 361. **5.** $5\frac{1}{4}$. **6.** $30\frac{1}{2}$. **7.** 40. **8.** $16\frac{5}{6}$. **9.** $177\frac{3}{4}$. **10.** 658. **11.** 300. **12.** 1,150. **13.** 1,692. **14.** $755\frac{1}{7}$. **15.** $472\frac{9}{7}$. **16.** $5,956\frac{4}{7}$. **408. 1.** 345. **2.** 642 sheep. **3.** $3,925. **4.** $8,050. **5.** 845. **6.** $4,077. **7.** $15,218. **8.** $2,440. **9.** $4,728. **10.** $315. **412. 1.** $\frac{1}{4}$. **2.** A, $22.50; B, $30.00; C, $45.00. **3.** $\frac{1}{6}$. **4.** $120. **5.** $\frac{3}{8}$. **6.** $\$5\frac{1}{8}$. **7.** $\frac{1}{5}$. **8.** $\frac{47}{80}$. **9.** $\frac{9}{16}$. **10.** $\frac{1}{3}$. **414. 1.** $42\frac{9}{7}$ days. **2.** $\$136\frac{1}{4}$. **3.** $\$65\frac{5}{8}$. **4.** $410\frac{8}{16}$ bushels. **5.** $\$3.12\frac{1}{2}$. **6.** $11\frac{5}{11}$ pieces. **7.** $3.90. **8.** $104\frac{1}{12}$ acres. **9.** House, $10.400; Stable, $2,600. **10.** $2,500. **11.** $.45. **12.** $20\frac{9}{10}$ hours. **13.** $2\frac{1}{12}$ days. **14.** $182\frac{1}{4}$. **15.** $1,508\frac{3}{7}$ times. **16.** $\frac{1}{7}$. **17.** $1\frac{1}{4}$. **18.** 8 chickens. **19.** $10\frac{9}{32}$. **20.** $42. **21.** 90. **22.** $\frac{1}{21}$. **23.** $13\frac{1}{8}$. **24.** $4\frac{1}{11}$. **25.** $219. **26.** $12,204. **27.** $\frac{9}{32}$. **28.** $155. **29.** $646. **30.** $11. **31.** 200. **32.** $\frac{3}{8}$. **33.** 9 yd. 11 in. **34.** 3 ft. 4 in. **35.** 27 A. 36 P. **36.** $673.50. **37.** $130.50. **38.** $98\frac{1}{6}$ days. **39.** 180 yards. **40.** $52\frac{13}{4}$. **419. 1.** $20\frac{3}{8}$ pence. **2.** $9\frac{3}{8}$ pence. **3.** 600 pence. **4.** 80 pence. **5.** $18\frac{3}{4}$ pwt. **6.** $17\frac{1}{2}$ pwt. **7.** 95 pwt. **8.** 540 pwt. **9.** 432 ℈. **10.** $17\frac{1}{2}$ ℈. **11.** 9 ℈. **12.** $57\frac{3}{8}$ ℈. **13.** $229\frac{1}{2}$ sq. ft. **14.** $2,075\frac{3}{32}$ sq. ft. **15.** 7,260 sq. ft. **16.** 10,890 sq. ft. **17.** 6 pints. **18.** 336 pints. **19.** 63 pints. **20.** 44 pints. **21.** 168 inches. **22.** $158\frac{3}{8}$ inches. **23.** $2\frac{2}{3}\frac{4}{3}$ inches. **24.** 2,970 inches. **420. 1.** 2 qt. $1\frac{1}{2}$ pt. **2.** 2 pk. 4 qt. **3.** 3 pk. 1 qt. $1\frac{1}{8}$ pt. **4.** 18 cu. ft. **5.** 6 cd. ft. $6\frac{3}{8}$ cu. ft. **6.** 864 cu. in. **7.** 2 ft. $7\frac{1}{2}$ in. **8.** 15 yd. 1 ft. 9 in. **9.** 3 furlongs. **10.** 42 lb. $13\frac{1}{2}$ oz. **11.** 4 cwt. **12.** 4 T. 5 cwt. **13.** 5 grains. **14.** 2 scruples. **15.** 6 ℥. 2 ℈. **16.** 60 grains. **17.** 12 pwt. **18.** 64 oz. 10 pwt. **421. 1.** $14.40. **2.** $16.00. **3.** $19.60. **4.** $5.10. **5.** $14,361.60. **6.** $739\frac{1}{8}$ rotations. **7.** 6,864 steps. **8.** $\frac{1}{4}$ mile. **9.** $2.67. **10.** 1 dozen. **11.** 30 cups. **12.** 100 lots. **13.** 40 dozen. **14.** 36 pieces. **424. 1.** $\frac{3}{4}$ pound. **2.** $\frac{1}{24}$ pound. **3.** $\frac{1}{240}$ pound. **4.** $\frac{1}{240}$ acre. **5.** $\frac{7}{320}$ acre. **6.** $\frac{1}{320}$ acres. **7.** $\frac{1}{24}$ pound. **8.** $\frac{13}{5760}$ pound. **9.** $\frac{1}{960}$ pound. **10.** $\frac{3}{4}$ gallon. **11.** $\frac{5}{6}$ gallon. **12.** $\frac{7}{32}$ gallon. **13.** $\frac{1}{144}$ pound. **14.** $\frac{1}{192}$ pound. **15.** $\frac{1}{4320}$ pound. **16.** $\frac{1}{44000}$ mile. **17.** $\frac{1}{132000}$ mile. **18.** $\frac{1}{2448}$ mile. **425. 1.** $\frac{5}{19}\frac{2}{3}\frac{2}{6}\frac{3}{0}$ acre. **2.** $\frac{2}{19}\frac{3}{8}\frac{4}{4}\frac{7}{0}$ acre. **3.** $\frac{8}{10}\frac{1}{8}\frac{7}{9}\frac{1}{0}$ acre. **4.** $3\frac{5}{11}\frac{2}{6}\frac{0}{6}\frac{9}{4}$ cu. yd. **5.** $5\frac{3}{23}\frac{2}{4}\frac{5}{0}\frac{9}{8}$ cu. yd. **6.** $10\frac{5}{23}\frac{2}{4}\frac{5}{8}$ cu. yd. **7.** $\frac{52}{165}$ mile. **8.** $\frac{373}{1584}$ mile. **9.** $\frac{4987}{12672}$ mile. **10.** $1\frac{23}{64}$ bushels. **11.** $2\frac{5}{64}$ bushels. **12.** $4\frac{19}{64}$ bushels. **13.** $\frac{1}{16}$ gallon. **14.** $4\frac{3}{8}$ gallons. **15.** $1\frac{5}{32}$ gallons. **16.** $\frac{1017}{6400}$ ton. **17.** $\frac{1017}{32000}$ ton. **18.** $3\frac{1453}{32000}$ tons. **426. 1.** $3\frac{1}{4}$ pounds. **2.** $\frac{1}{8}\frac{7}{2}$ bushel. **3.** $1,769.41\frac{3}{4}$. **4.** $5\frac{3}{4}$ cwt. **5.** $22.64. **6.** $\frac{35}{216}$ sq. yd. **7.** $\frac{1}{6}$ cu. yd. **8.** $26\frac{2}{3}$ yards. **9.** 60 cords. **428.** *3. 3 pk. 5 qt. 1 pt.* **4.** 12 mi. 254 rd. 3 yd. 2 ft. 6 in. **5.** 43 mi. 145 rd.

3 yd. 3 ft. 2 in. **6.** 79 mi. 14 rd. 4 yd. 2 ft. 8 in. **7.** 103 A. 2 R. 37 sq. rd. 27 sq. yd. 6 sq. ft. 108 sq. in. **8.** 60 mi. 128 rd. 4 yd. 1 ft. 8 in. **9.** 143 A. 3 R. 38 sq. rd. 12 sq. yd. 6 sq. ft. 72 sq. in. **429. 1.** 3 R. 38 P. 20 sq. yd. 1 sq. ft. 72 sq. in. **2.** 160 rd. 2 yd. 1 ft. $8\frac{7}{10}$ in. **3.** 7 cwt. 50 lb. **4.** 1 cd. $\frac{1}{8}$ cu. ft. **5.** 9 oz. 17 pwt. 4 gr. **6.** 1 pk. $6\frac{7}{10}$ qt. **7.** $3\frac{12}{60}$ qt. **8.** 320 A. 106 sq. rd. 20 sq. yd. 1 sq. ft. 72 sq. in. **9.** 21 cu. ft. $86\frac{2}{5}$ cu. in. **10.** 1 pk. 3 qt. $1\frac{1}{2}$ pt. **430. 3.** 319 rd. 2 yd. 2 ft. **4.** 5 mi. 317 rd. 1 yd. 2 ft. 4 in. **5.** 4 mi. 294 rd. 1 yd. 2 ft. 6 in. **6.** 6 A. 36 P. 26 sq. yd. 2 sq. ft. 36 sq. in. **7.** 14 mi. 256 rd. 1 yd. 2 ft. 1 in. **8.** 224 A. 1 R. 4 P. 5 sq. yd. 2 sq. ft. 36 sq. in. **9.** 30 A. 2 R. 27 sq. rd. 27 sq. yd. 2 sq. ft. 36 sq. in. **10.** 65 A. 6 R. 10 sq. rd. 24 sq. yd. 3 sq. ft. 36 sq. in. **11.** 57 A. 4 R. 37 sq. rd. 26 sq. yd. 2 sq. ft. 36 sq. in. **431. 1.** 79 sq. rd. 18 sq. yd. 8 sq. ft. $22\frac{1}{2}$ sq. in. **2.** 12 cwt. 53 lb. $5\frac{1}{3}$ oz. **3.** 5 cd. ft. $11\frac{3}{7}$ cu. ft. **4.** 39 sq. rd. 15 sq. yd. 1 sq. ft. 18 sq. in. **5.** $4\frac{1}{15}$ ʒ. **6.** 4 oz. 9 pwt. $13\frac{1}{4}$ gr. **7.** 1 qt. 1 pt. $1\frac{7}{8}$ gi. **8.** 3 pk. $1\frac{7}{16}$ pt. **9.** 255 rd. $2\frac{1}{16}$ yd. **10.** 13 cu. ft. 432 cu. in. **11.** 19 sq. yd. 7 sq. ft. 72 sq. in. **12.** 4 ʒ. 1 ℈. 18 gr. **13.** 4 rd. 1 ft. 6 in. **14.** 5 lb. 4 oz. 8 pwt. 16 gr. **15.** 6 bu. 2 pk. 2 qt. $1\frac{1}{2}$ pt. **16.** 2 gal. $1\frac{1}{10}$ pt. **432. 1.** $20,554. **2.** $43.51. **3.** $116.60. **4.** 6 lb. 10 oz. 10 pwt. **5.** 478.02\frac{1}{2}$. **6.** $3\frac{1}{32}$ hours. **7.** $154.69. **8.** 11 cu. yd. 24 cu. ft. 1,669 cu. in. **434. 3.** 37 mi. 8 rd. 3 yd. 2 ft. **4.** 146 mi. 83 rd. 1 yd. 1 ft. 6 in. **5.** 351 mi. 21 rd. 3 yd. 3 ft. 6 in. **6.** 50 lb. 2 oz. 2 pwt. 9 gr. **7.** 60 A. 2 R. 2 sq. rd. 9 sq. yd. 2 sq. ft. 36 sq. in. **8.** 253 A. 1 R. 24 sq. rd. 9 sq. yd. 4 sq. ft. 24 sq. in. **9.** 442 gal. 1 qt. 1 pt. $2\frac{3}{4}$ gi. **10.** 272 bu. $\frac{1}{2}$ pt. **11.** 107 sq. mi. 136 A. 340 sq. l. **435. 1.** 157 rd. 4 yd. 2 ft. $7\frac{1}{2}$ in. **2.** 33 A. 3 R. 7 P. 6 sq. yd. 5 sq. ft. 36 sq. in. **3.** 15 lb. 6 ℥. 4 ʒ. $1\frac{1}{2}$ ℈. **4.** 11 T. 9 cwt. 32 lb. **5.** 13 bu. 3 pk. $4\frac{1}{8}$ qt. **6.** 32 A. 35 P. 22 sq. yd. $64\frac{4}{9}$ sq. in. **7.** 122 gal. 2 qt. $1\frac{1}{2}$ gi. **8.** 100 T. 7 cwt. 68 lb. 2 oz. **436. 3.** 1 mi. 178 rd. 4 yd. 1 ft. 10 in. **4.** 192 rd. 3 yd. $1\frac{3}{8}$ in. **5.** £2, 19s. 10d. $1\frac{1}{12}$ far. **6.** 8 sq. mi. 371 A. 3 R. 35 sq. rd. 16 sq. yd. 5 sq. ft. $137\frac{1}{4}$ sq. in. **7.** 27 gal. 2 qt. 1 pt. $2\frac{3}{14}$ gi. **8.** 200 A. 2 R. 23 sq. rd. 28 sq. yd. 8 sq. ft. 64 sq. in. **9.** 2 mi. 270 rd. 4 yd. 1 ft. $10\frac{8}{9}$ in. **10.** 357 A. 2 R. 6 sq. rd. 12 sq. yd. 2 sq. ft. 128 sq. in **11.** 3 lb. 6 ℥. 2 ʒ. $2\frac{1}{2}$ gr. **437. 1.** 31 rd. 3 yd. 2 ft. $10\frac{8}{9}$ in. **2.** 11 A. 2 R. 28 sq. yd. 4 sq. ft. $33\frac{1}{9}$ sq. in. **3.** 4 sq. ch. 12 P. $178\frac{4}{9}$ sq. links. **4.** 7 cu. ft. $1,355\frac{1}{3}$ cu. in. **5.** 2 gal. 1 qt. $3\frac{9}{17}$ gi. **6.** 3 bu. $4\frac{152}{225}$ qt. **7.** 3 lb. 3 oz. 2 pwt. $14\frac{1}{3}$ gr. **8.** 2 ʒ. 1 ℈. $4\frac{1}{3}$ gr. **9.** 4 T. 13 cwt. 33 lb. $5\frac{1}{3}$ oz. **10.** £5. 14s. $4\frac{4}{9}$d. **11.** 2 mi. 235 rd. 1 yd. 1 ft. $10\frac{4}{7}$ in. **12.** 4 sq. rd. 28 sq. yd. 5 sq. ft. $52\frac{1}{12}$ sq in. **438. 1.** £6,445. 3s. 1d. 2 far. **2.** 4 mi. 10 rd. 3 yd. $2\frac{1}{40}$ ft. **3.** 55 hhd. 1 bbl. 23 gal. 1 qt. 1 pt. **4.** 18 lb. 10 oz. 12 pwt. **5.** 14 cwt. 12 lb. $8\frac{6}{7}$ oz. **6.** 166 rd. 5 yd. **7.** 2,377 bu. 3 pk. $2\frac{1}{2}$ qt. **8.** 3 lb. 1 oz. 10 pwt. $6\frac{4}{7}$ gr. **9.** 400 A. 40 sq. rd. 3 sq. yd. 6 sq. ft. **10.** 89 C. 4 cu. ft. **11.** 12 cwt. 60 lb. **439. 1.** 11 A. 3 R. 15 sq. rd. 29 sq. yd. 5 sq. ft. 36 sq. in. **2.** £24,000. 13s. **3.** 2 qt. $2\frac{1}{2}$ gi. **4.** 6 yd. 8 in. **5.** 7 da. 7 hr. **441. 1.** £90, 3s. 4d. **2.** 4 A. 1 R. 1 sq. rd. 7 sq. yd. 5 sq. ft. 9 sq. in. **3.** 417 mi. 311 rd. 2 yd. $2\frac{1}{2}$ ft. **4.** 2 lb. 10 oz. 16 pwt. **5.** 3 lb. 5 ʒ. 4 ʒ. 1 ℈. $6\frac{2}{3}$ gr. **6.** 5 lb. 5 oz. **7.** £136. 8s. 6d. $3\frac{9}{17}$ far. **8.** 10 cwt. 50 lb. **442. 3.** $\frac{12}{8}\frac{7}{0}\frac{7}{0}$ of a day. **4.** $\frac{8}{1}\frac{8}{2}\frac{3}{8}$ of a cord. **5.** $\frac{1}{2}\frac{1}{0}\frac{2}{1}\frac{1}{6}\frac{1}{6}\frac{1}{2}\frac{1}{8}$. **6.** $\frac{1}{3}\frac{1}{2}\frac{8}{1}$. **7.** $\frac{1}{1}\frac{1}{4}\frac{7}{4}$. **444. 1.** $\frac{1}{7}\frac{1}{6}\frac{1}{8}\frac{1}{0}$ of the farm. **2.** $\frac{1}{2}\frac{7}{4}\frac{7}{0}$ of a ream. **3.** $\frac{1}{7}\frac{1}{8}$ of a load. **4.** $\frac{1}{1}\frac{1}{2}\frac{9}{0}\frac{8}{8}\frac{7}{0}$ of each week. **446. 1.** $\frac{1}{4}\frac{1}{8}$ of a ream. **2.** 1 cu. yd. 8 cu. ft. 1701$\frac{3}{4}$ cu. in. **3.** $\frac{1}{8}$ of a shilling. **4.** 20 hr. 55 min. **5.** $\$\frac{4}{1}\frac{1}{7}$. 9ᵉᵉ **7.** 1,831$\frac{1}{2}$ times. **8.** $.50. **9.** $1,089,000. **10.** 132 cu. ft. **12.** $7.30. **13.** 235 lots. **14.** $.38. **15.** 15,111,36 **17.** $1. **18.** $\frac{8}{4}\frac{8}{0}\frac{3}{3}$. **19.** 25,600 square rods. **20.** 16.8

times. **21.** 14 da. 16 h. **22.** 56 square rods. **23.** $\frac{1603}{1728}$ of the cube. **24.** 720 seconds. **25.** $\frac{4375}{28800000}$. **487.** **1.** $5.68. **2.** $273.50. **3.** $2,565.95. **4.** $416.44. **5.** $433.15. **6.** $5,047.82. **7.** $3. **498.** **1.** $57,013.875. **2.** $742.997. **3.** $1,151.61¾. **4.** $12,562.50. **5.** $552.83. **511.** **1.** 28,125. **2.** $5,426. **3.** $15,646.25$\frac{2}{4}\frac{5}{9}$. **4.** 1,896 tons. **5.** $10.96. **513.** **1.** $125. **2.** $101.50. **3.** 96⅓ yards. **4.** $2.65. **5.** $12.50. **6.** 10 years. **7.** $1,032.33. **8.** $33.28. **9.** $243.75. **10.** 127 acres. **11.** $6,544.50. **12.** $3,076.50. **13.** .96,625. **14.** $23,305. **15.** $23,347,675. **16.** 4.125\frac{3}{20}$. **17.** 11.26\frac{9}{16}$. **18.** 336 barrels. **19.** 600 head. **20.** 211.47\frac{3}{50}$. **21.** $71.25. **22.** 139.49\frac{3}{4}$. **23.** 12.3467. **24.** $500. **25.** 89 cords. **26.** 4,180 pounds. **27.** 720 hours. **514.** **1.** $238.43¾. **2.** $815.58. **3.** $1,057.12½. **518.** **1.** $110. **2.** $140.62½. **3.** $31.05. **4.** $236.25. **5.** $200. **6.** $64.90. **526.** **1.** $31.68¾. **2.** $225. **3.** 628.32 yards. **4.** 6 barrels. **5.** $26,828+. **6.** $9.72½. **7.** $6.185. **8.** 85$\frac{5}{8}$ yards. **9.** 25 barrels. **10.** $160.36+. **11.** $257.03¼. **12.** 34$\frac{17}{25}$ inches. **13.** $14.06¼. **14.** $.015. **15.** .429. **16.** $25,000; $12,500. **17.** $57.54+Loss. **18.** $34.26+. **19.** 3,375 cu. ft. **20.** 20 yards. **21.** $6,712.50. **22.** $102.90. **23.** 840 pounds. **24.** 9,900 words. **25.** $\frac{1}{16}$. **26.** 7⅛. **530.** **2.** .00125 lb. **3.** .0004375 acre. **540.** **1.** .026+ mile. **2.** 5$\frac{7}{8}$ bushels. **3.** 12½ quarts. **4.** 10$\frac{5}{16}$ tons. **5.** $\frac{1}{5}$ pound. **6.** 26 A. 3 R. 4 rd. **7.** 4 C. 91$\frac{9}{25}$ cu. ft. **548.** **1.** $156.816. **2.** $144. **3.** 423.3 sq. ft. **4.** $4. **5.** 9 feet. **6.** $63,000. **7.** $5.51. **8.** $3,440. **9.** $17.10. **10.** 160 sq. yd. **560.** **1.** 1,350 bricks. **2.** $90.74+. **3.** 36$\frac{3}{4}$ ft. **4.** 8$\frac{45}{8}$ perches. **5.** 110,592 bricks. **569.** **1.** 75 bd. ft. **2.** 486. bd. ft. **3.** $20.925. **4.** 20 feet. **576.** **1.** $13.46+. **2.** 30,000 pounds. **3.** 53,859$\frac{37}{7}$ gallons. **4.** $871.87½. **5.** $.125$\frac{1}{3}$. **578.** **1.** 26$\frac{1}{4}$ miles. **2.** 65¼ acres. **3.** 20 feet. **4.** 8,448 steps. **5.** 60 bd. ft. **6.** 33$\frac{1}{6}$ miles an hour. **7.** $11.40. **8.** 48 feet. **9.** 3 A. 120 sq. rd. **10.** $.90. **11.** $5.33⅓. **12.** 233,887½ bricks. **13.** $33.075. **579.** **1.** $152.28. **2.** $446.93. **3.** $42.10. **4.** $42.28½. **5.** $115.57½. **599.** **1.** $8.75. **2.** 2 guineas, 7½ shillings. **3.** $1,350. **4.** 3⅜ shillings. **5.** 37,500 men. **6.** 1,000 barrels. **7.** $2,800. **8.** 1,000 trees. **9.** 1,500 pounds. **10.** 10,158¼. **604.** **1.** $3,150. **2.** 437 yards. **3.** 8,125 bushels. **4.** $599.25. **5.** $12½. **6.** 27 sq. yd. **7.** $2,100. **608.** **1.** 30 %. **2.** 52 %. **3.** 40 %; 60 %. **4.** 38$\frac{9}{17}$ %; 61$\frac{8}{17}$ %. **5.** 52$\frac{1}{2}$ %. **6.** 81⅛ %. **7.** 25 %. **8.** 37½ %. **9.** 59$\frac{1}{11}$ %. **10.** 42⅞ %. **612.** **1.** 1,250. **2.** $600. **3.** 1,900 sheep. **4.** 18,000 people. **5.** $9,000. **6.** 1,875 sheep. **7.** $14,000. **8.** $1,458. **9.** 3\frac{3}{4}$. **10.** $5. **616.** **1.** 888\frac{8}{9}$. **2.** $2,000. **3.** 1,500. **4.** 2,500. **5.** 1 87½. **6.** $200. **7.** $2.04⅛. **8.** 2,258$\frac{1}{3}$ bushels. **9.** $2½. **10.** $525. **618.** **1.** $3,291.30. **2.** $4,013.83½. **3.** $94. **4.** $247.69+. **5.** $206.25 Loss. **6.** $45. **7.** $1,474.28⅛. **8.** A, $4,620. B, $1,155. C, 4,725. **9.** 75 %. **10.** 7½ %. **11.** 1½ %. **12.** 3½ %. **13.** 70,000 men. **14.** $6,250. **15.** $1,000. **16.** 100 shares. **17.** $5 gain. **18.** .42+ mile. **19.** A, 60 %. B, 40 %. **20.** $216.22. **626.** **1.** $10,875. **2.** $153.13. **3.** $7,695. **4.** $1,150. **627.** **3.** 25 %. **4.** 80$\frac{3}{4}$ %. **5.** 9$\frac{1}{11}$ %. **631.** **1.** 4 %. **2.** 41$\frac{2}{7}$ %; 100 %. **3.** 0. **4.** 2$\frac{3}{4}$ %. **5.** 68$\frac{3}{4}$ %. **6.** 27$\frac{7}{9}$ %. **7.** 78¼ %; $883.93. **632.** **2.** $340,000. **3.** 46\frac{3}{4}$. **4.** $7,875. **636.** **1.** $130,400. **2.** $67,068.86. **3.** $6. **4.** $3,750. **5.** $186. **6.** $60. **637.** **1.** $72.50. **2.** $1.25. **3.** $2. **4.** $1.62½. **640.** **1.** $150. **2.** $400. **3.** $2,700. **4.** $4. **5.** $6.28¼. **6.** $7,000. **7.** $.09. **8.** $.02. **9.** $20.26. **10.** $45.56. **642.** **1.** $3. **2.** $212.50. **3.** 12$\frac{4}{9}$ %. **4.** 40 %. **5.** $11,000. **6.** $402.50. **7.** 15$\frac{6}{7}$ %. **8.** $6½. **9.** 7½ %. **10.** $.16. **11.** $6¼. **12.** $43. **13.** $4.60. **14.** $320. Loss.

647. 1. $296.75. 2. $437.63. 3. $89.25. 4. $148.75; $7,358.75. 5. $34.65; $831.60. **651.** 1. $51.56. 2. $187.69; $10,912.69. 3. $141; $2,679. 4. $2,890.63. **652.** 2. 2½%. 3. ¼%. 4. 2½%. 5. 10%. **656.** 1. 2½%. 2. 30%. 3. 3%. 4. 2%, com.: 3%, guarantee. **657.** 1. $12,500; $12,250. 2. $11,250; $11,475. 3. $36,540. 4. $5,850. **661.** 1. 40 bushels. 2. $15,000. 3. $1,900. 4. 100 barrels. **662.** 1. $15,000. 2. $14,491.84. 3. $3,760. 4. $12,948.72. **665.** 1. $2.25. 2. $6. 3. $10. 4. $30. 5. $50,330. 6. $85. **667.** 1. $2.90. 2. 8½%. 3. $37.50. 4. 1¾%. 5. $16,000. 6. $36,000. 7. 80 shares. 8. $375. 9. $5.50. 10. $12.50. 11. 2,000 pounds. 12. $800. 13. $525. 14. $4.13. 15. $6,000. 16. 1½%. 17. ½%. 18. 2,000 Louis d'or, 40,000 francs. 19. 24 yards. 20. $973.30. 21. $3,355.77. 22. $2,500. 23. $8,109.09. 24. ¼%; ¼%. 25. $4,092.74. **679.** 1. $3,459. 2. $72,135. 3. $19,557. 4. $4,235. 5. $6,990.00 6. $5,487.50. 7. 15½%. 8. $2,731.25. 9. $2,232.50. 10. $2,062.50. 11. $2,812.50. **680.** 1. 40 shares. 2. 200 shares. 3. 30 shares. 4. 60 shares. 5. 45 shares. 6. 500 shares. **681.** 1. $62,850. 2. $107,500. 3. $65,550. 4. $38,400. **682.** 2. $652.50. 3. $240. 4. $125. 5. $21,000. **683.** 2. 6¼%. 3. 6⅛%. 4. 6+%. 5. 5+%. **684.** 1. $83⅓. 2. $66⅔. 3. 25. 4. 14⅜%. 5. 18½⅛% above par. 6. 12½% below par. 7. $83⅓; $125. **685.** 2. $84. 3. $120. 4. $110. 5. $90. 6. $4,025. **687.** 1. $4,950. 2. $720. 3. 5%. 4. $150. 5. $75. 6. $25. 7. $320. 8. $20. 10. 900 shares. **694.** 2. ⅛%. **696.** 1. $50,000. 2. $175,000. 3. $65,201.78+. 4. $150,000. **698.** 1. $31.35. 2. $2.70. 3. $12,500. 4. $2,670. 5. 1½%. 6. $8,500. 7. 5%. 8. $4,000.00. 9. 2½%. 10. $6,200. 11. $2,400. 12. $10,000. **705.** 1. $75. 2. $450. 3. $367.50. 4. $1,425.60. 5. $107.10; $742.90. 6. $20. 7. 0. 8. $1.50. 9. $1. 10. $600. 11. 829.35. 12. $2,314.81+. **710.** 1. $76; .4½%. 2. $95,000. 3. $20. 4. 2%. 5. $21; 1⅜%. 6. $1,893,750. **716.** 1. $517.07. 2. $738.75. 3. $240. 4. £24; $116.80. 5. $76.43. 6. $570. 7. $276. 8. 1,020 francs; $196.86. 9. $219.52. 10. 10 hogsheads. 11. $1.45. 12. $882. **726.** 2. $678.82. 3. $409.50. 4. $4,512. 5. $1,465.80. 6. $1,100.25. 7. $2,769.20. **727.** 2. $738. 3. $1,546.67. 5. $120.87. 6. $273.38. 8. $20.53. 9. $7.20+. **733.** 2. $143.43. 3. $326.46+. 4. $806.23. **746.** 2. $44.73. 3. $56.10. 4. $51.37. **749.** 2. 4½%. 3. 4%. 4. 5%. 5. 9%. 6. 6%. **752.** 2. 3 yr. 3 mo. 3. 5 yr. 6 mo. 4. 4 yr. 3 mo. 20 da. 5. 10 yr. 4 mo. 16 da. **754.** 2. $8,000. 3. $500. 4. $9,625. 5. $153.84. **756.** 2. $2,600. 3. $850. 4. $1,200. **761.** 1. $21.47. 2. $.25. 3. $1,199. 4. $4.79 gain. 5. 4½%. 6. $57.06⅔. 7. $122.06. 8. $329.67. 9. 20%. 10. $20.52. 11. $.05. 12. $1,330. 13. 8½%. 14. $951.79. 15. 4½%. 16. 5%. 17. 12%. 18. 1 yr. 5 mo. 6 da. 19. 4½%. 20. 13 yr. 4 mo. 21. 6 yr. 7 mo. 12 da. 22. 8%. 23. $19.07. 24. $293.25. 25. 5%. 26. 7½%. 27. 4%. 28. $1,025. 29. $129.44 + int.; $1,280.04 + amt. 30. 10%. 31. $11.20. 32. 7%. 33. $263.58 + amt.: $13.58 + int. 34. $104.17. 35. 4½%. 36. $611.45. 37. $.82½. 38. $1,500. 39. 3 yr. 3 mo. 3 da. 40. 8½%. **788.** 3. $9.63; $1,490.37. **794.** 1. $1,790.10. 2. $5.27. 3. $1,500. 4. $6.86. 5. $22.75. 6. $600. 7. $9.98; $940.02. 8. $5,000. 9. $1,224.37. 10. $318.28. 11. $500. 12. $1,680. 13. $3.09; $554.65. 14. $149.95. 15. $125. 17. 1 month. **802.** 3. $1,**97.** 1. $7.50; $.39. 2. $36. 3. $75. 4. $3,200. 5. 1 yr. 5 mo. 12 da. 7. 2 yr. 4 mo. 8. 1 yr. 8 mo. 7 da. 9

$3,187.50. **10.** $3,025.60. **12.** 1 yr. 2 mo. **13.** 3 mo. 8 da. **814.** **2.** $34.29+. **3.** $38.29. **4.** $108.75+. **5.** $75.88. **6.** $31.20+. **7.** $29.10. **8.** $913.18. **9.** $67.92+. **10.** $471. **822.** **1.** March 8th. **2.** 59 days. **3.** Feb. 6th. **4.** May 4th. **5.** 10 mo. 27 da. **6.** 1 yr. 8 mo. **7.** May 1st. **8.** 9 mo. 13 da. **9.** July 13th. **10.** $1,641.52. **11.** $2,476.41. **12.** 7 yr. 5 mo. 4 da. **13.** $2,158.09. **825.** **1.** June 8, 1886. **2.** $300; Jan. 31, 1888. **3.** April 3, 1887. **4.** Oct. 14, 1885. $1,772.46. **839.** **3.** $2,475. **4.** $1,142.51. **5.** $985. **6.** $2,991. **7.** $2,473.75. **844.** **1.** $3,473.33. **2.** $4,950. **3.** $4,000. **4.** 1.01⅛. **5.** $6,500. **6.** $6,012. **7.** $7,826. **8.** $7,000. **9.** 1.01⅜. **10.** $1,999.67. **848.** **1** $565.20. **2.** £369. 17s. 6d. **3.** $1,476.52. **4.** £382. 14s. 4d. **5.** $389.32. **6.** 7,740 francs. **7.** 4,008 marks. **8.** $590.62½. **9.** $433.64. **10.** 11,544+ marks. **11.** $4.87. **12.** $976.43½. **13.** 5.193. **14.** 94½. **15.** 1,025 marks. **866.** **1.** $50. **2.** 54 bushels. **3.** $360. **4.** $125. **5.** 123 yards. **6.** 120 feet. **7.** 30 days. **8.** $106.80. **9.** $44. **10.** $2.72. **11.** 125 yards. **12.** $4.38. **13.** $4,767.58. **870.** **2.** 5 weeks. **3.** $1.80. **4.** 6 hours. **5.** $5.25. **6.** 208 acres. **7.** 34 days. **8.** 300 men. **9.** 8 men. **10.** 2,340 pounds. **873.** **2.** 24 men. **3.** 250 men. **4.** 40 days. **5.** 80 days. **6.** 40 barrels. **7.** 7 feet. **8.** 85 days. **9.** 8 ft. 3 in. **10.** 60 dozen pairs. **876.** **2.** 125 acres; 200 acres. **3.** 500 head; 750 head. **4.** $15,000. **877.** **1.** 208 feet. **2.** 125 reams. **3.** 6 men. **4.** 12; 24; 36. **5.** 5 ft. 10 in. **6.** $548.64. **7.** 28 sq. ft. 99 sq. in. **8.** $2,000; $3,000; $4,500. **9.** 13,038 bricks. **10.** $750. **11.** 6¾ hours. **12.** 15 years; 20 years; 30 years. **886.** **3.** A, $750; B, $975; C, 1,050. **4.** A, $166⅔; B, 55⅝; C, $27½. **5.** A, $24; B, $30; C, $36. **6.** A, £70. 2s. 6d.; B, £116. 17s. 6d. **7.** A, $4,000; B, $3,000; C, $2,000. **8.** A, $284.84; B, $569.68; C, $854.52. **9.** A, $570; B, $712.50; C, $760. **10.** A, $6,000; B, $12,600; C, $10,800; C, 13,200. **11.** A, $510; B, $520; C, $390. **891.** **1.** $525. **2.** 3,120 head. **3.** $1,100. **4.** 40 shares. **5.** $57,600. **6.** 1,200 spiders. **7.** Wife, $25,000; son, $10,000; daughter, $5,000. **8.** $150. **9.** $315. **10.** $10. **11.** $31.25; 20 pounds. **12.** $715; 77 acres. **13.** 16 days. **14.** 8 days. **15.** 3 days. **16.** 240½ days. **17.** 24 days. **18.** 5 weeks. **19.** 24 days. **20.** 9 laborers. **945.** **1.** 40 feet. **2.** 2.23+ inches. **3.** 179.65 feet. **4.** 40 yards. **5.** 39.37 inches. **947.** **1.** 388½ sq. ft. **2.** 300 sq. ft. **3.** 142.119 sq. ft. **4.** 7,370.4 sq. yds. **5.** 12.5664 sq. ft. **6.** 6,283.2 sq. ft. **7.** 667.59 sq. ft. **8.** 37,325.34 cu. ft. **949.** **1.** 99.6 cu. ft. **2.** 2,120.58 cu. ft. **3.** 12 cu. ft. **4.** 176.715 sq. ft. **5.** 904.7808 cu. ft. **6.** 1,414 ft.; $2\frac{9}{10}\frac{4}{8}\frac{7}{9}$ A. **950.** **1.** 75 feet; 200 feet. **2.** 477.5232 sq. ft. **3.** 1,285.7 bushels. **4.** 3,760.128 gallons. **5.** 1,176 sq. in. **6.** (1) 23.848 ft.; (2) 589.05 sq. ft.; (3) 1,404.76 cu. ft.; (4) 11.19 ft.; (5) 18.72 ft. **7.** 18.4078 cu. ft. **8.** 1,680.6 feet; 50.92 acres. **9.** 939⅛ sq. ft. **951.** **1.** $7,200. **2.** $8,020. **3.** $.37. **4.** $5,600. **5.** 683.98. **6.** $1.25. **7.** $1. **8.** 200 gallons. **9.** ⅛ %. **10.** $840. **11.** ⅜ %. **12.** 1 yr. 10 mo. 26 da. **13.** 5 mo. 10 da. **14.** 4 mo. 21 da. **15.** 7 mo. 18 da. **16.** 9 mo. 2 da. **17.** $172.91. **18.** $908.47. **19.** $7,909.99. **20.** $1,361.22. **21.** $183.25 due Oct. 15, 1889. **22.** August 13. **23.** Length, 18 ft.; width, 12 ft. **24.** $\frac{2}{17}$. **25.** Sept. 24, 1889; $2,314.57. **26.** Dec. 25, 1890. **27.** Jan. 24, 1890. **28.** 10 hours. **29.** $31,200. **30.** 30 hours. **31.** 33⅓ seconds. **32.** 420. **33.** 83+ feet. **34.** 320 bd. ft. **35.** 98. **36.** B, $30; C, $40. **37.** 805+ sq. ft. **38.** 14+ inches. **39.** 4.30 P. M. **40.** 110 feet. **41.** 201½. **42.** $\frac{1}{43}$. **43.** 8 feet. **44.** Mrs. D.'s age = ¼ of her husband's age. **45.** $12,530; $\frac{3}{17}$. **46.** 12 acres. **47.** $5.67. **48.** $168.33. **49.** $7,025.

50. $192. **51.** $1\frac{1}{2}$%. **52.** $360. **53.** 1 year. **54.** $241\frac{7}{15}\frac{13}{21}$ bushels. **55.** 16 acres. **56.** 44.68 yards. **57.** $13\frac{1}{2}$ cords. **58.** $6\frac{2}{3}\frac{2}{6}\frac{2}{7}$%. **59.** 10.8 feet. **60.** 1.34\frac{3}{4}$. **61.** 9 feet. **62.** $6\frac{8}{1}\frac{0}{7}\frac{2}{2}\frac{1}{8}\frac{7}{6}\frac{7}{8}$%. **63.** $160. **64.** 29 feet. **65.** $24\frac{3}{7}\frac{3}{6}$ miles; $29\frac{3}{7}\frac{3}{6}$ miles. **66.** $860. **67.** 21,600 sq. in. **68.** .000000004. **69.** $138.60. **70.** 6 feet. **71.** 8.5 feet. **72.** $4,500. **73.** $159\frac{4}{9}$ barrels. **74.** $42.21. **75.** 3,424 sq. ft. **76.** $1,537.50. **77.** £600. 10s. **78.** 17 rods. **79.** 135 ; 45. **80.** 9,500,000. **81.** $5\frac{1}{11}$ days. **82.** $74. **83.** 24.4 feet. **84.** 189.6 feet. **85.** $1,175\frac{1}{2}\frac{1}{5}$ gallons. **86.** 1,120 square feet. **87.** $3,020.83 gained. **88.** 9.4 chains. **89.** $81\frac{3}{7}$ barrels. **90.** $11,666\frac{2}{3}$. **91.** 280. **92.** 5% gained. **93.** 25.05 feet. **94.** Cash $100 better. **95.** $9.37\frac{1}{2}$. **96.** 35%. **97.** $300. **98.** 2,654.85. **99.** A, $1,125; B, $1,250; C, $625. **100.** $3,125; $4,375; $5,000. **101.** 23.7 feet. **102.** 5,096 sq. ft. **103.** 904.7808 cu. in. **104.** 81. **105.** 90 yards. **106.** 156. **107.** A, $600 ; B, $900. **108.** 2 lots. **109.** 512. **110.** $800. **111.** $1,350. **112.** $11.69. **113.** $29.89. **114.** $9.66; $910.34. **115.** 27 men. **116.** $15; $18; $27. **117.** 30 miles. **118.** 4:48 P.M. **119.** $871\frac{1}{2}$ tons. **120.** $31\frac{5}{7}\frac{5}{8}$ tons. **121.** $64,335. **122.** $1,782.90. **123.** $4\frac{1}{6}$%. **124.** 25%. **125.** $3.60; $4.48; $111.92. **126.** 20%. **127.** 3 A.M. **128.** $647.54. **129.** $1,496.10. **130.** 50%. **131.** $6,000. **132.** 5% bonds are $\frac{4}{9}\frac{5}{9}$% better. **133.** $2\frac{5}{1}\frac{5}{7}$ days; A, 7 days ; B, 10 days ; C, 15 days. **965.** **1.** $109.25, gain. **2.** $199.64. **3.** $12,001.32. **4.** 1,000 liters. **5.** 1,000,000,000,000,000 grams. **6.** 1,000,000,000 cu. mm. **7.** 1,000,000 grams. **8.** 9.88 A. **9.** $1.68. **10.** 305 francs. **11.** $16,375.78. **12.** 800 half dollars. **13.** 10 kilos. **14.** $385\frac{4}{5}$ lb. Troy. **15.** 35.67 steres. **16.** 5,648.98 kilos. **17.** $111\frac{7}{2}\frac{4}{5}$ miles. **18.** 2,377.575 gallons. **972.** **1.** 43. **2.** 68. **3.** $103. **4.** 505. **5.** 3. **6.** 5. **7.** $14\frac{3}{4}$. **8.** 21. **9.** 18. **10.** 2,485. **11.** $11\frac{7}{8}$. **12.** 37. **13.** 400. **14.** (1) 3,900 ; (2) 1,430 ; (3) $57\frac{1}{2}$; (4) $196\frac{3}{4}$. **15.** 852. **16.** (1) 18s. $5\frac{1}{2}$d.; (2) £51 3s. $9\frac{1}{2}$d. **17.** $19\frac{1}{8}\frac{1}{8}$ miles. **976.** **1.** 768. **2.** 2,187. **3.** $\frac{1}{2}\frac{1}{0}\frac{1}{2}\frac{1}{4}$. **5.** $1,276.23. **6.** 2. **7.** 2,044. **8.** $1\frac{1}{2}$. **9.** $\frac{1}{4}$. **10.** 224. **11.** 4. **12.** $\frac{1}{2}$, 1st term ; $174,762\frac{1}{2}$, sum of the terms. **13.** 3,280. **14.** $1\frac{1}{3}$. **15.** 15 fold.